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IDENTIFIERS	*CUBE System; *Mathematics Skills

ABSTRACT

Intended for use as a part of the adult basic education teaching/learning management system called CUBE (Continuity and Unity in Basic Education), these instructional materials consist of a math placement inventory, guide sheets, mastery tests, and eight sections of learning activities covering areas of basic mathematics. Topics addressed in the individual sections are fractions, decimals/percents, measurements, graphs, the rectangle and perimeters, formulas, algebra, and geometry. Designed to be used in individualized instruction programs, each section contains a variety of explanatory material as well as arithmetic, fill-in-the-blank, and word problems. Keys are provided for all exercises and tests. (A project report describing the development of these materials and manuals are available separately--see note.) (MN)

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MATH

GOAL II

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C.U.B.E.

GOAL II

MATH PLACEMENT

NOTEBOOK - YELLOW and/or RED MATH KIT

Note: CUBE has two good approaches for instruction in math. You may choose to use one approach alone, both approaches together (OUR RECOMMENDATION), or even to combine one or both with the approach you are presently using. The taxonomy in the Red Math Kit provides a skills list of all math materials being used in ABE programs throughout Indiana. Chances are, unless your program is new, you'll have at least some of the materials listed.

MATERIALS:
- -Math Placement Inventory and answer key
- -Steck-Vaughn Series, "Working With Numbers", "Steps to Mathematics", "Number Power" books, or see taxonomy, Red Math Kit
- -Math Guidesheets
- -Math supplementary handouts
- -Math Kit

PROCEDURES: -Identify the skills needing remediation by administering:

Math Survey I
for
(Math Kit)

1. For use with all new students, see "Instructions for Use", Pg. 6, Manual (Math Kit). All new students start with Survey I.
2. For use with currently enrolled students, see "Instructions for Use", Pg. 6, Manual (Math Kit). Students may use any or all of the surveys, depending on teacher's discretion.
3. For use with returning students, (same as Step 2).

Math Placement Inventory
for
(Steck Vaughn Series)

1. A student is placed in the "Working With Numbers" series according to the Math Placement Inventory Answer Key. A student missing 50% or more of any one skill area should be placed in the corresponding book listed on the left-hand margin of the answer key. The table of contents will direct you to the needed sections.
2. A math guidesheet is used to monitor progress in each "Working With Numbers" books.
3. Supplementary handouts are given as indicated on guidesheet and are recorded on progress sheets.
4. Mastery tests are given upon completion of each book.
5. Upon completion of a book and successful completion of the Mastery Test, information is recorded on the progress sheet.

* See PAM Notebook for complete list of materials, publishers, and prices in all subject areas.

6. Upon successful completion of a Mastery Test, student is placed in the next book.

7. We have in our program the Steck-Vaughn Algebra book and a geometry handout. However, you may wish to give your student the Pre-GED math test upon completion of the Heptagon book, depending on that student's goals and abilities.

Note: For percentages of skill areas covered in math test of GED, see "New GED Test Information" in the white PAM notebook.

MATH PLACEMENT INVENTORY

1. Jim works 8 hours a day for 5 days a week. If he is paid $4.00 an hour, how much does Jim earn each week? ______________________

2. Bob works from 7:30 a.m. to 4:30 p.m. He takes an hour off for lunch. He earns $24.88 per day. How much is this per hour? _______________

3. Sue has $8.76. She pays $2.35 for vegetables, $1.20 for milk, and $4.10 for meat. How much money does she have left? _______________

ADD

$$\begin{array}{r} 7 \\ +3 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ 8 \\ +7 \\ \hline \end{array} \qquad \begin{array}{r} 33 \\ 25 \\ +41 \\ \hline \end{array} \qquad \begin{array}{r} 408 \\ +281 \\ \hline \end{array}$$

SUBTRACT

$$\begin{array}{r} 7 \\ -5 \\ \hline \end{array} \qquad \begin{array}{r} 12 \\ -7 \\ \hline \end{array} \qquad \begin{array}{r} 87 \\ -34 \\ \hline \end{array} \qquad \begin{array}{r} 895 \\ -332 \\ \hline \end{array}$$

MULTIPLY

$$\begin{array}{r} 7 \\ \times 3 \\ \hline \end{array} \qquad \begin{array}{r} 93 \\ \times 3 \\ \hline \end{array} \qquad \begin{array}{r} 301 \\ \times 5 \\ \hline \end{array}$$

DIVIDE

$$3\overline{)96} \qquad 7\overline{)147} \qquad 4\overline{)284}$$

With a full load, Bob's truck weighs 57,423 pounds. The truck weighs 26,780 pounds. How much does the load weigh? ______________________

Bill earned \$417.00 in May, \$398.00 in June, and \$365.00 in July. How much did he make in these three months? ____________

WATCH THE SIGN

$$\begin{array}{r} \$43.06 \\ 21.34 \\ 17.85 \\ +\ 6.02 \\ \hline \end{array} \qquad \begin{array}{r} \$17.34 \\ -\ 10.38 \\ \hline \end{array} \qquad \begin{array}{r} \$43.89 \\ \times\ \ \ 4 \\ \hline \end{array}$$

$$4\overline{)6276} \qquad 5\overline{)2735} \qquad 6\overline{)\$78.96}$$

Dave paid \$87.32 for 2 new tires. How much did each tire cost? ______________

Tires for Janet's car cost \$26.85 each. How much will she pay for 4 new tires? _________

ADD

$$\begin{array}{r} 1542 \\ 2617 \\ 1928 \\ 2437 \\ +\ 1658 \\ \hline \end{array} \qquad \begin{array}{r} .1604 \\ 124.6 \\ 9.307 \\ +\ 17.08 \\ \hline \end{array} \qquad \begin{array}{r} 15\frac{1}{4} \\ +\ 23\frac{1}{3} \\ \hline \end{array}$$

SUBTRACT

$$\begin{array}{r} 48\frac{1}{4} \\ -\ 15\frac{3}{4} \\ \hline \end{array} \qquad \begin{array}{r} 1462.5 \\ -\ 476.352 \\ \hline \end{array} \qquad \begin{array}{r} 706090 \\ -\ 370804 \\ \hline \end{array}$$

MULTIPLY

$$\begin{array}{r} 738 \\ \times\ 635 \\ \hline \end{array} \qquad \begin{array}{r} 1095 \\ \times\ 38 \\ \hline \end{array}$$

DIVIDE

$$82\overline{)1968} \qquad 234\overline{)28782}$$

Change to mixed number: $\frac{16}{3} =$

Change to improper fractions: $3\frac{1}{3} =$

Change as indicated: $\frac{1}{2} = \frac{\quad}{4}$ $\frac{2}{5} = \frac{\quad}{10}$

Change to decimals: $\frac{1}{10} =$ $\frac{7}{10} =$ $\frac{1}{2} =$ $\frac{3}{4} =$

Change to common fractions and reduce:

.5 = .25 = .75 = .03 =

Michael drove his car 207 miles on 9 gallons of gas. How many miles did he get to the gallon? ________________

From St. Louis to Little Rock is 348.9 miles. From St. Louis to San Antonio is 923.6 miles. How far is it from Little Rock to San Antonio? ________________

SOLVE:

$\frac{3}{5} \times \frac{15}{16} =$ $\frac{3}{4} \times 6\frac{2}{3} =$ $2\frac{4}{5} \times 7\frac{1}{2} =$ $64 \times \frac{3}{4} =$

$5 \div \frac{1}{5} =$ $\frac{1}{4} \div 2 =$ $1\frac{1}{2} \div 1\frac{1}{5} =$ $\frac{2}{3} \div \frac{4}{9} =$

$$\begin{array}{r} 2.604 \\ \times\ .03 \\ \hline \end{array}$$

$$.15\overline{)22.5}$$

$$\begin{array}{r} 5\text{ ft. } 9\text{ in.} \\ +\ 2\text{ ft. } 6\text{ in.} \\ \hline \end{array}$$

$$\begin{array}{r} 21\text{ yr. } 4\text{ mo.} \\ -\ 12\text{ yr. } 6\text{ mo.} \\ \hline \end{array}$$

$$\begin{array}{r} 5\text{ lb. } 12\text{ oz.} \\ \times\qquad 4 \\ \hline \end{array}$$

$$5\overline{)7\text{ min. } 5\text{ sec.}}$$

Fill in the missing numbers:

A square foot = 12 x _______ = __________________ square inches.

A square yard = 3 x ________ = __________________ square feet.

A cubic foot = 12 x _______ x 12 = _______________ cubic inches.

A cubic yard = 3 x _______ x _______ = ____________ cubic feet.

A living room measures 18 ft. by 20 ft. How many square yards of carpet would be needed to cover this floor?

A cubic foot of water equals 7.5 gallons. How many gallons are there in a full tank 12 feet long, 6 feet wide, and 5 feet deep?

How much fencing would be needed to enclose a lot measuring 50 ft. by 150 ft.?

Change to percents

.20 =

Change to fractions

25% =

Change to percents

$\frac{2}{3}$ =

$\frac{3}{10}$ =

Change to decimals

50% =

Change to percents

$\frac{1}{3}$ =

$\frac{3}{8}$ =

Change to decimals

$1\frac{1}{2}\%$ =

$\frac{1}{4}\%$ =

Change to decimals:

$\frac{1}{10}\%$ =

$\frac{1}{2}\%$ =

Change to percents:

.015 =

.075 =

Change to mixed decimals:

210% =

125% =

Change to percents:

2.25 =

1.10 =

15% of 60 =

Using the circumference formula $C = \pi D$, find C when D is 28, ($\pi = \frac{22}{7}$).

Using the formula $V = LWH$, find V when L is 30, W is 20, and H is 10.

Using the circle area formula $A = \pi R^2$, find A when R is 14.

Using the triangle formula $A = \frac{1}{2}AB$, find A when A is 24 and B is 10.

Using the area formula $A = LW$, find A if L is 40 and W is 20.

A circle whose diameter is 28 has what radius?

In the perimeter formula $P = 2L + 2W$, find P when L is 300 and W is 200.

Using the cylinder volume formula $V = \pi R^2 H$, find V when R is 7 and H is 10.

Wilbur borrowed $500.00 at 6% for 90 days. How much interest did he owe? ________

Agatha bought a hat for $7.50, which was at 25% discount. What was the original price of the hat? ________

10 is what percent of 40? ________

Kathryn bought a $40.00 suit at a discount of 25%. How much did she pay for the suit?

____

Dan's bank pays 5½% interest. How much interest will he have in his savings account of $360.00 at the end of a year? ________

How much interest will be due on a note made out for $480.00 at 8% for 9 months? ________

Mr. Earnest shipped 450 crates of strawberries at $3.50 per crate. The market agent, Mr. James earned a commission of 8%. How much did Mr. Earnest receive?

12½% of 400 =

½% of 200 =

Student Guide Sheet
(Steps to Mathematics Bk. #1)

Name ____________________ Date given ____________

Mastery Test ________________

Date M.T. Completed ______________ **Date Completed**

Step 1: Cyclo-teacher M4 (addition) ____________

Step 2: Cyclo-teacher M5 (additon) ____________

Step 3: Work pages 1-8 ____________

Step 4: Cyclo-teacher M13 (subtraction) ____________

Step 5: Cyclo-teacher M14 (subtraction) ____________

Step 6: Work pages 9-14 ____________

Step 7: Cyclo-teacher M6 (addition two-digit) ____________

Step 8: Cyclo-teacher M7 (addition) ____________

Step 9: Cyclo-teacher M8 (addition) ____________

Step 10: Cyclo-teacher M9 (addition) ____________

Step 11: Cyclo-teacher M10 (addition) ____________

Step 12: Cyclo-teacher M11 (addition) ____________

(And-or)

Step 13: Math Power Pac A-Lesson #2 (Addition Facts through 20) ____________

Step 14: Work pages 15-24 ____________

Step 15: Cyclo-teacher M15 (two-digit subtraction) ____________

Step 16: Cyclo-teacher M16 (Subtraction) ____________

Step 17: Cyclo-teacher M17 (Subtraction) ____________

Step 18: Cyclo-teacher M18 ____________

Step 19: Cyclo-teacher M19 (Subtraction) ____________

Date Completed

Step 20: Cyclo-teacher M20
(Subtraction) ____________

(And-or)
Step 21: Math Power Pac A-Lesson 3 ____________
(Subtraction Facts through 20)

Step 22: Work pages 25-41 ____________

Step 23: Cyclo-teacher M22
(Multiplication Table 1's) ____________

Step 24: Cyclo-teacher M23
(Multiplication tables 2's) ____________

Step 25: Cyclo-teacher M24
(Multiplication table 3's) ____________

Step 26: Cyclo-teacher M25
(Multiplication table 4's) ____________

Step 27: Cyclo-teacher M26
(multiplication table 5's) ____________

And-or
Step 28: Math Power Pac A-Lesson 4 ____________
(Multiplication facts through 5's)

Step 29: Work pages 42-43 ____________

Step 30: Cyclo-teacher M32
(Division by 1-2) ____________

Step 31: Cyclo-teacher M33
(Div. by 3) ____________

Step 32: Cyclo-teacher M34
(Div. by 4) ____________

Step 33: Cyclo-teacher M35 Side 1
(Div. by 5) ____________

Step 34: Work pages 44-48 ____________

Step 35: Cyclo-teacher M27
(Multiplication by 6) ____________

Step 36: Cyclo-teacher M28
(Multiplication by 7) ____________

Step 37: Cyclo-teacher M29
(Multiplication by 8) ____________

Step 38: Cyclo-teacher M30
(Multiplication by 9) ____________

And-or
Step 39: Math Power pac A-Lesson 5 ____________
(Multiplication Facts through 9's)

Date Completed ____________________

Step 40: Work pages 49-51 ____________________

Step 41: Cyclo-teacher M35 (Side 2)
(Division by 6) ____________________

Step 42: Cyclo-teacher M36
(Div. by 6) ____________________

Step 43: Cyclo-teacher M37
(Div. by 7) ____________________

Step 44: Cyclo-teacher M38
(Div. by 8) ____________________

Step 45: Cyclo-teacher M39
(Div. by 9) ____________________

And-or

Step 46: Math Powerpac A-Lesson 6
(6-9 as Divisors) ____________________

Step 47: Finish Working Book 52-60 ____________________

Step 48: Mastery Test ____________________

THE END

Student Guide Sheet
(Steps to Mathematics #2)

Name ____________________ Date Given ____________________

Mastery Test ____________________

Date M.T. Completed ____________________

Date Completed

Step 1: Work pages 1-10 ____________________

Step 2: Cyclo-teacher M12
(Addition: Renaming) ____________________

Step 3: Math Powerpac B-Lesson 2 ____________________

Step 4: Work pages 11-22 ____________________

Step 5: Cyclo-teacher M21
(Subtraction: Renaming) ____________________

And or
Step 6: Math Power Pac B-Lesson 3
(Subtraction with Renaming) ____________________

Step 7: Work pages 23-32 ____________________

Step 8: Cyclo-teacher M31
(Multiplication Renaming) ____________________

Step 9: Work pages 33-39 ____________________

Step 10: Cyclo-teacher M40
(Division: remainders) ____________________

Step 11: Cyclo-teacher M41
(Division: Two-place
Dividends) ____________________

Step 12: Cyclo-teacher M42
(Division: Two-place
Dividends) ____________________

Step 13: Cyclo-teacher M43
(Division: Three-place
Dividends)

Step 14: Cyclo-teacher M44
(Division: Using zero) ____________________

Step 15: Math Power Pac B-Lesson #5
(Uneven Division-One-digit) ____________________

Step 16: Cyclo-teacher M58
(Linear Measure) ____________________

Step 17: Work pages 39-52 ____________________

		Date Completed
Step 18:	Cyclo-teacher M55 (Telling Time)	
Step 19:	Cyclo-teacher M56 (Telling time)	
Step 20:	Work pages 53-60	
Step 21:	Math Power Pac F (Lesson 1) (Writing Math Sentences	
Step 22:	Mastery Test (pgs. 1-60)	

THE END

STUDENT GUIDE SHEET
(Pentagon)

Name: ____________________ Date Given: ____________________

Page Started: ____________________

	Date Completed
Step 1: Roman numerals p. 3 (Supplement: Cyclo M46,M47 - Roman numerals)	____________________
Step 2: Review of basic addition and subtraction	____________________
Step 3: Multiplication pp. 14-16 (Supplement: Cyclo M31– Multiplication renaming)	____________________
Step 4: Measurement p. 17 (Supplement: Cyclo M57 - Liquid Measure)	____________________
Step 5: Review pp. 18-19	____________________
Step 6: Working with Four Digit or Larger Numbers pp 20-25 (Supplement: Cyclo M12 - Addition renaming)	
Step 7: Subtracting Four Digit or Larger Numbers pp. 26-29 (Supplement: Cyclo M21 - Subtracting renaming)	____________________
Step 8: Multiplying Four Digit or Larger Numbers pp. 30-36	____________________
Step 9: Review pp. 37-42	____________________
Step 10: One Figure Divisors pp. 43-46 (Supplement: Cyclo M44 - Division using zero)	____________________
Step 11: Two Figure Divisors pp. 47-55	____________________
Step 12: Review pp. 56-62	____________________
Step 13: More Division Work pp. 63-70	____________________
Step 14: Dividing Dollars and Cents p. 71	____________________
Step 15: More Division Work pp. 72-75	____________________
Step 16: Three Figure Divisors pp. 76-77	____________________
Step 17: Review pp. 78-81	____________________

Step 18: Meaning of Fractions pp. 82-84 ______________________
(Supplement: Cyclo M67 - Understanding Fractions,
M68 - Fraction Terms, M72 - Fractions: Part of a Group)

Step 19: Adding Fractions and Imperfect Fractions ______________________
pp. 85-87
(Supplement: Cyclo M71 - Kinds of Fractions, M73 - Adding and Subtracting Like
Fractions, M74 - Improper Fractions)

Step 20: Equal Fractions p. 88 ______________________
(Supplement: Cyclo M70 - Equal Fractions

Step 21: Reducing Fractions p. 89 ______________________
(Supplement: Cyclo M75 - Fractions to Lowest Terms)

Step 22: Adding and Subtracting Fractions pp. 90-97 ______________________

Step 23: Subtracting Fractions with Unlike
Denominators pp. 98-100 ______________________
(Supplement: Cyclo M77-Fractions: Common Denominator, handout: Fractions -
Subtracting, handout: Fractions-Addition)

Step 24: Review pp. 101-103 ______________________
(Supplement: handout: Fractions)

Step 25: Meaning of Decimals pp. 104-105 ______________________
(Introduction to Decimals I & II Handouts)

Step 26: Adding Decimals pp. 106-107 ______________________

Step 27: Decimals and Fractions, Comparing ______________________
Decimals pp. 108-109

Step 28: Subtracting Decimals pp. 110-112 ______________________

Step 29: Changing Decimals to Fractions pp. 113-115 ______________________
(Supplement: Lesson 1 & 2 of handout: Changing
Decimals to Fractions)

Step Review pp. 116-117 ______________________

Step 31: Review of Pentagon book ______________________

Step 32: Mastery Test ______________________

THE END

STUDENT GUIDE SHEET
(HEXAGON)

Name: ____________________ Date Given: ____________________

Page Started: ______________

Date Completed:

Step 1: Thinking about Math pp. 3-4

Step 2: Process and Math pp. 5-9

Step 3: Reviewing Addition pp. 10-12

Step 4: Reviewing Subtraction pp. 13-15

Step 5: Reviewing Multiplication pp. 16-18

Step 6: Reviewing Division pp. 19-22

Step 7: Review pp. 23-24

Step 8: Meaning of Fractions p. 25

Step 9: Equal Fractions p. 26
(Supplement: Cyclo M70 -Equal Fractions)

Step 10: Review Adding & Subtracting like Denominators
p. 27
(Supplement: Cyclo M73 - Adding and
Subtracting like Fractions

Step 11: Fractions & Unlike Denominators
(Supplement: Cyclo M77 - Fractions:
Common Denominators)

Step 12: Mixed Fractions pp. 32-35

Step 13: Subtracting Fractions pp. 36-41
(Supplement: handout on Fractions -
Subtraction)

Step 14: More Work with Fractions pp. 42-43

Step 15: Review pp. 44-45

Step 16: Multiplying Fractions pp. 46-54
(DO NOT DO PAGE 51)
(Supplement: Cyclo M76 - Multiplying
Fractions; handout on Multiplying
Fractions)

Step 17: Review p. 55

Step 18: Division of Fractions pp. 56-61 (Supplement: Cyclo M78 - Dividing Fractions; handout on Division of Fractions)

Step 19: Weights and Measures p. 62

Step 20: Review pp. 63-65

Step 21: Reviewing & Understanding Decimals pp.66-70 (Supplement: Cyclo M86 - Changing Decimals to Fractions; M89 - Changing Fractions to Decimals)

Step 22: Multiplying Decimals pp. 71-76 (Supplement: Cyclo M85 - Multiplying Decimals; handout on Multiplying Decimals)

Step 23: Review pp. 77-79

Step 24: Dividing Decimals pp. 80-84 (Supplement: Cyclo M87 - Dividing Decimals; Cyclo M88 - Dividing by Decimals; handout on Dividing Decimals)

Step 25: Review pp. 85-87

Step 26: Checks & Accounts pp. 88-91

Step 27: Review pp. 92-93

Step 28: Adding, Subtracting, Multiplying Dividing Weights & Measures pp. 94-100 (Supplement: Handout - Introduction to Measurements

Step 29: Cyclo M61 - Perimeters of Rectangles & Squares

Step 30: Perimeters pp. 101-106

Step 31: Cyclo M62 - Areas of Rectangles & Squares

Step 32: Area pp. 107-108

Step 33: Volume pp. 109-111

Step 34: Review p. 112

Step 35: Graphs pp. 113-117 (Handout - Use of Graphs)

Step 36: Review of Hexagon Book pp. 118-127

Step 37: Mastery Test

STUDENT GUIDE SHEET
(Heptagon)

NAME: ______________________ DATE GIVEN: ______________________

PAGE STARTED: ______________________

Date Completed

NOTE: **Perimeters,** Area, Volume and Capacity Review Handouts may be given at this time ______________________

Step 1: Math vocabulary and symbols p. 3 ______________________

Step 2: Process in Math pp. 4-5 ______________________

Step 3: Reviewing Math Basics pp. 6-17 ______________________

Step 4: Review of Formula in Measurement pp. 18-21 (Supplement: handout on Perimeters, Areas, etc.) ______________________

Step 5: Review pp. 22-25 ______________________

Step 6: Lesson #3-9 (Percent Handout) ______________________

Step 7: Meaning of Percent p. 26 (Supplement: Math Power Pac E - tape 3 - Meaning of Percent) ______________________

Step 8: Problems with Percent p. 27 (Supplement: Math Power Pac E - tape 4 - Percent of a Number) ______________________

Step 9: Decimals and Fractions to Percents p. 28-31; (Supplement: Math Power Pac E - tape 5 - Equations to Find Percents) ______________________

Step 10: Working with Percents p. 32-37 (Supplement: Math Power Pac E - tape 6 - More Problems with Percents) ______________________

Step 11: Percents pp. 38-40 ______________________

Step 12: Review pp. 41-43 ______________________

Step 13: Commissions pp. 44-48 ______________________

Step 14: Increasing and Decreasing Value with Percent pp. 49-50 ______________________

Step 15: Meaning of Interest pp. 51-58 ______________________

Step 16: Review pp. 59-61 ______________________

Step 17: Review of Rectangle pp. 62-63 (Supplement: Cyclo M61 - Perimeters of Rectangles and Squares; M62- Areas of Rectangles and Squares) ______________________

Step 18: Review of Volume pp. 64-65 ______________________

Step 19: Shortcuts in Formulas pp. 66-67 ______________________

Step 20: Triangle pp. 68-70 ______________________
(Supplement: Cyclo M63 Perimeter of Triangle) ______________________

Step 21: Circles pp. 71-75 ______________________
(Supplement: Cyclo M65 - C = d;
Cyclo M66 - Area of Circles) ______________________

Step 22: Review pp. 76-79 (Formula Worksheet) ______________________

Step 23: Banking Processes pp. 80-83 ______________________

Step 24: Difference between Dates (loans) pp.84-86 ______________________

Step 25: More on Percents pp. 87-93 ______________________

Step 26: Review pp. 94-96 ______________________

Step 27: Buying and Budgeting pp. 97-106 ______________________

Step 28: Using Utilities pp. 107-110 ______________________

Step 29: Review of Book pp. 111-124 ______________________

Step 30: Mastery Test ______________________

THE END

Student's Guide
(Octagon)

Name________________________ Date Given_______________

Mastery Test ____________________

Date M.T. Completed

Date Completed

Step 1: Work pages 1-85 ____________________

Step 2: Cyclo-teacher M59
(D=RT) ____________________

Step 3: Work page 86 ____________________

Step 4: Math Power pac-Lesson 1
(Prime and Composite numbers) ____________________

Step 5: Math Powerpac Lesson 2 ____________________

Step 6: Mastery Test #1 and #2 ____________________

Mastery #2 only work
Problems 1-12
Problem 26
Problem 30

* While student may do entire Octagon book, material after p. 86 is in the algebra book.

STUDENT GUIDE SHEET
(ALGEBRA)

Steps		Date Finished
1:	Work pages 1-21	
2:	Cyclo-teacher M93 & 93a (Literal Numbers & Symbols)	
3:	Cyclo-teacher M93 & 94a (Kinds of Expressions)	
4:	Work pp. 22	
5:	Cyclo-teacher M-95 & 95a (Signed Numbers)	
6:	Explanation of pg. 23	
7:	Work pg. 23	
8:	Cyclo-teacher M-97 & 97a (Adding Signed Nos.)	
9:	Cyclo-teacher M-98 & 98a (Adding Monomials)	
10:	Cyclo-teacher M-99 & 99a (Adding Monomials)	
11:	Explanation of pg. 24	
12:	Work pg. 24	
13:	Handout: Adding & Subtracting Signed Numbers	
14:	Cyclo-teacher M-101 & (Subtracting Signed Numbers)	
15:	Cyclo-teacher M-102 & 102a (Subtracting Monomials)	
16:	Cyclo-teacher M-103 & 103a (Subtracting Monomials)	
17:	Explanation of pg. 25-27 (Subtracting, Multiplying, Dividing of Monomials)	
18:	Work pg. 25-27	
19:	Explanation of pgs. 28-32 (Simple Equations)	
20:	Work pgs. 28-32	
21:	Work pgs. 1 & 2 in Algebra Handout - Equations	
22:	Explanation of pgs. 33-36 (Changing equations)	
23:	Work pgs. 33-36	

	Date Finished
24. Skills Test #1 (pgs. 23-36)	
25. Explanation of pgs. 37-39 (Clearing Fractions)	
26. Work pgs. 37-39	
27. Handout, Multiplying and Dividing Signed Numbers	
28. Cyclo-teacher M-104 & 10a (Multiplying Signed Numbers)	
29. Cyclo-teacher M-105 & 105a (Multiplying Monomials)	
30. Cyclo-teacher M-110 & 110a (Simplifying Expressions)	
31. Explanation of pgs. 40-41 (Parenthesis)	
32. Work pgs. 40-41	
33. Work pg. 3 in Algebra Handout	
34. Explanation of pgs. 42-43 (Problem Solving)	
35. Work pgs. 42-43	
36. Work pgs. 5-7 in Algebra Handout	
37. Work pgs. 44-46 (Review)	
38. Skills Test #2 pgs. 1-46	
39. Cyclo-teacher M-106 & 106a (Multiplying Monomials)	
40. Explanation of pgs. 47-49 (Multiplication with Exponents)	
41. Explanation of pgs. 50-51 (Division with Exponents)	
42. Work pgs. 47-51	
43. Skills Test #3 - pgs. 47-51	
44. Explanation of pgs. 52-54 (Equations with Exponents and Cubes)	
45. Work pgs. 52-54	
46. Skills Test #4 pgs. 52-54	
47. Explanation of pgs, 55-57 (Square Root)	
48. Work pgs. 55-58	
49. Cyclo-teacher M-107 & 107a (Dividing Signed Numbers)	

Date Finished

50. Cyclo-teacher M-108 & 108a (Dividing Monomials) ____________

51. Cyclo-teacher M-109 & 109a (Dividing Monomials) ____________

52. Explanation of pgs. 59-63 (Multiplying & Dividing of Signed Numbers ____________

53. Work pgs. 59-66 ____________

54. Skills Test #5 ____________

55. Cyclos M-111 & M-112 (Adding Polynomials) ____________

56. Cyclos M-113 & M-114 (Subtracting Polynomials) ____________

57. Explanation of pgs. 67-71 (Adding & Subtracting of Bynomials) ____________

58. Work pgs. 67-71 ____________

59. Skills Test #6 (pgs. 1-71) ____________

60. Explanation of pgs. 72-74 (Simultaneous Equations) ____________

61. Work pgs. 72-75 ____________

62. Explanation of 76-85 (Simultaneous Equations Changing Coefficients) ____________

63. Work pgs. 76-85 ____________

64. Work pgs. 88-90 (Review) ____________

65. Skills Test #7 (pgs. 1-85, 88-90) ____________

66. Explanation of pgs.86-87 (Graphing a Linear Equation) ____________

67. Work pgs. 86-87 ____________

68. Work Rectangular Coordinates Handout ____________

69. Explanation of pgs. 91-93 (Fractional Simultaneous Equations) ____________

70. Work pgs. 91-94 ____________

71. Explanation of pg. 95 (Fractional Equations) (Work pg. 95) ____________

72. Skills Test #8 pgs. 91-95 ____________

Date Finished

73. Explanation of pgs. 96-97 (Multiplying & Division of Fractions)
74. Work pgs. 96-98
75. Cyclo M-203 (Ratio)
76. Explanation pgs. 99-100 (Ratio)
77. Work pgs. 99-100
78. Work pgs. 8-9 in Algebra Handout
79. Work Cyclo 204 (Proportion)
80. Explanation pgs. 101-104 (Proportion)
81. Work pgs. 101-108 (Work & Review)
82. Work pgs. 10-21 Algebra in Handout
83. Skills Test #9 (pgs. 1-108)
84. Explanation pgs. 109-110 (Multiplying & Dividing Polynomials by Monomials)
85. Work pgs. 109-110
86. Explanation pgs. 111-112 (Reducing Fractions & Common Factors)
87. Work pgs. 111-112
88. Explanation pgs. 113-114 (Multiplying Polynomials)
89. Work pgs. 113-114
90. Explanation of pgs. 115-116 (Dividing Polynomials)
91. Work pgs. 115-117
92. Explanation of pgs. 118-120 (Squares, Square Roots & Factoring)
93. Work pgs. 118-120
94. Skills Test #10 (pgs. 1-120)
95. Explanation of pg. 121 (Quadratic Equations 1)
96. Work pg. 121

Date Finished

97. Explanation of pgs. 122-123 (Square of Differences Factoring) ________________

98. Work pgs. 122-123 ________________

99. Explantion of pg. 124 (Quadratic Equations 11) ________________

100. Work pg. 124 ________________

101. Explanation of pgs. 125-126 (Product of Sum & Difference Factoring) ________________

102. Work pgs. 125-126 ________________

103. Explanation of pg. 127 (Quadratic Equation IV) ________________

104. Work pg. 127 ________________

105. Explanation of pgs. 128-130 (Two Binomials with Common Term Factoring) ________________

106. Work pgs. 131-135 ________________

107. Explanation of pgs. 136-138 (Completing Square) ________________

108. Work pgs. 136-138 ________________

109. Skills Test #11 (pgs. 1-138) ________________

110. Explanation pgs. 139-141 (Quadratic Formula) ________________

111. Work pgs. 139-141 ________________

112. Work pgs. 142-145 (Review) ________________

113. Explanation pgs. 146-147 (Inequalities) ________________

114. Work pgs. 146-147 ________________

115. Skills Test #12 (pgs. 1-147) ________________

116. Work pgs. 148-160 (Review) ________________

117. Mastery Test #1 & #2 (pgs. 1-160) ________________

Student Guide Sheet

Refresher

Since Refresher is a combination of all previous Working With Numbers Math books and since few students would do the entire book, a guide sheet covering the contents was not made. Refresher is primarily utilized by students who have good basic Math concepts, but need a quick review (as determined by the Math Placement Inventory/Math Kit).

MASTERY TEST #1

Steps To Mathematics #1

Add:

1. 4
2

2. 8
0

3. 3
6
7

4. 3
8
4

5. 4
3
2
4

6. 5
1
3
9

7. 2
10

8. 53
56

9. 841
634

10. 801
12
163
322

11. 502
612
34
351

12. 253
432
214

13. 201
128
660

14. 298
326
438
525

15. 124 + 421 + 332 + 29 =

Subtract:

1. 8
-2

2. 9
-5

3. 15
-8

4. 57
-32

5. 65
-32

6. 499
-263

7. 568
-224

8. 735
-621

9. 479
-135

10. 331
-211

Multiply:

1. $\begin{array}{r} 3 \\ \underline{\times 1} \end{array}$ 2. $\begin{array}{r} 80 \\ \underline{\times 6} \end{array}$ 3. $\begin{array}{r} 11 \\ \underline{\times 9} \end{array}$ 4. $\begin{array}{r} 71 \\ \underline{\times 7} \end{array}$

5. $\begin{array}{r} 300 \\ \underline{\times 4} \end{array}$ 6. $\begin{array}{r} 901 \\ \underline{\times 9} \end{array}$ 7. $\begin{array}{r} 911 \\ \underline{\times 8} \end{array}$ 8. $\begin{array}{r} 321 \\ \underline{\times 4} \end{array}$

9. $\begin{array}{r} 800 \\ \underline{\times 9} \end{array}$ 10. $\begin{array}{r} 201 \\ \underline{\times 6} \end{array}$

Divide:

1. $3\overline{)\,15}$ 2. $4\overline{)\,28}$ 3. $4\overline{)\,84}$ 4. $4\overline{)\,48}$

5. $9\overline{)\,819}$ 6. $4\overline{)\,168}$ 7. $7\overline{)\,637}$ 8. $7\overline{)\,147}$

9. $3\overline{)\,129}$ 10. $4\overline{)\,368}$

1. Joe drives 300 miles in 5 hours. How many miles did he drive in 1 hour?

2. Bob bought 5 shirts. Each shirt cost $2.21. How much does he pay for all 5 shirts?

3. Mary has $25.00. She bought groceries for $15.00. How much money does she have left?

4. How much money would you have if you had a quarter, one dime, 2 nickels, and 4 pennies?

5. Tom bought a hammer for $3, a saw for $5, and a drill for $21. How much was his bill?

THE END

Steps to Mathematics #2

Add:

1\.
31
3
71
54

2\.
$17.95
21.03
60.00

3\.
131
312
342
216

4\.
35.04
12.38
.26
32.14

5\.
$52.43
10.76
3.07
22.32

Subtract:

1\.
$47.32
-25.89

2\.
$17.05
- 3.98

3\.
$46.60
-38.81

4\.
$27.55
-17.36

5\.
$6.82
-4.75

Multiply:

1\.
$43.65
8

2\.
$43.89
4

3\.
$10.03
7

4\.
$12.98
4

5\.
$23.14
4

Divide:

1. 8) 1872

2. 6) 7896

3. 7) 749

4. 6) $8.46

5. 5) 98.75

1. **1 cup = __________ ounces**

2. **2 cups = __________ pints**

3. **1 pint = __________ ounces**

4. **1 quart = __________ pints**

5. **1 quart = __________ ounces**

6. **1 gallon = __________ quarts**

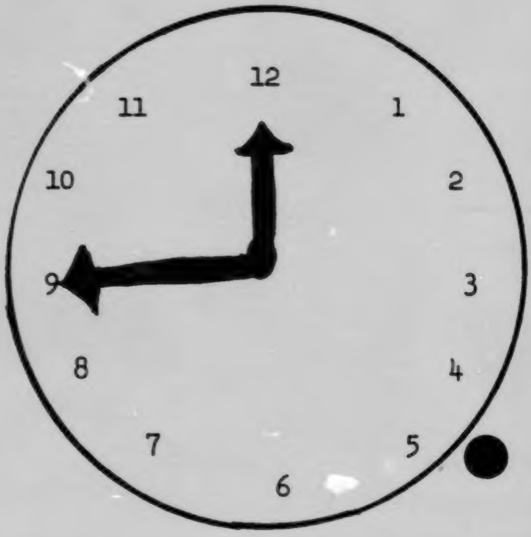
7. Beans

Large Can	Small Can
24 ounces $1.50 each	6 ounces $.45 each

Vi bought a small can of beans. How many ounces did she buy? ____________________

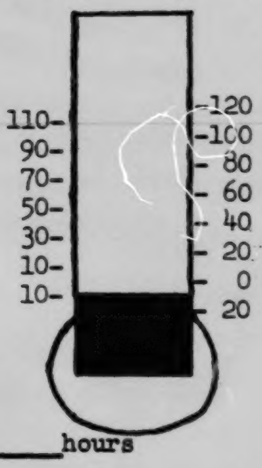
How much money could she have saved by buying one large can? ____________________

8.


12
1
2
3
4
5
6
7
8
9
10
11

What time does this clock show? ____________

9.


110-
90-
70-
50-
30-
10-
10-
-120
-100
- 80
- 60
- 40
- 20
- 0
- 20

What temperature is shown on this thermometer in degrees Fahrenheit? ____________________

10. 1 day = __________hours

1 year = _________days

1 year = _________weeks

THE END

Mastery tests for Pentagon, Hexagon, Heptagon, Octagon, Algebra and Refresher are available through the publisher:

Steck-Vaughn Co.
Box 2028
Austin, Texas 78768

THE END

Algebra Test #1
pgs. 23-36

Add:

1. $\begin{array}{r} 2A \\ 7A \\ 5A \\ \underline{12A} \end{array}$

2. $\begin{array}{r} 3Ab \\ 4Ab \\ 32Ab \\ \underline{9Ab} \end{array}$

3. $\begin{array}{r} 16x^2y \\ 9x^2y \\ 14x^2y \\ \underline{23x^2y} \end{array}$

4. $\begin{array}{r} -15a \\ +12a \\ -3a \\ \underline{+9a} \end{array}$

5. $\begin{array}{r} -15a \\ -12a \\ -3a \\ \underline{-9a} \end{array}$

6. $\begin{array}{r} 15abc \\ -6abc \\ -9abc \\ \underline{3abc} \end{array}$

****

Subtract:

1. $\begin{array}{r} 15x \\ \underline{7x} \end{array}$

2. $\begin{array}{r} 165xy \\ \underline{39xy} \end{array}$

3. $\begin{array}{r} 45ab \\ \underline{-19ab} \end{array}$

4. $\begin{array}{r} -45ab \\ \underline{-19ab} \end{array}$

5. $\begin{array}{r} -45ab \\ \underline{19ab} \end{array}$

6. $\begin{array}{r} 126x^2y \\ \underline{-120x^2y} \end{array}$

****

Multiply:

1. $\begin{array}{r} 15a \\ \underline{3} \end{array}$

2. $\begin{array}{r} 64ab \\ \underline{3} \end{array}$

3. $\begin{array}{r} 19x^2 \\ \underline{4} \end{array}$

4. $\begin{array}{r} 32xy \\ \underline{5} \end{array}$

5. $\begin{array}{r} 18m^2n \\ \underline{10} \end{array}$

6. $\begin{array}{r} 4.5a^2 \\ \underline{2} \end{array}$

Divide:

1. $\frac{15x}{5}$ = ______
2. $\frac{63xy}{7}$ = ______
3. $\frac{92a^2}{4}$ = ______
4. $\frac{25a^2b^2}{25}$ = ______
5. $\frac{225xy^2}{25}$ = ______
6. $\frac{1.6x}{4}$ = ______

****

Solve for the unknown:

1. $M + 13 = 32$
2. $x + 17 = 19$
3. $x + 3x = 18 + 6$
4. $8x - 6x = 3.8$
5. $15a - 3a = 60$
6. $10x = 115$
7. $15a = 45$
8. $\frac{x}{2} = 115$
9. $\frac{2x}{3} = 4$
10. $\frac{3y}{8} = 6$
11. $3x - 5 = x + 7$
12. $15x - 5 = 10x + 15$

1. Together, Earl and Roy have $4.80. Earl has three times as much as Roy. How much does each have?

2. George and Harry inherited an estate of $1500. George is to receive three times as much as Harry. How much does each receive?

3. Dorothy and Jane have $4.80. Dorothy has only one-fifth as much as Jane has. How much does each have?

4. Mary is twice as old as her sister. The sum of their ages is 63 years. What is the age of each?

THE END

Algebra Test #2
pgs. 1-46

1. $A = lw$
$l = 30$
$w = 20$
$A =$ ________

2. $V = lwh$
$l = 15$
$w = 10$
$h = 5$
$V =$ ________

3. $A = \pi r^2$
$r = 7$
$A =$ ________

4. $V = \pi r^2 h$
$D = 14$
$h = 10$
$V =$ ________

5. $I = PRT$
$P = \$1000$
$R = 6\%$
$T = 4$ mos.
$I =$ ________

6. $I = PRT$
$P = \$1200$
$R = 6\%$
$T = 18$ mos.
$I =$ ________

7. From the formula $A = Lw$, write the formula for finding L.

8. From the formula $I = PRT$, write a formula for finding R.

9. From the formula $V = Lwh$, write a formula for finding h.

10. From the formula $C = \pi d$, write a formula for finding d.

Solve each of the following equations:

11. $5x = x + 20$

12. $3x + 5 = 23$

13. $4x - 5 = 3x + 1$

14. $2(3x + 4) = 3x + 23$

15. $3(2x - 4) = 2(2x + 2)$

16. $\frac{2x}{4} = 16$

17. $\frac{x}{5} = 10$

18. $\frac{3x}{3} = 7$

19. $12x + 5x = 51$

20. $14b - 6b = 48$

21. Helen's father is three times as old as she. The sum of their ages is 56 years. How old is each?

22. Henry is twice as old as his sister. The sum of their ages is 21 years. How old is each?

23. The perimeter of a rectangle is 90. The length is twice as great as the width. Find the length and width.

24. We have a lot whose perimeter is 360 ft. The length is 3.5 times the width. Find the length and width of the lot.

25. I am thinking of a number. Five times that number equals 240. What is the number?

Solve these equations:

1. $\left(\frac{x}{2} + 3\right) = \left(\frac{x}{8} + 7\right)$

2. $\left(\frac{x}{2} - 5\right) = \left(\frac{x}{5} - 11\right)$

3. $\left(x + 5\right) = \left(\frac{2x}{7} + 10\right)$

4. $\frac{x}{3} = \frac{2}{5}$

5. $\frac{2x}{7} = \frac{6}{5}$

6. $\frac{3x}{8} = \frac{6}{4}$

THE END

Algebra Test #3
pgs. 47-51

1. When multiplying expressions which contain exponents, you (add, subtract) the exponents.

2. When dividing expressions which contain exponents, you (add, subtract) the exponents.

Multiply:

3. b^2de
$\underline{\quad bd^2 \quad}$

4. $2xy^3$
$\underline{\quad 6yz^4 \quad}$

5. $14b^3$
$\underline{\quad 4cd \quad}$

6. $19R^3N^3$
$\underline{\quad 4MRN \quad}$

7. $12d^3$
$\underline{\quad 6ade \quad}$

8. $10bc^3d$
$\underline{\quad 5acd \quad}$

9. $6ab^2$
$\underline{\quad 9bc^2 \quad}$

10. $5x^2y^2$
$\underline{\quad 4x^3y^3 \quad}$

Divide:

1. $\frac{d^5e^6f}{d^2ef} =$

2. $\frac{a^5b^2c}{a^3bc} =$

3. $\frac{cde^6f}{de} =$

4. $\frac{8y^2}{16y} =$

5. $\frac{45x^2y^2}{5xy} =$

6. $\frac{72x^2y}{8xy} =$

Divide: (continued)

7. $\frac{14y^4z^3}{28y^3z^4}$ =

8. $\frac{25c^2d^6}{75c^6d^2}$ =

9. $\frac{18x^2}{2x^3}$ =

10. $\frac{16a^2b^3}{32a^3b^2}$ =

THE END

Algebra Test #4
pgs. 51-63

Solve for the unknown:

1. $6x = 36$
2. $15a = 45$
3. $12x + 5x = 51$
4. $14x - 6x = 48$
5. $25m - 14m = 77$
6. $\frac{2n}{3} = 14$
7. $\frac{3z}{10} = 9$
8. $\frac{2R}{3} = 48$
9. $3(x + 2) = 2(x + 5)$
10. $5(3x + 1) = 5(2x + 3)$
11. Mrs. Lopez and her sister inherited $20,000. Mrs. Lopez received 4 times as much as her sister. How much did each receive?
12. Five times a number is 35. What is the number?

Test #4 (continued)

13. The area of a rectangle is 192. The width is one-third of the length. Find the length and width.

14. Find the dimensions of the rectangular prism with a volume of 162, if the base width is one-half the base length and the height is three times the base width.

15. $5x^3 - 17 = 3x^3 + 111$

16. $2x^3 + 5 = x^3 + 221$

17. $4x^3 - 6 = 3x^3 + 119$

18. $7x^2 - 15 = 5x^2 + 17$

Solve:

1. The area of a rectangular field is 1200 square rods. The length of the field is 40 rods. What is the width?

2. The area of a rectangle is 75. The length is three times the width. Find the two dimensions.

3. The width of a rectangle is one-third the length, and the area is 108. Find the two dimensions of the rectangle.

4. Find the three dimensions of the rectangular prism with a volume of 500, the length and height each being twice the width.

5. Find the three dimensions of the cube whose volume is 216.

THE END

Algebra Test #5
pgs. 1-63

Add:

1. 28xy
-14xy
-3xy

2. -14ab
2ab
ab

3. +12m
-3m
-6m

4. -14x
2x
10x

5. -6a
5a

Subtract:

1. 15a
-3a

2. 26bc
13bc

3. 4a
-6a

4. 16xy
8xy

5. $\frac{1}{2}$a
$-\frac{1}{2}$a

Multiply:

1. $-2.5a$
   $\underline{\quad 2a \quad}$

2. $-16b^2$
   $\underline{\quad -5b \quad}$

3. $-3xy^3$
   $\underline{\quad 2x \quad}$

4. $1.5b$
   $\underline{\quad -.7b \quad}$

Divide:

1. $\frac{18b}{2} =$

2. $\frac{15ac}{5bc} =$

3. $\frac{-32p}{8} =$

4. $\frac{-60rs}{-5s} =$

5. $\frac{-16r}{4} =$

Solve:

1. $9x = 54$

2. $4x + 2x = 30$

3. $4x + 2 = 10$

4. $10x = 1,000$

5. $2x + 2 (x+3) = x + 15$

6. $4 + 2x - 1 = 4 (x+1)$

7. If 2 is added to twice a number, the result is equal to the number increased by 12. Find the number.

8. If five times a number is increased by four, then decreased by ten, the result will equal three times the number increased by four. What is the number?

9. If 14 is added to 3 times a number, the result is 53. What is the number?

10. 12 less than twice a number is equal to the number increased by 6. What is the number?

11. The length of a playground is 5 times the width. The perimeter is 300 rods. Find the length and the width of the playground.

12. The width of a rectangle is $\frac{1}{4}$ its length. The perimeter is 50 rods. What are the two dimensions?

13. The basement of a house is a rectangle with a perimeter of 144 ft. The length is $1\frac{1}{2}$ times the width. Find the width.

14. The perimeter of a rectangle is 80 yards. The length equals 3 times the width. What are the dimensions?

15. Jane is $2\frac{1}{2}$ times as old as her sister. The sum of their ages is 63 years. How old is Jane and how old is her sister?

MT-21
THE END

Algebra Test #6
pgs. 1-71

Solve:

1. When 8 is subtracted from 4 times a number, the result is 36.

2. If 9 times a number is decreased by 10, the result is equal to 4 times the number increased by 30.

3. Three less than a number equals 42.

4. If 60 is decreased by 3 times a number, the result is 12 less than the number. Find the number.

5. The length of a rectangle is one less than twice the width. If the perimeter of the rectangle is 34", find the dimensions of the rectangle.

Solve for X:

1. $6x = 78$

2. $x - 3 = 42$

3. $2x + 4 = 10$

4. $5x + 13 = 43$

5. $9x - 10 = 4x + 30$

Add:

1. $$\begin{array}{r} +6b^2 \\ -5b^2 \\ \underline{+8b^2} \end{array}$$

2. $$\begin{array}{r} -10xy \\ +12xy \\ \underline{-8xy} \end{array}$$

3. $$\begin{array}{r} +9mx^3 \\ -11mx^3 \\ +9mx^3 \\ \underline{-6mx^3} \end{array}$$

4. $$\begin{array}{r} -3/4cd^4 \\ \underline{+3/8cd^4} \end{array}$$

5. $$\begin{array}{r} -x^2y \\ \underline{-4x^2y} \end{array}$$

Subtract:

1. $$\begin{array}{r} +8x \\ \underline{-5x} \end{array}$$

2. $$\begin{array}{r} -2m \\ \underline{-6m} \end{array}$$

3. $$\begin{array}{r} 9x^2y \\ \underline{-12x^2y} \end{array}$$

4. $$\begin{array}{r} +5b^4 \\ \underline{-5b^4} \end{array}$$

5. $$\begin{array}{r} +23a \\ \underline{-29a} \end{array}$$

Multiply the following monomials:

1. $a^7 \cdot a^9 =$

2. $b^6 \cdot b^5 =$

3. $e \cdot e^5 \cdot e^7 =$

Multiply the following monomials: (continued)

4. $t^9 \cdot t^7 \cdot t^4 \cdot t^2 =$

5. $a^m \cdot a^{m2} =$

6. $\begin{array}{r} 2ab \\ \underline{-5b} \end{array}$

7. $\begin{array}{r} -3x^2y^2 \\ \underline{-2x^2y} \end{array}$

8. $\begin{array}{r} +6rst \\ \underline{+3rst} \end{array}$

9. $\begin{array}{r} -2x^2c \\ \underline{+3xc^2} \end{array}$

10. $\begin{array}{r} 2x^2c^3d^2 \\ \underline{6xcd} \end{array}$

Add the following polynomials:

1. $\begin{array}{r} 7ab+4x^2 \\ \underline{2ab-2x^2} \end{array}$

2. $\begin{array}{r} 5a+3b-2c \\ \underline{-2a+3b+2c} \end{array}$

3. $\begin{array}{r} +2y^2-9 \\ -5y^2+4 \\ \underline{-6y^2-8} \end{array}$

Subtract the following polynominals:

1. $\begin{array}{r} x^2+y^2+z^2 \\ \underline{-3x^2+2y^2+z^2} \end{array}$

2. $\begin{array}{r} 5b^2+9 \\ \underline{+2b^2+9} \end{array}$

3. $\begin{array}{r} -7y^2+3xy-2 \\ \underline{-y^2+xy+1} \end{array}$

Divide:

1. $\frac{15ac}{5bc} =$

2. $\frac{x^4y^3z^2}{xyz^2} =$

3. $\frac{10a^2b^2}{5ab} =$

Divide: (continued)

4. $\dfrac{8x^2y^2}{16x^2y} =$

5. $\dfrac{12xyz}{48x^2y^2z^2} =$

Add the following binomials:

1. $\begin{array}{c} 2a+b \\ \underline{\quad a-b \quad} \end{array}$

2. $\begin{array}{c} x+y \\ \underline{\quad x-y \quad} \end{array}$

3. $\begin{array}{c} 2x+2y \\ \underline{\quad 3x-2y \quad} \end{array}$

4. $\begin{array}{c} 3x+2 \\ \underline{\quad x-2 \quad} \end{array}$

5. $\begin{array}{c} 4a-b \\ \underline{\quad a-b \quad} \end{array}$

Solve for X:

1. $x+3\ (x-2) = 2x + 4$

2. $\dfrac{3x}{4} = 12$

3. $3x + (x-4) = 3x + 1$

4. $4x - 2\ (x + 3) = x - 4$

5. $\dfrac{3x}{8} = 6$

Solve:

1. How many square feet of carpet are needed to cover a living room 18 feet long and 12 feet wide?

Solve: (continued)

2. The interest paid on $800 at 4% for 3 years.

3. The interest paid on $1,200 at 6% for 2 years.

4. The square of a number increased by 4 equals 40. Find the number.

5. Find V when L = 25, w = 20, and h = 10.

6. In the formula $A = \pi r^2$, find A when r = 7.

7. The length of a rectangle is twice the width. The area is 450. Find the two dimensions.

Solve: (continued)

8. Find the three dimensions of the cube whose volume is 125.

9. The width of a rectangle is only one-third the length, and the area is 4332; find the dimensions of the rectangle.

THE END

Algebra Test #7
pgs. 1-85

Solve for X:

1. $\frac{x}{2} = 2$

2. $13x = 39$

3. $x + 25 = 45$

4. $x - 21 = 6$

5. $\frac{2x}{2} = 14$

6. $9x - 11 = 70$

7. $16x + 16 = 32$

8. $7 + 5x = 42$

9. $8x - 3x = 24 - 4$

10. $3x - 4 = x + 8$

11. $15x + 5 = 10x + 15$

12. $5x - 7 = 2x + 11$

13. $5x - (x + 6) = 10$

Solve for X: (continued)

14. $4(2x - 5) - 3(x + 10) = -15$

15. $5(2x - 1) - 3(x + 2) = 31$

Read and Solve:

1. The product of 6 and a number equals 78. Find the number.

2. Twice a number increased by 4 equals 10. Find the number.

3. The sum of five times a number and 13 is 43. Find the number.

4. If 5 times a number is decreased by 3 the result is the same as when twice the number is increased by 21.

5. The length of a rectangle is represented by $3x + 5$ and the width by $6x - 20$. Find the length and the width if the perimeter is 78 feet.

Test #7

Multiply:

1. $$\begin{array}{r} 6x^3y^3 \\ \underline{4x^2y} \end{array}$$

2. $$\begin{array}{r} -15x^2y \\ \underline{6xy^3} \end{array}$$

3. $$\begin{array}{r} -24x^2 \\ \underline{-4x^3y} \end{array}$$

4. $$\begin{array}{r} -19xy^2 \\ \underline{-20x^2y^3z} \end{array}$$

5. $$\begin{array}{r} -23ab^2c \\ \underline{-4a^2bc^2} \end{array}$$

Divide:

1. $$\frac{25x^2y^2z}{5xy^3z^4}$$

2. $$\frac{5x^4y^4c^4}{15x^6y^6c^6}$$

3. $$\frac{-15x^2y^2}{5xy}$$

4. $$\frac{-6x^2y^4c^4}{-2xy^3c^5}$$

5. $$\frac{8x^3y^4c^6}{-16x^4y^8c^{10}}$$

Add:

1. $$\begin{array}{r} 7a - 5b - 3c \\ -9a + b - 5c \\ -4a + 3b - 2c \\ \underline{6a - 4b + c} \end{array}$$

2. $$\begin{array}{r} 32a - 17b + 15c \\ -45a + 23b - 19c \\ 16a - 19b + 7c \\ -20a + 10b + 10c \\ 13a - 10b + 2c \\ \underline{-20a + 20b - 14c} \end{array}$$

Subtract:

1. $$\begin{array}{r} -32b - 16c + 13d \\ \underline{-37b + 4c - 7d} \end{array}$$

2. $$\begin{array}{r} -26a + 17b - 33c \\ \underline{-9a + 6b - 16c} \end{array}$$

Solve for x and y:

1. $$\begin{array}{r} x + y = 35 \\ \underline{x - y = 5} \end{array}$$

2. $$\begin{array}{r} x + y = 25 \\ \underline{x - y = 15} \end{array}$$

3. $$\begin{array}{r} x - y = 2 \\ \underline{4x + y = 18} \end{array}$$

4. $$\begin{array}{r} 3x - 5y = 11 \\ \underline{x + 5y = 17} \end{array}$$

5. $$\begin{array}{r} x + 3y = 45 \\ \underline{x - 3y = -9} \end{array}$$

Solve for the unknowns:

1. $$\begin{array}{r} x + y = 5 \\ \underline{3x + 2y = 13} \end{array}$$

2. $$\begin{array}{r} 2m + 3n = 9 \\ \underline{m - 2n = 1} \end{array}$$

3. $$\begin{array}{r} a + 2b = 10 \\ \underline{a - b = 1} \end{array}$$

4. $$\begin{array}{r} 4a + 3b = 14 \\ \underline{2a + 3b = 10} \end{array}$$

5. $$\begin{array}{r} 2x - 3y = 1 \\ \underline{3x - 2y = 9} \end{array}$$

Solve:

1. The difference of two numbers is -3. Twice the first number increased by three times the second is 29. Find the numbers.

2. Four times a number increased by a second number equals 29. Three times the first number decreased by the second number equals 13. What are the numbers?

3. The width of a rectangle is one-fourth the length, and the area is 256. Find the dimensions.

4. The length of a rectangle is four times the width. The area is 900. Find the dimensions.

5. The difference of two numbers is 8. The sum of the two numbers is 24. Find the numbers.

THE END

Algebra Test #8
pgs. 91-95

Add:

1. $\begin{array}{r} \frac{a}{2} + y \\ \frac{a}{2} - y \\ \hline \end{array}$

2. $\begin{array}{r} \frac{2x}{3} - y \\ \frac{x}{3} - y \\ \hline \end{array}$

3. $\begin{array}{r} \frac{5x}{8} - \frac{2y}{3} \\ \frac{3x}{8} - \frac{y}{3} \\ \hline \end{array}$

4. $\begin{array}{r} \frac{m}{3} - N \\ \frac{2m}{3} + N \\ \hline \end{array}$

5. $\begin{array}{r} \frac{3a}{4} + b \\ \frac{a}{4} - b \\ \hline \end{array}$

Subtract:

1. $\begin{array}{r} \frac{x}{3} + y \\ \frac{x}{3} - y \\ \hline \end{array}$

2. $\begin{array}{r} \frac{3x}{8} - y \\ \frac{2x}{8} + y \\ \hline \end{array}$

3. $\begin{array}{r} \frac{5x}{9} + \frac{7y}{8} \\ \frac{x}{9} - \frac{y}{8} \\ \hline \end{array}$

4. $\begin{array}{r} \frac{3a}{4} + b \\ \frac{a}{4} - b \\ \hline \end{array}$

5. $\begin{array}{r} \frac{2a}{5} + \frac{2b}{3} \\ \frac{2a}{5} + \frac{b}{3} \\ \hline \end{array}$

Solve:

1. Five-eighths of a certain number increased by a second number equals 12. Three-eighths of the same number decreased by the second number equals -4. Find the numbers.

2. One-half a certain number increased by a second number equals 5. One-half the same number decreased by the second number equals 1. Find the two numbers.

3. One-half of a number decreased by 4 equals one-ninth of the number increased by 3. What is the number?

4. Three-fifths of a number added to a second number equals 11. Two-fifths of the first number decreased by the second number equals -1. Find the numbers.

5. Two-thirds of a number added to a second number equals zero. Two-thirds of the first number decreased by the second number equals 8. Find the numbers.

Solve for the unknowns:

1. $\frac{1}{x} + \frac{1}{y} = 3/4$

$\frac{1}{x} - \frac{1}{y} = 1/4$

2. $\frac{1}{x} - \frac{1}{y} = 2$

$\frac{1}{x} + \frac{1}{y} = 4$

THE END

pgs. 1-107

Solve:

1. If 4 times a number is increased by 30, the result is the same as when 95 is decreased by the number; find the number.

2. The perimeter of a rectangle is 70 feet. The width and length are in ratio of 2 to 3. Find the dimensions.

3. The area of a rectangle is 200 square feet. The length and width are in the ratio of 2 to 1. Find the dimensions.

4. If Jim earned $8.25 in 3 hours, how much would he earn in 40 hours?

5. A baseball player makes 24 hits in 98 times at bat. Express this ratio as a fraction, and then change it to a decimal correct to three places.

6. If it takes 16 gallons of gas to drive a car 288 miles, how many gallons will it take to drive 558 miles at the same rate?

7. $\frac{AB}{EF} = \frac{BC}{FG}$

AB = 2
BC = 6
EF = 1
Find FG

8. x:14 = 6:21

Review:

Add:

1.
$$\begin{array}{r} -15x \\ +6x \\ -20x \\ +26x \\ \underline{+14x} \end{array}$$

2.
$$\begin{array}{r} 1/3a \\ \underline{1/3a} \end{array}$$

3.
$$\begin{array}{l} \frac{m}{3} - N \\ \underline{\frac{2m}{3} + N} \end{array}$$

Subtract:

1.
$$\begin{array}{r} -14a^2b^3c^4 \\ \underline{-9a^2b^3c^4} \end{array}$$

2.
$$\begin{array}{r} +16x - 12y + 9z \\ \underline{+ 7x + 5y - 4z} \end{array}$$

3.
$$\begin{array}{ll} \frac{-4b}{5} & \frac{-4c}{3} \\ \underline{\frac{3b}{5}} & \underline{\frac{c}{3}} \end{array}$$

Multiply:

1.
$$\begin{array}{r} -9y^3z^2 \\ \underline{-3y^2z^5} \end{array}$$

2.
$$\begin{array}{r} +8a^3z^2 \\ \underline{-2a^2z^4} \end{array}$$

Divide:

1. $\dfrac{-12x^4}{15x^2}$

2. $\dfrac{-25x^2y^3}{-50x^3y^2}$

Solve for the unknown:

1. $.15m = 30$

2. $\dfrac{3x}{4} = 3$

3. $a - 4 = 7$

4. $x + 2 = 5$

5. $2x + 3 = x + 5$

6. $4x - 2(x+3) = x - 4$

7. $5y + 7 - 3y - 5 = y + 4$

8. $4x^3 - 6 = 3x^3 + 119$

9. $5x^2 + 3 = 2x^2 + 30$

10. $a + b = 15$
    $\underline{a = b = 5}$

11. $4r + 6s = 58$
    $\underline{4r - 6s = -2}$

12. $x - 3y = 3$
    $\underline{2x + y = 13}$

Add and Solve:

13. $\frac{3x}{4} + y = 6$

$\frac{x}{4} - y = -2$

14. $\frac{x}{2} + 3 = \frac{x}{8} + 7$

15. $\frac{x}{2} - 5 = \frac{x}{5} + 1$

16. $\frac{x}{3} = \frac{2}{5}$

17. $\frac{2x}{7} = \frac{6}{5}$

18. $\frac{3}{a} + \frac{2}{b} = 4$

$\frac{3}{a} - \frac{2}{b} = 2$

19. $x:16 = 4:8$

20. $7:14 = 4:x$

MT-39
THE END

Algebra Test #10
pgs. 1-119

1. Five-eighths of a certain number increased by a second number equals 12. Three-eighths of the same number decreased by the second number equals -4. Find the numbers.

2. A man works for \$2.80 per hour. What are his earnings for 80 hours of work?

3. The length of a rectangle is 30 feet and the width is 20 feet. A second rectangle with the same proportions has a width of 8 feet. What is the length of the second rectangle?

4. A man is 6' tall and casts a shadow 3' long and a tree nearby casts a shadow of 15'. How tall is the tree?

5. Combine the following: $5d - 2c + 6d + 3c - 5d$.

6. A family spends 23.5% of their income on food. Their income is \$135.00 per week. How much is their food bill per week?

7. Find two consecutive integers whose sum is 85.

8. $x^2y^3 \div xy^2 =$

9. $14x^5y^3 \div 7x^3y^2 =$

10. The area of a rectangle is 720 square feet, and the width is 1/5 the length. Find the dimensions.

11. $x + 4y = 12$
$\underline{2x - y = 6}$

12. Add: $2a + 3b$
$\underline{3a + 4b}$

13. Find the distance traveled by a car going an average speed of 43.6 miles per hour for 3.4 hours.

14. Find the dimensions of a rectangle if the area is 144 square inches and the length and width are in the ratio of 4 to 1.

15. The taxes on a piece of property valued at $6,000.00 are $150.00. At the same rate, what would be the taxes on property valued at $10,000.00?

16. $x^2 + 3y^2 = 12$

$x = 3y$

17. $x^2 + 2y^2 = 24$

$x = 2y$

18. $2x^2 + 3y^2 = 53$

$y = \frac{x}{5}$

19. $2x^2 + 3y^2 = 126$

$x = \frac{y}{2}$

20. The sum of two squares is 125. The first number is one-half the second. Find the numbers.

Solve:

1. $6a \overline{)\,12a^5}$

2. $-3xy \overline{)\,-27x^2y^3}$

3. $-6B^2c \overline{)\,+48B^3c^4}$

4. $\frac{4a}{x^2 + y^2} \cdot \frac{2(x^2 + y^2)}{5} =$

5. $\frac{x - y}{2x} \cdot \frac{4x^2}{x - y} =$

6. $\frac{5x^2}{12a} \cdot \frac{6a^3}{10x^4} \cdot \frac{-8x}{3a} =$

7. $\frac{-4a^2}{x} \cdot \frac{2x^3}{a} \cdot \frac{3a}{16x} =$

8. $\frac{x}{a} \cdot \frac{a^2}{x^3} \cdot \frac{a}{x} =$

9. $\frac{a^2}{x^2} \cdot \frac{x^4}{a^3} \cdot \frac{3x}{a} =$

10. $\frac{-5x}{a} \cdot \frac{-a^2}{10x^2} \cdot \frac{-2a}{x} =$

11. $\frac{a^2}{x} \div \frac{a}{x^2} =$

12. $\frac{2(a-b)}{x} \div \frac{a-b}{x^2} =$

13. $\frac{a + b}{2} \div \frac{a + b}{4} =$

14. $\frac{5}{2x} \div \frac{15}{4x^2} =$

15. $\frac{3x}{4} \div \frac{x^2}{2} =$

16. $\frac{10x^2y}{3} \div 5xy =$

17. $\frac{-12a^3}{5} \div 4 =$

Multiply:

18. $\begin{array}{r} -a + b + 2c \\ \underline{\qquad\qquad +3b} \end{array}$

19. $\begin{array}{r} a + b \\ \underline{\quad -a} \end{array}$

20. $\begin{array}{r} a - b \\ \underline{\quad a} \end{array}$

21. $\begin{array}{r} 5a + 4b + 2c \\ \underline{\qquad\qquad -3b} \end{array}$

22. $\begin{array}{r} 5x - 2y + 3z \\ \underline{\qquad\qquad -2xy} \end{array}$

23. $\begin{array}{r} a + b \\ \underline{a + b} \end{array}$

24. $\begin{array}{r} x + y \\ \underline{x - y} \end{array}$

25. $\begin{array}{r} 2x - y \\ \underline{x - y} \end{array}$

26. $\begin{array}{r} 3x + 2y \\ \underline{2x - 3y} \end{array}$

27. $\begin{array}{r} 2x + 4y \\ \underline{3x - 2y} \end{array}$

28. $\begin{array}{r} a + b \\ \underline{c + d} \end{array}$

29. $\begin{array}{r} x - y + z \\ \underline{x + y - z} \end{array}$

30. $\begin{array}{r} 6 + 2a - 3a^2 \\ \underline{2 - 3a + a^2} \end{array}$

MT-44

THE END

Algebra Test #11

pgs. 1-123

1. Find l when a = lw.

2. $\begin{array}{r} -3.5a \\ \underline{\times 2} \end{array}$

3. Solve for x: x-3 = 18.

4. ab X ab =

5. Find three consecutive even integers whose sum is 54.

6. If 22 is added to 2 times a certain number, the result is 88. What is the number?

7. Find the square root of 2401.

8. Add: $\begin{array}{r} -6a - 2b \\ \underline{-2a + \ \ b} \end{array}$

9. $x + 1 \overline{\smash{)}\, x^2 + 3x + 2}$

10. $x - 4 \overline{\smash{)}\, 2x^2 - 9x + 4}$

11. $2x - y \overline{\smash{)}\, 2x^2 + 5xy - 3y^2}$

Multiply:

12. $\begin{array}{r} x^2 + 2xy + y^2 \\ \underline{\quad\quad x + y} \end{array}$

13. $\begin{array}{l} a + b + c \\ \underline{a - b - c\quad\quad} \end{array}$

14. The length of a rectangular lot is two times the width. The area of the lot is is 5,000 square feet. Find the dimensions.

15. John bowled three games and had scores of 159, 168, and 173. What was his average score for the three games?

16. The balance owed on a kitchen stove is $234.60 after a down payment. If it is paid for in 12 equal payments, how much is each payment?

17. $\frac{22xyz}{11} =$

18. Solve for a:

$9a = 63$

19. Find the interest paid on $2,200.00 at 1½% for 6 years.

20. Sue is 3 times as old as her sister. The sum of their ages is 16. How old is Sue and how old is her sister?

21. The perimeter of a rectangle is 60 yards. The length is 4 times the width. What are the dimensions?

22. Solve for x: $4 + 2x - 3 + 6(x + 1)$

23. Write the exponent for $x \cdot x \cdot x \cdot x \cdot x =$

Multiply:

24. $$\begin{array}{r} x^2y^2z \\ \underline{x^2yz} \end{array}$$

25. $$\begin{array}{r} 4x - y \\ \underline{3x + 2y} \end{array}$$

Subtract:

26. $$\begin{array}{r} 4a + 3b \\ \underline{3a - 2b} \end{array}$$

27. Find the square root of 16,384.

28. What does $A^2 \times A^5 =$ ____________?

29. $$\begin{array}{l} 3x + y = 8 \\ \underline{4x + 2y = 26} \end{array}$$

Solve for x and y:

30. $$\begin{array}{l} 4x + y + 15 \\ \underline{x - 2y = -12} \end{array}$$

Solve for x and y:

Add:

31. $$\begin{array}{r} 25xy \\ -14xy \\ 70xy \\ \underline{-25xy} \end{array}$$

32. $$\begin{array}{r} 7a - 5b - 9c \\ -6a + 9b - 7c \\ 4a - 8b + 6c \\ \underline{-3a + b - c} \end{array}$$

33. $\dfrac{-18c^2d - 24c^2d + 12cd^3}{-6cd} =$

34. $\dfrac{-35ab^3}{-7a^2b} =$

35. **Reduce:** $\dfrac{15ab^2c^3}{10a^3b^2c} =$

36. $\dfrac{x}{y} \div \dfrac{x}{2} =$

37. $\dfrac{2x}{3} \div \dfrac{x^2}{6} =$

38. $\dfrac{3x}{8} \div \dfrac{4}{x^2} =$

39. $\dfrac{5x}{3} \div \dfrac{9}{10x^3} =$

Remove common factors:

40. $-9x^2 + 6xy - 18y^2 =$

41. $16a^2 - 24a^2b + 28a^3b^3 =$

Multiply:

42. $$\begin{array}{r} 2a + 3a^2 + 4a^3 \\ \underline{a - a^2} \end{array}$$

43. $$\begin{array}{r} -3a + 2a^2 - a^3 \\ \underline{-2a - 3a^2} \end{array}$$

44. $x + 1 \overline{\smash{)}\,3x^2 + x - 2}$

45. $3-2x \overline{\smash{)}\,6 + 5x - 6x^2}$

46. $(-9ab)^2 =$

47. $(7x^2y^2z^2)^2 =$

48. $\sqrt[2]{16x^2y^4}$

49. $\sqrt[2]{x^4y^2}$

50. $\sqrt[2]{25x^4y^2z^6}$

THE END

Algebra Test #12
pgs. 124-131

Perform the indicated operation:

1. $(x + 5)^2 =$

2. $(3x + 4)^2 =$

3. $(2x + 5)^2 =$

4. $(x - 3)^2 =$

5. $(2x - 3)^2 =$

6. $(3x - 5)^2 =$

7. $(2x + 1)\ (x + 3) =$

8. $(3x - 2)\ (x + 1) =$

Solve the equations:

9. $x^2 + 6x + 9 = 0$

10. $x^2 - 25 = 0$

11. $x^2 - 8x = -12$

12. $x^2 + 18x + 81 = 0$

13. $x^2 - 100 = 0$

14. $2x^2 + 11x + 5 = 0$

15. $x^2 - 4x = 12$

16. $x^2 - 81 = 0$

17. $\frac{2x + 4y}{3} \div \frac{3x + 6y}{9} =$

18. $\frac{2(a-b)}{x} \div \frac{a-b}{x^2} =$

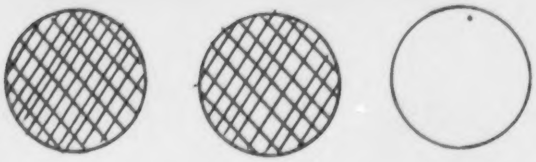
19. $2\sqrt{25x^2}$

20. $(2xy)^3 =$

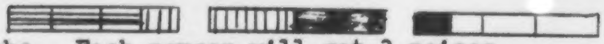
THE END

FRACTIONAL NUMBERS
(For use with Pentagon pp. 101-103)

This bar is divided into 4 equal parts.
3 of the 4 or 3/4 are plain.
1 of the 4 or 1/4 is shaded.



There are 3 items in this group (set).
2 of the 3 or 2/3 are marked.
1 of the 3 or 1/3 is plain.

3 candy bars divided among 4 people. 
The candy bars are divided into fourths. Each person will get 3 peices.
3 of 4 equal pieces = 3/4 of a candy bar.

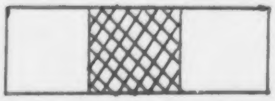
In each fraction, there is a numerator and a denominator.

2 numerator - number of parts you are using
3 denominator - number of parts in a whole thing

The top number is always the numerator; the bottom the denominator. The numerator and denominator are called the terms of a fraction.

Practice 1: Name the shaded part with a fraction.

1.



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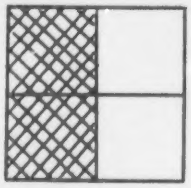
2.



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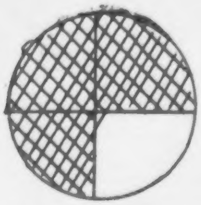
3.



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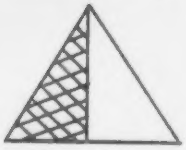
4.



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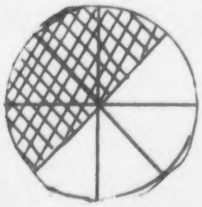
5.



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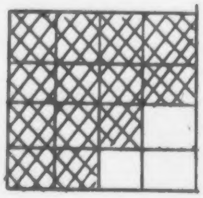
6.



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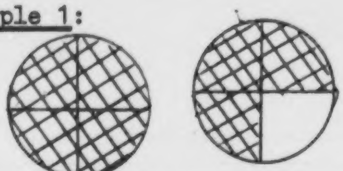
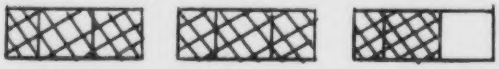
Fractions greater than one:

Example 1:



Example 2:



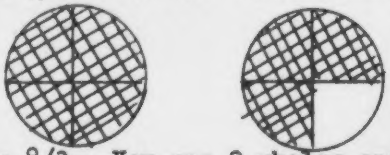
How many fourths are colored in example 1? ______________________________

This may be expressed a 7/4. The 4 shows how many pieces it takes to make a whole. The 7 shows how many pieces are colored.

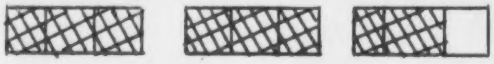
In example 2 there are eight thirds colored. This may be expressed as 8/3. The 3 shows how many pieces it takes to make a whole. The 8 shows how many pieces are colored.

The fractions 7/4 and 8/3 are examples of improper fractions because they are greater than or equal to one whole.

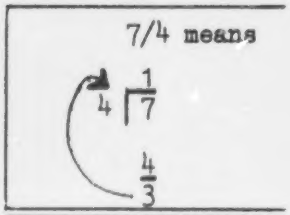
The picture shows 7/4. You see 1 whole and 3/4 of another. So 7/4 = 1 3/4.



The picture shows 8/3. You see 2 wholes and 2/3 of another. So 8/3 = 2 2/3.

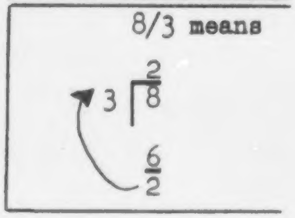


1 3/4 and 2 2/3 are mixed numbers. Improper fractions such as 7/4 and 8/3 may be changed to mixed numbers by the following method. Divide the numerator by the denominator and put any remainder over the denominator.



7 divided by 4 = 1 with a remainder of 3. The 3 is put over the divisor 4 to make the fraction 3/4. There is 1 whole with 3 fourths left over.

So 7/4 = 1 3/4



8 divided by 3 = 2 with a remainder of 2. The 2 is put over the divisor 3 to make the fraction 2/3. There are 2 wholes with 2 thirds left over.

So 8/3 = 2 2/3

Practice 2: Change these improper fractions into mixed numbers.

1. 7/2
2. 7/3
3. 13/10
4. 17/8
5. 17/4
6. 11/2
7. 32/5
8. 47/8
9. 6/5
10. 9/5
11. 23/2
12. 33/8
13. 25/4
14. 13/3
15. 12/7
16. 15/4
17. 11/6
18. 9/2
19. 8/5
20. 21/4

Mixed numbers such as 6 1/3 may be changed to improper fractions by another method.

Multiply the denominator times the whole number. Add the numerator.

6 1/3

Think: There are 3/3 in one whole
6 wholes and 3 thirds = 18 thirds (18/3)
$\frac{18}{3} + \frac{1}{3} = \frac{19}{3}$

7 1/2

7 wholes X 2 halves = 14 halves (14/2)
$\frac{14}{2} + \frac{1}{2} = \frac{15}{2}$

Shortcut: 6 1/3   3 X 6 = 18 + 1 = 19/3

7 1/2   2 X 7 = 14 + 1 = 15/3

Practice 3

Change these mixed numbers to improper fractions.

1. 1 3/4
2. 5 1/8
3. 7 1/8
4. 3 1/3
3. 2 1/2
6. 5 1/2
7. 8 1/3
8. 4 5/8
9. 6 1/8
10. 4 2/3
11. 9 1/4
12. 1 3/5
13. 8 3/4
14. 3 7/8
15. 5 4/5
16. 1 7/8
17. 9 1/3
18. 6 2/3
19. 4 3/5
20. 2 3/5

Comparing Fractions

Which is larger 1/2 or 1/3?	Which is smaller 1/3, 1/4, 1/5?
A.	C.
B.	D.
1/2 is larger because the whole A is divided into fewer pieces, so pieces are larger.	1/3 is smaller because the whole D is divided into more pieces so the pieces are smaller.

In each fraction 1/2, 1/3, 1/4, and 1/5, all of the numerators are the same. The denominators are getting larger. As you can see from the examples, if the numerators are the same and the denominator gets larger, the fraction gets smaller. 1/2 is the largest; 1/5 is the smallest.

Expressing equivalent or equal fractions

1/2

2/4

4/8

Fractions such as 1/2, 2/4, and 4/8 are called equivalent fractions because they represent the smae part of a whole.

You may express a fractions equivalent in higher or lower terms by multiplying or dividing the numerator and denominator by the same number.

Exmaple 1:

Express 1/2 in fourths

1/2 = ?/4 Think: to change 2 to 4 I must multiply 2 by 2.

$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$ I must therfore multiply 1 by 2.

Example 2:

Express 2/4 in eighths

2/4 = ?/8

$\frac{2}{4} \times \frac{2}{2} = \frac{4}{8}$

Think: to change 4 to 8 I must multiply 4 by 2.

I must also multiply 2 by 2.

Example 3:

Express 3/4 as thirty-seconds

3/4 = ?/32

$\frac{3}{4} \times \frac{8}{8} = \frac{24}{32}$

Think: to change 4 to 32 I must multiply 4 by 8.

I must also multiply 3 by 8.

Practice 4:

Change the following to their equivalents.

1. 3/4 = ?/12
2. 2/3 = ?/27
3. 5/8 = ?/16
4. 3/5 = ?/10
5. 5/6 = ?/12
6. 1/2 = ?/6
7. 7/8 = ?/24
8. 1/9 = ?/27
9. 2/7 = ?/21
10. 2/5 = ?/15

Example 4:

Express 9/12 as fourths

9/12 = ?/4

$\frac{9}{12} \div \frac{3}{3} = \frac{3}{9}$

Think: to change 12 to 4 I must divide 12 by 3.

I must also divide 9 by 3.

Example 5:

Express 12/36 as ninths

12/36 = ?/9

$\frac{12}{36} \div \frac{4}{4} = \frac{3}{9}$

Think: to change 36 to 9 I must divide 36 by 4.

I must also divide 12 by 4.

Practice 5: Change these fractions to their equivalents.

1. 12/36 = ?/3
2. 14/32 = ?/16
3. 9/15 = ?/5
4. 6/24 = ?/8
5. 16/64 = ?/4
6. 8/24 = ?/3
7. 9/18 = ?/2
8. 14/21 = ?/3
9. 7/28 = ?/4
10. 3/21 = ?/7

Like Fractions, Unlike Fractions, and Common Denominators

1/7, 2/7, 5/7 are like fractions

1/2, and 1/3 are not like fractions

Like fractions are fractions with common denominators. In order to express unlike fractions as like fractions we must give them a common denominators. All fractions may be expressed as like fractions by finding the lowest common denominator (sometimes called LCD).

Example 1:

1/2 and 1/3 are unlike. However, both 1/2 and 1/3 can be expressed as sixths.

$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$

$\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$

3/6 and 2/6 are like fractions.

Example 2:

2/3 and 3/4 are unlike fractions.

However $\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$

and $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$

8/12 and 9/12 are like fractions.

How to find a common denominator.

1. Divide one denominator into the other

1/2 and 1/4   2 will divide into 4 so use fourths as the denominator.

2. Multiply the two denominators.

1/3 and 1/4 $3 \text{ X } 4 = 12$

use twelfths as denominator

3. Think of a common multiple.

1/12 and 1/18

$12 \text{ X } 1 = 12$ $18 \text{ X } 1 = 18$
$12 \text{ X } 2 = 24$ $\boxed{18 \text{ X } 2 = 36}$
$\boxed{12 \text{ X } 3 = 36}$

Use 36ths as denominator

1/6 and 1/9

$6 \text{ X } 1 = 6$ $9 \text{ X } 1 = 9$
$6 \text{ X } 2 = 12$ $\boxed{9 \text{ X } 2 = 18}$
$\boxed{6 \text{ X } 3 = 18}$
$6 \text{ X } 4 = 24$

Use 18ths as denominator

Practice 6:

Find a common denominator for each group of fractions.

1. 1/3 and 1/9
2. 1/3 and 1/7
3. 1/12 and 1/24
4. 1/5 and 1/8
5. 1/8 and 1/4
6. 1/3 and 1/12
7. 1/4 and 1/6
8. 1/10 and 1/8
9. 1/2, 1/4, and 1/8
10. 1/3, 1/4, and 1/6

Reducing Fractions to Lowest Terms

When you work a problem in fractions, you should always try to reduce the answer to lowest terms.

Rule 1. Any fraction with a numerator of 1 (1/3, 1/64, 1/102) is already in lowest terms.

Rule 2. Any fraction with consecutive numbers in the numerator and the denominator (2/3, 5/6, 72/73, 101/102) is already in lowest terms. Consecutive numbers are like counting: 1, 2, 3, 4, or 72, 73, 74.

Rule 3. To find lowest terms for other fractions, review the section on equivalent fractions. A fraction may be changed to an equivalent fractions with lower terms by dividing the top and the bottom by the same number.

Example 1:

9/12 — Think: both 9 and 12 are divisible by 3.

$\frac{9 \div 3}{12 \div 3} = \frac{3}{4}$ — Since 3 and 4 are consecutive, this is in lowest terms.

Example 2:

12/36 — Both 12 and 36 are divisible by 2.

$\frac{12 \div 2}{36 \div 2} = \frac{3}{9}$ — Both 6 and 18 are divisible by 2 (not yet in lowest terms)

$\frac{6 \div 2}{18 \div 2} = \frac{3}{9}$ — Both 3 and 9 are divisible by 3 (not in lowest terms)

$\frac{3 \div 3}{9 \div 3} = \frac{1}{3}$ — Numerator is 1. Fraction is in lowest terms.

Shortcut: Try to think of the largest number which will divide evenly into both the top and the bottom.

12/36 — Both 12 and 36 are divisible by 12

$\frac{12 \div 12}{36 \div 12} = \frac{1}{3}$ — 1/3 is lowest terms

Example 3:

75/100 — Both 75 and 100 are divisible by 25.

$\frac{75 \div 25}{100 \div 25} = \frac{3}{4}$

Practice 7: Reduce these fractions to lowest terms.

1. 15/18 =
2. 10/12 =
3. 18/24 =
4. 7/21 =
5. 14/35 =
6. 6/12 =
7. 9/15 =
8. 4/20 =
9. 80/100 =
10. 45/50 =

Circle those which are already in lowest terms

11. 1/75 14/15 18/22 17/34 1/104
12. 2/104 6/7 17/31 17/51 9/12

FRACTIONAL NUMBERS - ANSWER KEY

Practice 1

1. 1/3
2. 2/4 = 1/2
3. 3/4
4. 1/2
5. 4/8 = 1/2
6. 13/16

Practice 2

1. 3 1/2
2. 2 1/3
3. 1 3/10
4. 2 1/8
5. 4 1/4
6. 5 2/5
7. 6 2/5
8. 5 7/8
9. 1 1/5
10. 1 4/5
11. 11 1/2
12. 4 1/8
13. 6 1/4
14. 4 1/3
15. 1 5/7
16. 3 3/4
17. 1 5/6
18. 4 1/2
19. 1 3/5
20. 5 1/4

Practice 3

1. 7/4
2. 41/8
3. 57/8
4. 10/3
5. 5/2
6. 11/2
7. 25/3
8. 37/8
9. 49/8
10. 14/3
11. 37/4
12. 8/5
13. 35/4
14. 31/8
15. 29/5
16. 15/8
17. 28/3
18. 20/3
19. 23/5
20. 13/5

Practice 4

1. 9/12
2. 18/27
3. 10/16
4. 6/10
5. 10/12
6. 3/6
7. 21/24
8. 3/27
9. 6/21
10. 6/15

Practice 5

1. 1/3
2. 7/16
3. 3/5
4. 2/3 = 1/4
5. 1/4
6. 1/3
7. 1/2
8. 2/3
9. 1/4
10. 1/7

Practice 6

1. 9
2. 21
3. 24
4. 40
5. 8
6. 12
7. 12 or 24
8. 40 or 80
9. 8
10. 12 or 72
    (other possibilities)

Practice 7

1. 5/6
2. 5/6
3. 3/4
4. 1/3
5. 2/5
6. 1/2
7. 3/5
8. 1/5
9. 4/5
10. 9/10
11. 1/75, 14/15, 1/104
12. 6/7, 17/31

FRACTIONS - ADDITION

(For use with Pentagon pp. 90-100)

When adding fractions, you need to be able to

1.) reduce fractions to lowest terms,
2.) find common dinominators,
3.) change to equivalent fractions, and
4.) change an improper fraction to a mixed number

If you need help with these, talk to your instructor before continuing.

Like denominators

Example 1: $\frac{1}{12} + \frac{5}{12} = \frac{6}{12}$ or $\frac{1}{2}$ or

$$\begin{array}{r} \frac{1}{12} \\ + \\ \frac{5}{12} \\ \hline \frac{6}{12} = \frac{1}{2} \end{array}$$

Example 2: $\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$ or

$$\begin{array}{r} \frac{1}{3} \\ + \frac{1}{3} \\ \hline \frac{2}{3} \end{array}$$

Example 3: $\frac{2}{9} + \frac{8}{9} = \frac{10}{9}$ or

$$\begin{array}{r} \frac{2}{9} \\ + \frac{8}{9} \\ \hline \frac{10}{9} \end{array} = 9\overline{)10}^{\,1} \;\; \begin{array}{r} 9 \\ \hline 1 \end{array} = 1\ 1/9$$

> Rule: When adding fractions with like denominators, all you do is add the numerators. The denominator stays the same.

Practice 1: Add these fractions. Reduce answers to lowest terms or to mixed numbers if necessary.

1. $\begin{array}{r} \frac{1}{4} \\ \frac{1}{4} \\ \hline \end{array}$ 2. $\begin{array}{r} \frac{1}{8} \\ \frac{5}{8} \\ \hline \end{array}$ 3. $\begin{array}{r} \frac{3}{5} \\ \frac{1}{5} \\ \hline \end{array}$ 4. $\begin{array}{r} \frac{7}{9} \\ \frac{1}{9} \\ \hline \end{array}$ 5. $\begin{array}{r} \frac{1}{3} \\ \frac{2}{3} \\ \hline \end{array}$ 6. $\begin{array}{r} \frac{2}{7} \\ \frac{6}{7} \\ \hline \end{array}$ 7. $\begin{array}{r} \frac{7}{10} \\ \frac{7}{10} \\ \hline \end{array}$

8. $$\begin{array}{c} \frac{3}{16} \\ \frac{15}{16} \\ \hline \end{array}$$

9. $$\begin{array}{c} \frac{3}{11} \\ \frac{6}{11} \\ \hline \end{array}$$

10. $$\begin{array}{c} \frac{1}{7} \\ \frac{2}{7} \\ \hline \end{array}$$

Unlike Denominators

Denominators must be alike before you can add fractions. If the denominators are different then you must make all the denominators equal.

Example 1: Add 1/2 and 3/8

Since 2 will divide into 8, we can use eighths as the common denominator.

$\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$

Remember to multiply the top and the bottom by the same number

1/2 = 4/8

So 4/8 + 3/8 = 7/8

Example 2:

$$\begin{array}{c} \frac{1}{4} \\ \frac{1}{3} \\ \hline \end{array}$$

If we multiply 3 X 4 we get 12. Use 12 for the denominator.

$$\begin{array}{rcl} \frac{1 \times 3}{4 \times 3} & = & \frac{3}{12} \\ \frac{1 \times 4}{3 \times 4} & = & \frac{4}{12} \\ \hline & & \frac{7}{12} \end{array}$$

Example 3:

$$\begin{array}{rcl} \frac{1 \times 3}{4 \times 3} & = & \frac{3}{12} \\ \frac{1 \times 2}{6 \times 2} & = & \frac{2}{12} \\ \hline & & \frac{5}{12} \end{array}$$

Find a common multiple.

$4 \times 1 = 4$
$4 \times 2 = 8$
$\boxed{4 \times 3 = 12}$

$6 \times 1 = 6$
$\boxed{6 \times 2 = 12}$

Example 4:

$$\begin{array}{l} \frac{1 \times 6}{2 \times 6} = \frac{6}{12} \\[1em] \frac{2 \times 4}{3 \times 4} = \frac{8}{12} \\[1em] \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \\ \hline \qquad\qquad \frac{23}{12} = 1\ 11/12 \end{array}$$

To find a common denominator, multiply all together 2 X 3 = 6 X 4 = 24

To find lowest common denominator think of multiples starting with the larger numbers.

$$\begin{array}{ll} 3 \times 1 = 3 & 4 \times 1 = 4 \\ 3 \times 2 = 6 & 4 \times 2 = 8 \\ 3 \times 3 = 9 & \boxed{4 \times 3 = 12} \\ \boxed{3 \times 4 = 12} & \end{array}$$

Since 2 (our other denominator) will divide into 12, 12 can be our LCD (lowest common denominator)

Practice 2:

1. $\begin{array}{c} \frac{5}{12} \\ \frac{2}{3} \\ \hline \end{array}$

2. $\begin{array}{c} \frac{5}{6} \\ \frac{1}{12} \\ \hline \end{array}$

3. $\begin{array}{c} \frac{1}{4} \\ \frac{5}{8} \\ \hline \end{array}$

4. $\begin{array}{c} \frac{2}{5} \\ \frac{3}{10} \\ \hline \end{array}$

5. $\begin{array}{c} \frac{3}{4} \\ \frac{3}{16} \\ \hline \end{array}$

6. $\begin{array}{c} \frac{2}{3} \\ \frac{2}{9} \\ \hline \end{array}$

7. $\begin{array}{c} \frac{2}{15} \\ \frac{1}{30} \\ \hline \end{array}$

8. $\begin{array}{c} \frac{1}{4} \\ \frac{3}{5} \\ \hline \end{array}$

9. $\begin{array}{c} \frac{1}{2} \\ \frac{5}{9} \\ \hline \end{array}$

10. $\begin{array}{c} \frac{2}{3} \\ \frac{3}{8} \\ \hline \end{array}$

11. $\begin{array}{c} \frac{1}{5} \\ \frac{1}{2} \\ \hline \end{array}$

12. $\begin{array}{c} \frac{3}{8} \\ \frac{1}{6} \\ \hline \end{array}$

13. $\begin{array}{c} \frac{1}{7} \\ \frac{1}{2} \\ \hline \end{array}$

14. $\begin{array}{c} \frac{4}{9} \\ \frac{1}{2} \\ \hline \end{array}$

15. $\begin{array}{c} \frac{2}{5} \\ \frac{2}{3} \\ \hline \end{array}$

16. $\begin{array}{c} \frac{1}{4} \\ \frac{5}{6} \\ \hline \end{array}$

17. $\begin{array}{c} \frac{2}{3} \\ \frac{3}{4} \\ \hline \end{array}$

18. $\begin{array}{c} \frac{5}{6} \\ \frac{3}{4} \\ \hline \end{array}$

19. $\begin{array}{c} \frac{5}{8} \\ \frac{2}{3} \\ \hline \end{array}$

20. $\begin{array}{c} \frac{1}{10} \\ \frac{3}{5} \\ \hline \end{array}$

21. $\begin{array}{c} \frac{1}{2} \\ \frac{2}{3} \\ \frac{3}{5} \\ \hline \end{array}$

22. $\begin{array}{c} \frac{1}{4} \\ \frac{1}{8} \\ \frac{5}{16} \\ \hline \end{array}$

23. $\begin{array}{c} \frac{2}{3} \\ \frac{1}{4} \\ \frac{1}{8} \\ \hline \end{array}$

24. $\begin{array}{c} \frac{2}{3} \\ \frac{3}{4} \\ \frac{1}{6} \\ \hline \end{array}$

25. $\begin{array}{c} \frac{5}{16} \\ \frac{1}{2} \\ \frac{1}{8} \\ \hline \end{array}$

Addition of Mixed Numbers:

Step 1: Change denominators if necessary
Step 2: Add fractions
Step 3: Reduced fractions or change to mixed number if necessary
Step 4: Add whole numbers
Step 5: Add answers from Step 3 (mixed numbers) if necessary

Example 1:

$$\begin{array}{r} 20\ 7/16 \\ \underline{17\ 1/4} \end{array} \qquad \frac{1 \times 4}{4 \times 4} = \frac{4}{16} \leftarrow \text{Step 1}$$

$$\begin{array}{r} 20\ 7/16 \\ \underline{17\ 4/16} \\ 11/16 \end{array}$$

Step 2
Step 3 not needed since 11/16 is in lowest terms

$$\begin{array}{r} 20\ 7/16 \\ \underline{17\ 4/16} \\ 37\ 11/16 \end{array}$$

Step 4
Step 5 not needed since fraction was not changed to mixed number.

Example 2:

$$\begin{array}{rl} 12 & \dfrac{4 \times 2}{7 \times 2} = \dfrac{8}{14} \\ 5 & \dfrac{1 \times 7}{2 \times 7} = \dfrac{7}{14} \end{array}$$

Step 1
Common denominator: 7 X 2 = 14

$$\begin{array}{r} 12\ 8/14 \\ \underline{5\ 7/14} \\ 15/14 \end{array}$$

Step 2

15/14 = 1 1/14

Step 3 $\begin{array}{r} 1 \\ 14\overline{)15} \\ \underline{14} \\ 1 \end{array} = 1\ 1/14$

$$\begin{array}{r} 12\ 8/14 \\ \underline{5\ 7/14} \\ 17\ 15/14 = 1\ 1/4 \end{array}$$

Step 4

$$\begin{array}{r} 17 \\ \underline{1\ 1/14} \\ 18\ 1/14 \end{array}$$

Step 5

Shortcut: Example 1:

$$\overset{1}{16}\quad \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

$$\underline{8\quad \frac{3 \times 3}{4 \times 3} = \frac{9}{12}}$$

$$\underline{\frac{17}{12} = 1\ 5/12}$$

$$25\ 5/12$$

Step 1 through 3

Step 4 and 5 combined 25 5/12

NOW: Bring down 5/12 to the final answer.
Carry 1 to ones column in whole numbers.

Example 2:

$$\overset{1}{13}\quad \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

$$\underline{9\quad \frac{2 \times 5}{3 \times 5} = \frac{10}{15}}$$

$$\underline{\frac{22}{15} = 1\ 7/15}$$

$$23\ 7/15$$

Practice 3:

1. 8 2/3 5 3/5	2. 5 7/8 9 2/3	3. 7 4/5 6 1/2	4. 2 2/3 7 1/2	5. 3 1/3 3 1/4
6. 7 3/5 8 1/2	7. 2 7/9 2 1/3	8. 9 5/6 2 1/4	9. 8 2/3 2 1/2	10. 6 5/12 2 2/3
11. 1 2/5 3 1/10 4 3/5	12. 1 1/3 6 1/4 5 1/2	13. 2 3/4 2 2/3 7 1/6	14. 3 2/3 6 1/2 4 7/10	15. 7 2/3 3 3/4 9 1/6

FRACTIONS - ADDITION

ANSWER KEY

Practice 1:

1. 1/2
2. 3/4
3. 4/5
4. 8/9
5. 1
6. 1 1/7
7. 1 2/5
8. 1 1/8
9. 9/11
10. 3/7

Practice 2:

1. 1 1/12
2. 11/12
3. 7/8
4. 7/10
5. 15/16
6. 8/9
7. 1/6
8. 17/20
9. 1 1/18
10. 1 1/24
11. 7/10
12. 13/24
13. 9/14
14. 17/18
15. 1 1/15
16. 1 1/12
17. 1 5/12
18. 1 7/12
19. 1 7/24
20. 7/10
21. 1 23/30
22. 11/16
23. 1 1/24
24. 1 7/12
25. 1 7/12

Practice 3:

1. 14 4/15
2. 15 13/24
3. 14 3/10
4. 10 1/6
5. 6 7/12
6. 16 1/10
7. 5 1/9
8. 12 1/12
9. 11 1/6
10. 9 1/12
11. 9 1/10
12. 13 1/12
13. 12 7/12
14. 14 13/15
15. 20 7/12

FRACTIONS: SUBTRACTION

(For Use with Pentagon pp. 90-100)

To subtract fractions, you must be able to

1. Find common denominators,
2. Change fractions to equivalents, and
3. Reduce answers to lowest terms.

If you need help with these, see your instructor.

Like Denominators

As in addition, you work only with the numerators when the denominators are the same.

Examples:

$$\begin{array}{rl} & \frac{5}{6} \\ - & \frac{1}{6} \\ \hline & \frac{4}{6} = \frac{2}{3} \end{array} \qquad \begin{array}{rl} & \frac{7}{8} \\ - & \frac{1}{8} \\ \hline & \frac{6}{8} = \frac{3}{4} \end{array} \qquad \begin{array}{rl} & \frac{5}{7} \\ - & \frac{1}{7} \\ \hline & \frac{4}{7} \end{array}$$

and, when the denominators are different, you must find common denominators.

$$\begin{array}{rr} & \frac{3}{4} = \frac{6}{8} \\ - & \frac{5}{8} = \frac{5}{8} \\ \hline & \frac{1}{8} \end{array} \qquad \begin{array}{rr} & \frac{2}{3} = \frac{8}{12} \\ - & \frac{1}{4} = \frac{3}{12} \\ \hline & \frac{5}{12} \end{array} \qquad \begin{array}{rr} & \frac{13}{16} = \frac{13}{16} \\ - & \frac{3}{4} = \frac{12}{16} \\ \hline & \frac{1}{16} \end{array}$$

Practice 1

Subtract and reduce answers if necessary.

1. $\frac{3}{4} - \frac{3}{8}$
2. $\frac{1}{3} - \frac{3}{10}$
3. $\frac{5}{12} - \frac{1}{3}$
4. $\frac{2}{5} - \frac{1}{3}$
5. $\frac{5}{8} - \frac{1}{2}$
6. $\frac{3}{4} - \frac{2}{3}$
7. $\frac{7}{8} - \frac{1}{8}$
8. $\frac{3}{4} - \frac{9}{20}$
9. $\frac{3}{4} - \frac{7}{12}$
10. $\frac{5}{6} - \frac{1}{2}$
11. $\frac{6}{7} - \frac{1}{2}$
12. $\frac{1}{3} - \frac{1}{4}$
13. $\frac{5}{6} - \frac{4}{5}$
14. $\frac{25}{27} - \frac{21}{27}$
15. $\frac{1}{3} - \frac{1}{9}$

Subtraction with Mixed Numbers -- No Borrowing

Step 1: Find common denominator and change to equivalents

Step 2: Subtract fractions and reduce

Step 3: Subtract whole numbers

Example 1

$$\begin{array}{lll} 7\ 1/2 = 3/6 & & \\ 2\ 1/3 = 2/6 & \leftarrow & \text{Step 1} \\ \hline 5 \qquad\quad 1/6 & \leftarrow & \text{Step 2} \\ \uparrow & & \\ \text{Step 3} & & \end{array}$$

Example 2

$$\begin{array}{lll} 7\ 2/5 = 6/15 & & \\ 3\ 1/3 = 5/15 & \leftarrow & \text{Step 1} \\ \hline 4 \qquad\quad 1/15 & \leftarrow & \text{Step 2} \\ \uparrow & & \\ \text{Step 3} & & \end{array}$$

Practice 2

Subtract and reduce fractions if necessary.

1. $\begin{array}{r} 7\ 4/5 \\ -\ 6\ 1/2 \\ \hline \end{array}$	2. $\begin{array}{r} 18\ 2/3 \\ -\ 10\ 1/2 \\ \hline \end{array}$	3. $\begin{array}{r} 15\ 7/8 \\ -\ 9\ 2/3 \\ \hline \end{array}$	4. $\begin{array}{r} 8\ 2/3 \\ -\ 2\ 3/5 \\ \hline \end{array}$
5. $\begin{array}{r} 9\ 3/4 \\ -\ 6\ 2/5 \\ \hline \end{array}$	6. $\begin{array}{r} 5\ 1/2 \\ -\ 3\ 1/3 \\ \hline \end{array}$	7. $\begin{array}{r} 6\ 1/2 \\ -\ 4\ 2/9 \\ \hline \end{array}$	8. $\begin{array}{r} 3\ 1/3 \\ -\ 3\ 1/4 \\ \hline \end{array}$
9. $\begin{array}{r} 2\ 2/3 \\ -\ 1\ 3/10 \\ \hline \end{array}$	10. $\begin{array}{r} 2\ 7/8 \\ -\ 1\ 3/8 \\ \hline \end{array}$	11. $\begin{array}{r} 24\ 3/5 \\ -\ 21\ 1/5 \\ \hline \end{array}$	12. $\begin{array}{r} 27\ 7/8 \\ -\ 23\ 3/4 \\ \hline \end{array}$

Subtracting Mixed Numbers - Borrowing Like Denominators

$$\begin{array}{r} 2\ 3/8 \\ -\ 1\ 7/8 \\ \hline \end{array} \quad ①$$

③ 1 = 8/8

②

$$\begin{array}{r} \overset{1}{\cancel{2}}\ 3/8 \\ -\ 1\ 7/8 \\ \hline \end{array}$$

④

$$\begin{array}{rcl} 1\ 3/8 + 8/8 & = & 1\ 11/8 \\ -\ 1\ 7/8 & = & 1\ 7/8 \\ \hline \end{array}$$

So: $\begin{array}{r} 1\ 11/8 \\ -\ 1\ 7/8 \\ \hline 4/8 = 1/2 \end{array}$ ⑤

1. 7/8 cannot be subtracted from 3/8
2. So, we must borrow from the ones column
3. The one we borrow must be changed to a fraction using the denominator in the problem. 1 whole = 8/8
4. This 8/8 is added to the 3/8
5. The problem is completed. Fraction is reduced.

Example 2: $\begin{array}{r} -22\ 1/5 \\ \underline{11\ 2/5} \end{array}$ ①

1. 2/5 is larger than 1/5
2. We must borrow from the ones column

②

$\begin{array}{r} 22\ 1/5 \\ \underline{-\ 11\ 2/5} \end{array}$ $1 = 5/5$ ③

3. Change the 1 we borrowed to fifths
   1 = 5/5

④

$\begin{array}{l} 22\ 1/5 + 5/5 = 21\ 6/5 \\ \underline{11\ 2/5 \qquad\quad\ = 11\ 2/5} \end{array}$

4. We add 5/5 to 1/5

$\begin{array}{r} 21\ 6/5 \\ \underline{-\ 11\ 2/5} \\ 10\ 4/5 \end{array}$ ⑤

5. Complete the problem.

Practice 3: Subtract and reduce fractions if necessary.

1. $\begin{array}{r} 7\ 1/2 \\ \underline{3\ 5/12} \end{array}$
2. $\begin{array}{r} 20\ 1/3 \\ \underline{19\ 2/3} \end{array}$
3. $\begin{array}{r} 12\ 1/4 \\ \underline{5\ 3/4} \end{array}$
4. $\begin{array}{r} 16\ 2/7 \\ \underline{8\ 5/7} \end{array}$
5. $\begin{array}{r} 19\ 1/8 \\ \underline{14\ 7/8} \end{array}$
6. $\begin{array}{r} 11\ 5/24 \\ \underline{9\ 13/24} \end{array}$
7. $\begin{array}{r} 15\ 3/10 \\ \underline{8\ 7/10} \end{array}$
8. $\begin{array}{r} 4\ 1/5 \\ \underline{2\ 3/5} \end{array}$
9. $\begin{array}{r} 21\ 4/9 \\ \underline{8\ 7/9} \end{array}$
10. $\begin{array}{r} 9\ 1/6 \\ \underline{7\ 5/6} \end{array}$
11. $\begin{array}{r} 3\ 5/8 \\ \underline{1\ 7/8} \end{array}$
12. $\begin{array}{r} 27\ 1/4 \\ \underline{14\ 3/4} \end{array}$
13. $\begin{array}{r} 14\ 3/7 \\ \underline{7\ 5/7} \end{array}$
14. $\begin{array}{r} 1\ 3/8 \\ \underline{5/8} \end{array}$
15. $\begin{array}{r} 108\ 1/3 \\ \underline{17\ 2/3} \end{array}$

Subtraction - Borrowing - Unlike Denominators

When the denominators are not the same, you must first find a common denominator. Then proceed as in the previous practice set.

Example 1:

14 1/3 X 4/4 = 4/12 ①
6 3/4 X 3/3 = 9/12

1. 3 X 4 is 12, the common denominator.
2. 9/12 is larger than 4/12 so you must borrow.

② 14 4/12 + 12/12 ③
6 9/12

3. The one you borrow is changed to twelfths and added to the 4/12. 1 = 12/12

13 16/12 ④
6 9/12
7 7/12

4. Complete the problem.

Example 2:

6 1/5 X 2/2 = 2/10 ①
4 3/10

1. Since 10 is divisible by 5, use 10 as the common denominator.

② 6 2/10 + 10/10 = 12/10
4 3/10 ③

2. 3/10 is larger than 2/10 so you must borrow.

5 12/10 ④
4 3/10
1 9/10

3. The one you borrow is changed to tenths and added to the 2/10. 1 = 10/10
4. Complete the problem.

Practice 4: Subtract and reduce fractions if necessary.

1. $\begin{array}{r} 4\ 1/3 \\ \underline{2\ 3/4} \end{array}$ 2. $\begin{array}{r} 7\ 1/2 \\ \underline{4\ 4/5} \end{array}$ 3. $\begin{array}{r} 6\ 1/5 \\ \underline{3\ 3/10} \end{array}$ 4. $\begin{array}{r} 4\ 3/4 \\ \underline{7/8} \end{array}$

5. $\begin{array}{r} 5\ 2/5 \\ \underline{1\ 1/2} \end{array}$ 6. $\begin{array}{r} 5\ 1/6 \\ \underline{1\ 3/8} \end{array}$ 7. $\begin{array}{r} 9\ 1/5 \\ \underline{4\ 1/2} \end{array}$ 8. $\begin{array}{r} 8\ 1/3 \\ \underline{5\ 7/8} \end{array}$

9. $\begin{array}{r} 7\ 2/5 \\ \underline{3\ 1/2} \end{array}$ 10. $\begin{array}{r} 7\ 2/3 \\ \underline{3\ 3/4} \end{array}$ 11. $\begin{array}{r} 8\ 1/3 \\ \underline{3\ 1/2} \end{array}$ 12. $\begin{array}{r} 4\ 1/5 \\ \underline{1\ 2/3} \end{array}$

13. $\begin{array}{r} 8\ 1/2 \\ \underline{4\ 3/8} \end{array}$ 14. $\begin{array}{r} 7\ 1/2 \\ \underline{4\ 3/4} \end{array}$ 15. $\begin{array}{r} 4\ 2/5 \\ \underline{5/6} \end{array}$

Special Case

Can You Do This Problem?

$$\begin{array}{r} 6 \\ \underline{3\ 3/4} \end{array}$$

If your answer is 2 1/4, you may check your answers. If not, do the following practice set.

Example: $\begin{array}{r} 6 \\ \underline{3\ 3/4} \end{array}$

1. Since 3/4 cannot be subtracted from nothing, you must borrow.

① $\begin{array}{r} \overset{5}{\cancel{6}} \\ \underline{3\ 3/4} \end{array}$

2. The one you borrowed is changed to fourths and put them with the whole number. 1 = 4/4

② $\begin{array}{r} 5\ 4/4 \\ \underline{3\ 3/4} \\ 2\ 1/4 \end{array}$ ③

3. Complete the problem.

Example 2:

7
5 1/6

1. 1/6 cannot be subtracted so you must borrow.

7
5 1/6

2. The one you borrowed is changed to sixths and put them with the whole number.
1 = 6/6

6 6/6
5 1/6

3. Complete the problem.

Practice 5: Subtract and reduce fractions if needed.

1. 8
1 1/3

2. 6
1 3/4

3. 14
2/3

4. 8
4 1/8

5. 3
1 1/2

6. 4
1 5/6

7. 7
3 2/3

8. 5
1 2/5

9. 8
3 2/7

10. 4
2 1/9

Fractions - Subtraction - Answer Key

Practice 1

1. 3/8
2. 1/30
3. 1/12
4. 1/15
5. 1/8
6. 1/12
7. 6/8 = 3/4
8. 6/20 = 3/10
9. 2/12 = 1/6
10. 2/6 = 1/3
11. 5/14
12. 1/12
13. 1/30
14. 4/27
15. 2/9

Practice 2

1. 1 3/10
2. 8 2/12 = 8 1/6
3. 6 5/24
4. 6 1/15
5. 3 7/20
6. 2 1/6
7. 2 5/18
8. 1/12
9. 1 11/30
10. 1 4/8 = 1 1/2
11. 3 2/5
12. 4 1/8

Practice 3

1. 3 8/12 = 3 2/3
2. 2/3
3. 6 2/4 = 6 1/2
4. 7 4/7
5. 4 2/8 = 4 1/4
6. 1 16/24 = 1 2/3
7. 6 6/10 = 6 3/5
8. 1 3/5
9. 12 6/9 = 12 2/3
10. 1 2/6 = 1 1/3
11. 1 6/8 = 1 3/4
12. 12 2/4 = 12 1/2
13. 6 5/7
14. 3/4
15. 90 2/3

Practice 4

1. 1 7/12
2. 2 7/10
3. 2 9/10
4. 3 7/8
5. 3 9/10
6. 3 19/24
7. 4 7/10
8. 2 11/24
9. 3 9/10
10. 3 11/12
11. 4 5/6
12. 2 8/15
13. 4 1/8
14. 2 3/4
15. 3 17/30

Practice 5

1. 6 2/3
2. 4 1/4
3. 13 1/3
4. 3 7/8
5. 1 1/2
6. 2 1/6
7. 3 1/3
8. 3 3/5
9. 4 5/7
10. 1 8/9

Fractions - Multiplication

(For use with Hexagon pp. 46-54)

In order to be able to multiply fractions, you must be able to:

1. change mixed numbers to improper fractions
2. change improper fractions to mixed numbers
3. reduce fractions to lowest terms

If you are not sure about these, ask your instructors

To multiply one fraction by another, multiply the numerators then multiply the denominators making a new fraction. Then change to mixed numbers or reduce to lowest terms if necessary.

Type 1: A whole number times a fraction.

Example 1:

3 x 1/2

1. Put the whole number over 1.
Note: (Any whole number over 1 equals the same whole number.)

① $\frac{3}{1} \times \frac{1}{2}$

2. Multiply numerators.

3. Multiply denominators.

$\frac{3}{1} \times \frac{1}{2} = \frac{3}{2}$ ② ③

$\frac{3}{2} = \begin{array}{r} 1 \\ 2\overline{)3} \\ \underline{2} \\ 1 \end{array} = 1\ 1/2$ ④

4. Change improper fractions 3/2 to mixed number.

Example 2: 1/2 x 5

$\frac{1}{2} \times \frac{5}{1}$ ①

1. Put the whole number over one.

$\frac{1}{2} \times \frac{5}{1} = \frac{5}{2}$ ② ③

2. Multiply numerators.

3. Multiply denominators.

$5 = \begin{array}{r} 2\ 5 \\ \underline{4} \\ 1 \end{array} = 2\ 1/2$ ④

4. Change improper fractions 5/2 to mixed number.

Practice 1: Multiply and change to mixed numbers.

1. 1/4 x 3   2. 5 x 7/8   3. 8 x 2/5

4. 3/8 x 7   5. 9 x 1/2   6. 3/4 x 5

7. 15 x 1/8   8. 8 x 3/5   9. 7 x 1/4

10. 2/3 x 8

Type 2: A fraction times another fraction.

Example 1: 1/2 x 1/2

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

1. Multiply numerators.
2. Multiply denominators.

Example 2: 1/4 x 1/2 x 1/3

$$\frac{1}{4} \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{24}$$

1. Multiply numerators.
2. Multiply denominators.

Special Note   Cancellation

Cancellation is dividing a numerator and denominator by a common divisor. You must divide both by the same number.

Example 1: 3/4 x 9/12

$$\frac{3}{4} \times \frac{9}{12}$$

1. Check for common divisors or factors. Since 3 will divide into 3 (a numerator) and 12 (a denominator) evenly you may use 3.

So

$$\frac{\cancel{3}}{4} \times \frac{9}{\cancel{12}}$$

2. Think 3 ÷ 3 = 1
   12 ÷ 3 = 4

$$\frac{1}{4} \times \frac{9}{4} = \frac{9}{16}$$

3. 4 and 9 have no common divisor.

4. Complete problem as before, multiplying numerator and denominators.

Example 2:

$\frac{8}{9} \times \frac{3}{4}$

1. 8 and 4 have a common divisor, 4. Think 8 ÷ 4 = 2; 4 ÷ 4 = 1

① $\frac{\overset{2}{\cancel{8}}}{9} \times \frac{3}{\underset{1}{\cancel{4}}}$

2. 9 and 3 have a common divisor, 3. Think 9 ÷ 3 = 3; 3 ÷ 3 = 1

② $\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} = \frac{2}{3}$ ③

3. Multiply numerators
Multiply denominators

Example 3: 2/3 x 8/9 x 27/28

① $\frac{\overset{1}{\cancel{2}}}{3} \times \frac{8}{9} \times \frac{27}{\underset{14}{\cancel{28}}}$

1. Sometimes numerators and denominators with common divisors are not next to each other. However, 2 (a numerator) and 28 (a denominator) are divisible by 2. 2 ÷ 2 = 1
28 ÷ 2 = 14

② $\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{8}{9} \times \frac{\overset{9}{27}}{\underset{14}{\cancel{28}}}$

2. Next, 3 (a denominator) and 27 (a numerator) are divisible by 3. 3 ÷ 3 = 1
27 ÷ 3 = 9

③ $\frac{\overset{1}{2}}{\underset{1}{\cancel{3}}} \times \frac{8}{\underset{1}{\cancel{9}}} \times \frac{\overset{1}{\cancel{27}}}{28}$

3. Now the 9 in the numerator and the 9 in the denominator are divisible by 9.
9 ÷ 9 = 1
9 ÷ 9 = 1

④ $\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{9}}} \times \frac{\overset{1}{\cancel{27}}}{\underset{\underset{7}{\cancel{14}}}{\cancel{28}}} = \frac{4}{7}$ ⑤

4. Since 8 will not divide into 14, you might feel finished. However, 8 and 14 do have a common divisor, 2. So you can cancel. Think 8 ÷ 2 = 4
14 ÷ 2 = 7

5. Complete problem by multiplying numerator and denominator.

NOTE: 1. Thinks of your multiplication table when the common divisor is not given. 9 and 6 will not divide into each other. But, both are part of the 3's multiplication table and are divisible by 3.

2. As long as you divide (cancel) one numerator and one denominator at a time, you may skip across a fraction to the next one to cancel.

Example 4: $\frac{4}{9} \times \frac{3}{5} \times \frac{5}{12}$

1. The 4 and 12 will cancel. (Divide by 4)
2. The 5's will cancel. (Divide by 5)
3. The 3 and 9 will cancel. (Divide by 3)

$\frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{12}}} = \frac{1}{9}$

Practice 2: Multiply. Show cancellation where possible. Reduce when necessary. Numbers 17, 20, and 21 are more difficult.

1. 4/9 x 27/32
2. 2/3 x 3/4
3. 5/6 x 18/25
4. 7/16 x 32/49
5. 5/8 x 24/29
6. 9/16 x 8/9
7. 7/8 x 7/16
8. 6/11 x 7/12
9. 2/3 x 5/9
10. 8/9 x 27/40
11. 4/9 x 12/17
12. 2/3 x 3/5
13. 5/6 x 18/25
14. 4/9 x 12/17 x 3/32
15. 9 x 2/3 x 5/12
16. 3/12 x 11/16 x 4/22
17. 21/25 x 5/6 x 18/35
18. 11/16 x 32/55
19. 2/3 x 3/5 x 5/8
20. 21/32 x 24/35
21. 64/75 x 50/72
22. 5/8 x 4/5
23. 3/16 x 32/39
24. 9/28 x 7/27
25. 7/8 x 16/21

Type 3: Multiplication of mixed numbers.

Only one extra step is involved: Change the mixed numbers to improper fractions.

Example 1: 8 2/3 x 5 1/4

1. $\frac{26}{3} \times \frac{21}{4}$ $4 \times 5 = 20 + 1 = 21$	1. To change to improper fractions multiply denominator x whole number, and add the numerator. Put over denominator.
2. $\frac{\cancel{26}}{3} \times \frac{21}{\cancel{4}}$	2. Cancel where possible. 26 and 4 can be divided by 2 26 ÷ 2 = 13, 4 ÷ 2 = 2
3. $\frac{\cancel{26}}{\cancel{3}} \times \frac{\cancel{21}}{\cancel{4}} = \frac{91}{2}$	3. 3 and 21 can be divided by 3 3 ÷ 3 = 1, 21 ÷ 3 = 7
4. $\frac{\cancel{26}}{\cancel{3}} \times \frac{\cancel{21}}{\cancel{4}} = \frac{91}{2}$	4. Complete problem. Multiply numerator and denominators. Change to mixed numbers or reduce if necessary.

$$\begin{array}{r} 45 \\ 2\,\overline{)91} \\ \underline{8} \\ 11 \\ \underline{10} \\ 1 \end{array} = 45\ 1/2$$

Example 2: 3 3/8 x 2 1/3

① $\frac{27}{8} \times \frac{7}{3}$

② $\frac{\overset{9}{27}}{8} \times \frac{7}{\underset{1}{3}}$

③ $\frac{\overset{9}{\cancel{27}}}{8} \times \frac{7}{\underset{1}{\cancel{3}}} = \frac{63}{8} =$ ④ $\begin{array}{r} 7 \\ 8\,\overline{)63} \\ \underline{56} \\ 7 \end{array} = 7\ 7/8$

Practice 3: Multiply. Show cancellation when possible. Reduce when necessary.

1. 2 1/4 x 3 1/2
2. 4 1/5 x 2 1/2
3. 7 1/3 x 3/8
4. 4 2/3 x 4 1/2
5. 3 1/3 x 4 1/2
6. 1 7/8 x 3 2/3
7. 3 1/2 x 4 2/3
8. 30 x 2 2/5
9. 5 7/8 x 2 2/3
10. 2 3/11 x 9/25
11. 4 7/8 x 6 2/3
12. 6 2/5 x 4 1/5
13. 2 1/3 x 4 1/5
14. 6 2/3 x 5 2/5
15. 4 1/6 x 3 3/5
16. 1 1/4 x 2 1/2
17. 8 1/3 x 3 1/5
18. 2 4/5 x 1 3/7
19. 1 1/8 x 2 2/3
20. 2 3/5 x 1 1/6

FRACTIONS - MULTIPLICATION - ANSWER KEY

PRACTICE 1

1. 3/4
2. 35/8 = 4 3/8
3. 16/5 = 2 1/5
4. 21/8 = 2 5/8
5. 9/2 = 4 1/2
6. 15/4 = 3 3/4
7. 15/8 = 1 7/8
8. 24/5 = 4 4/5
9. 7/4 = 1 3/4
10. 16/3 = 5 1/3

PRACTICE 2

1. 3/8
2. 1/2
3. 3/5
4. 2/7
5. 15/29
6. 1/2
7. 49/128
8. 7/22
9. 10/27
10. 3/5
11. 16/51
12. 2/5
13. 3/5
14. 1/34
15. 2 1/2
16. 1/32
17. 9/25
18. 2/5
19. 1/4
20. 9/20
21. 16/27
22. 1/2
23. 2/13
24. 1/12
25. 2/3

PRACTICE 3

1. 63/8 = 7 7/8
2. 21/2 = 10 1/2
3. 11/4 = 2 3/4
4. 21
5. 15
6. 55/8 = 6 7/8
7. 49/3 = 16 1/3
8. 72
9. 47/3 = 15 2/3
10. 9/11
11. 65/2 = 32 1/2
12. 672/25 = 26 22/25
13. 49/5 = 9 4/5
14. 36
15. 15
16. 25/8 = 3 1/8
17. 80/3 = 26 2/3
18. 4
19. 3
20. 91/30 = 3 1/30

Multiplication of Fractions II

(For use with Hexagon beyond p. 54)

Multiplication of fractions may take three forms.

1. Multiplication of a whole number by a fraction.

   Example: 3 x 1/2   3/1 x 1/2 = 3/2 = 1 1/2

2. Multiplication of a fraction by a whole number.

   Example: 1/2 x 3   1/2 x 3/1 = 3/2 = 1 1/2

3. Multiplication of a fraction by a fraction.

   Example: 1/2 x 1/2   1/2 x 1/2 = 1/4

Note: All whole numbers (integers) have a denominator of 1.

Example: 3 = 3/1

The one in the denominator does not change the value of the whole number.

To multiply one fraction by another, multiply the numerators and then multiply the denominators, arriving at a new fraction; then reduce it to lowest terms when possible.

Example: Multiply 2/3 times 6/7

2/3 x 6/7 = 12/21 = 4/7

Example: Multiply 3/7 times 2/3 times 4/9

$$3/7 \times 2/3 \times 4/9 = \frac{3 \times 2 \times 4}{7 \times 3 \times 9} = \frac{24}{189} = \frac{8}{63}$$

Cancellation in Multiplying Fractions

Dividing a factor of the numerator and a factor of the denominator by a common divisor (factor) is called cancellation.

Example: Multiply 3/4 x 8/12

1st. Will one of the numerators go into one of the denominators? Yes, 3 will go into 12, so we can use 3 as a factor to cancel from the numbers 3 and 12.

Example: (3 goes into 3, 1 time) $\overset{1}{\cancel{3}}/4 \times 8/\underset{4}{\cancel{12}}$

(3 goes into 12, 4 times)

2nd. Now do the same for 8 and 4. Four will go into eight, so it can be used as the common factor.

(4 goes into 4, 1 time) $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{4}{\cancel{12}}}$ (4 goes into 8, 2 times)

3rd. Multiply the numerators and denominators and reduce the fraction.

a. $\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{4}{\cancel{12}}} = \frac{1 \times 2}{1 \times 4} = \frac{2}{4}$

b. 2/4 2/2 = 1/2

Sometimes, however, none of the numerators or denominators can be used for divisors in cancellation. For example: 21/24 x 16/18

Cancellation is still possible, though, if you remember your multiplication tables.

Try to think of a number which will divide 21 evenly (3, 7). Will either of these numbers divide evenly into 18? Of course, 3 will.

(3 goes into 21, 7 times) $\overset{7}{\cancel{21}}/24 \times 16/\underset{6}{\cancel{18}}$ (3 goes into 18, 6 times)

The same process will work for 16 and 24. 2, 4, 8 will divide 16 evenly. Since they also will divide 24 evenly, use 8 (always use the largest number possible).

(8 goes into 24, 3 times) $\overset{7}{\cancel{21}}/\underset{3}{\cancel{24}} \times \overset{2}{\cancel{16}}/\cancel{18}$ (8 goes into 16, 2 times)

Multiply the numerators $\frac{\overset{7}{\cancel{21}}}{\underset{3}{\cancel{24}}} \times \frac{\overset{2}{\cancel{16}}}{\underset{6}{\cancel{18}}} = \frac{14}{18}$

Reduce 14/18 ÷ 2/2 = 7/9

Use the same process when multiplying more than 2 fractions.
Remember that you can cancel from any top number and any bottom number.
They needn't be side by side.

Example 1: 4/9 x 3/5 x 5/12

1st. The 4 will cancel with the 12
2nd. The fives will cancel.
3rd. The 3 will cancel with the 9.

$\overset{1}{\cancel{4}}/\underset{3}{\cancel{9}} \times \overset{1}{\cancel{3}}/\underset{1}{\cancel{5}} \times \overset{1}{\cancel{5}}/\underset{3}{\cancel{12}} = \frac{1}{3} \times \frac{1}{1} \times \frac{1}{3} = \frac{1}{9}$

Example 2: 8/9 x 6/7 x 7/12

1st. 4 will go into both 8 and 12
2nd. 3 will go into both 6 and 9
3rd. The sevens will cancel.

$$8/9 \times 6/7 \times 7/12 = \frac{2 \times 2 \times 1}{3 \quad 1 \quad 3} = \frac{4}{9}$$

Practice Exercises for Multiplication of Fractions

Note: Use cancellation if possible.

1. 4/9 x 12/17 x 3/32 =
2. 9 x 2/3 x 5/12 =
3. 2/3 x 6/10 x 3/5 =
4. 3/12 x 11/16 x 4/24 =
5. 4/9 x 27/32 =
6. 6/11 x 7/12 =
7. 7/8 x 7/16 =
8. 7/16 x 32/49 =
9. 9/1 x 8/9 =
10. 8/9 x 27/40 =
11. 21 x 5/6 x 18/25 =
12. 11/16 x 32/55 x 16/33 =

Multiplication of Mixed Numbers

Example: Multiply 3 2/3 x 5 1/4

1st. Change both mixed numbers to improper fractions.

8 2/3 = 26/3 5 1/4 = 21/4 (larger number on top, 21/4)

2nd. Cancel where possible.

26/3 x 21/4

3rd. Multiply numerators and denominators.

$$26/3 \times 21/4 = \frac{13 \times 7}{1 \times 2} = \frac{91}{2}$$

4th. Reduce or change to mixed number.

91/2 = 45 1/2

Practice Exercises for Multiplying Mixed Numbers

1. 3 3/8 x 2 1/3
2. 6 2/9 x 3/4
3. 15 x 6 1/3
4. 30 x 2 2/5
5. 4 1/6 x 3 3/5
6. 5 7/8 x 2 2/3
7. 2 3/11 x 9/25
8. 6 2/3 x 5 2/5
9. 4 7/8 x 6 2/3
10. 6 2/5 x 2 1/8
11. 6 2/5 x 5 3/8
12. 2 1/3 x 4 1/5

MULTIPLICATION OF FRACTIONS II

ANSWER KEY

Page F 33	Page F-34
1. 1/34	1. 7 7/8
2. 2 1/2	2. 4 2/3
3. 6/25	3. 95
4. 11/384	4. 72
5. 3/8	5. 15
6. 7/22	6. 15 2/3
7. 49/128	7. 9/11
8. 2/7	8. 36
9. 8	9. 32.5 (32 1/2)
10. 3/5	10. 13 3/5
11. 12 3/5	11. 34 2/5
12. 32/165	12. 9 4/5

Fractions - Division
(For use with Hexagon pp. 56-61)

In order to be able to divide fractions you must be able to:

1. change mixed numbers to improper fractions
2. change improper fractions to mixed numbers
3. cancel
4. reduce fractions to lowest terms

Division

Division problems may be written in 3 ways.

1.	$9 \div 2$	All may be read 9 divided by 2. It is important to know that the 2 is the divisor
2.	$2\overline{)9}$	
3.	$\frac{9}{2}$	

Division of fraction problems are usually written in the first form- the standard form for fractions.

$6 \div 1/2$ division or $1/2 \div 2/3$ division or $1\ 5/6 \div 7\ 1/3$ division

However you will sometimes see problems written in form 3.

$\frac{6}{1/2}$ division or $\frac{1/2}{2/3}$ division or $\frac{1\ 5/6}{7\ 1/3}$ division

As in regular division, the right hand or bottom fraction is the divisor. $6 \div 1/2$ and $\frac{6}{1/2}$ are read 6 divided by 1/2.

$1/2 \div 2/3$ and $\frac{1/2}{2/3}$ are read 1/2 divided by 2/3. And $1\ 5/6 \div 7\ 1/3$ and $\frac{1\ 5/6}{7\ 1/3}$ are read 1 5/6 divided by 7 1/3.

Remember that this form means the same thing. But, always simplify by putting the problem in standard form.

In other words, if you see $\frac{1\ 5/6}{7\ 1/3}$, write $1\ 5/6 \div 7\ 1/3$ to work the problem.

Working the problems

Rule of Division of Fractions: Invert the divisor (right hand fraction) and multiply. Invert means turn it upside down or reverse the numerator and the denominator.

Type 1 Whole number divided by a fraction

Example 1: 6 ÷ 3/8

1. $\frac{6}{1} \div \frac{3}{8}$	1. As in multiplication, put the whole number over 1.
2. $\frac{3}{8} \rightarrow \frac{8}{3}$	2. Invert the divisor
3. $\frac{6}{1} \times \frac{8}{3}$	3. Put in multiplication form.
4. $\frac{\overset{2}{\cancel{6}}}{1} \times \frac{8}{\underset{1}{\cancel{3}}} = \frac{16}{1} = 16$	4. Cancel if you can and complete the problem.

Example 2: 8 ÷ 2/3

1. $\frac{8}{1} \div \frac{2}{3}$	1. Place whole number over 1.
2. $\frac{2}{3} \rightarrow \frac{3}{2}$	2. Invert divisor.
3. $\frac{8}{1} \times \frac{3}{2}$	3. Put in multiplication form.
4. $\frac{\overset{4}{8}}{1} \times \frac{3}{\underset{1}{2}} = \frac{12}{1} = 12$	4. Cancel and complete.

Practice 1: Divide. Reduce answers if necessary.

1. 3 ÷ 2/5
2. 10 ÷ 5/8
3. 12 ÷ 2/3
4. 9 ÷ 3/4
5. 12 ÷ 6/7
6. 7 ÷ 1/2
7. 18 ÷ 9/10
8. 12 ÷ 3/4
9. $\frac{18}{6/5}$
10. $\frac{14}{7/2}$

Type 2: Division of a fraction by a whole number.

Example 1: $3/4 \div 4$

1. $3/4 \div 4/1$	1. Place whole number over 1.
2. $4/1 \rightarrow 1/4$	2. Invert the divisor.
3. $3/4 \times 1/4$	3. Put in multiplication form.
4. $\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$	4. Complete problem.

Example 2: $15/16 \div 3$

1. $15/16 \div 3/1$	1. Place whole number over 1.
2. $3/1 \rightarrow 1/3$	2. Invert the divisor.
3. $15/16 \times 1/3$	3. Put in multiplication form.
4. $\frac{{}^{5}\cancel{15}}{16} \times \frac{1}{\cancel{3}_{1}} = \frac{5}{16}$	4. Cancel and complete.

Practice 2: Divide. Reduce if necessary.

1. $2/7 \div 4$
2. $2/3 \div 9$
3. $2/3 \div 2$
4. $5/12 \div 5$
5. $5/8 \div 10$
6. $2/3 \div 6$
7. $3/5 \div 9$
8. $5/18 \div 10$
9. $\frac{5/6}{15}$
10. $\frac{5/8}{20}$

Type 3: Division of a fraction by a fraction.

Example 1: $5/8 \div 1/4$

1. $1/4 \rightarrow 4/1$	1. Invert divisor
2. $5/8 \times 4/1$	2. Multiply
3. $5/\underset{2}{\cancel{8}} \times \overset{1}{\cancel{4}}/1 = 5/2 = 2\ 1/2$	3. Cancel and complete problem by reducing fraction or changing to mixed number.

Example 2: 5/6 ÷ 5/9

1. 5/9 → 9/5	1. Invert divisor
2. 5/6 x 9/5	2. Multiply
3. $^{1}\frac{\cancel{5}}{\cancel{6}} \times \frac{\cancel{9}}{\cancel{5}}^{3}$	3. Cancel
4. $^{2}\frac{\cancel{5}}{_{2}\cancel{6}} \times \frac{\cancel{9}^{3}}{\cancel{5}_{1}} = \frac{3}{2}$ = 1 1/2	4. Complete problem

Practice 3: Divide. Reduce or change to mixed numbers as needed.

1. 5/8 ÷ 3/4
2. 3/5 ÷ 3/4
3. 2/3 ÷ 3/5
4. 5/9 ÷ 5/6
5. 4/9 ÷ 16/27
6. 6/11 ÷ 28/33
7. 1/2 ÷ 2/3
8. 7/8 ÷ 7/12
9. 2/3 ÷ 3/4
10. 1/5 ÷ 5/8
11. 4/5 ÷ 2/3
12. 3/8 ÷ 3/4
13. 5/6 ÷ 3/8
14. 7/12 ÷ 1/8
15. 2/3 ÷ 2/5

Type 4: Division with mixed numbers.

Mixed Numbers need to be changed to improper fractions.

Example 1: $3\ 1/2 \div 2\ 3/4$

1. $\frac{7}{2} \div \frac{11}{4}$	1. Change both mixed numbers to improper fractions.
2. $\frac{11}{4} \quad \frac{4}{11}$	2. Invert divisor
3. $\frac{7}{2} \times \frac{4}{11}$	3. Put in multiplication form.
4. $\frac{7}{\cancel{2}} \times \frac{\cancel{4}}{11} = \frac{14}{11} = 1\ 3/11$	4. Cancel and complete problem.

Example 2: $3\ 3/4 \div 2\ 1/2$

1. $\frac{15}{4} \div \frac{5}{2}$	1. Change both to improper fractions.
2. $\frac{5}{2} \quad \frac{2}{5}$	2. Invert divisor.
3. $\frac{15}{4} \times \frac{2}{5}$	3. Put in multiplication form.
4. $\frac{\cancel{15}}{\cancel{4}} \times \frac{\cancel{2}}{\cancel{5}} = \frac{3}{2} = 1\ 1/2$	4. Cancel and complete problem.

Practice 4:

1. $1\ 1/8 \div 6\ 3/5$
2. $2\ 5/8 \div 1\ 1/2$
3. $3\ 1/3 \div 2\ 1/2$
4. $4\ 1/5 + 2\ 2/5$
5. $7 \div 1\ 1/2$
6. $2\ 5/6 \div 1\ 1/12$
7. $6\ 2/5 \div 8$
8. $3\ 5/6 \div 1\ 2/3$

9. 1 1/4 ÷ 2 1/2

10. 1 1/5 ÷ 6 2/5

11. 3/5 ÷ 1 1/3

12. 4 2/3 ÷ 2 1/3

13. 5 1/2 ÷ 7 1/2

14. 5 1/5 ÷ 4/5

15. 1 1/3 ÷ 3 2/3

16. 5 1/4 ÷ 4 3/8

17. 5 1/10 ÷ 3 3/10

18. 4 8/9 ÷ 2 2/3

19. 1 1/2 ÷ 3 3/8

20. 15 ÷ 1 1/2

FRACTIONS--DIVISION--ANSWER KEY

Practice 1

1. 15/2=7 1/2
2. 16
3. 18
4. 12
5. 14
6. 14
7. 20
8. 16
9. 15
10. 4

Practice 2

1. 1/14
2. 2/27
3. 1/3
4. 1/12
5. 1/16
6. 1/9
7. 1/15
8. 1/36
9. 1/18
10. 1/32

Practice 3

1. 5/6
2. 4/5
3. 10/9=1 1/9
4. 2/3
5. 3/4
6. 9/14
7. 3/4
8. 3/2=1 1/2
9. 8/9
10. 8/25
11. 6/5=1 1/5
12. 1/2
13. 20/9=2 2/9
14. 14/3 = 4 2/3
15. 5/3 = 1 2/3

Practice 4

1. 15/88
2. 7/4 = 1 3/4
3. 4/3 = 1 1/3
4. 7/4 = 1 3/4
5. 14/3 = 4 2/3
6. 34/13 = 2 8/13
7. 4/5
8. 23/10 = 2 3/10
9. 1/2
10. 3/16
11. 9/20
12. 2
13. 11/15
14. 13/2 = 6 1/2
15. 4/11
16. 6/5 = 1 1/5
17. 17/11 = 1 6/11
18. 11/6 = 1 5/6
19. 4/9
20. 10

INTRODUCTION TO DECIMALS I

(For use with Pentagon 104-115)

I Reading Decimals

Decimal fractions are another way of writing regular fractions.

Regular or Common Fraction		Deciaml Fraction
$\frac{3}{10}$	=	.3

Both are read three tenths.

$\frac{3}{100}$	=	.03

Both are read three hundredths.

How Decimals are Read

.1	Read one tenth	$= \frac{1}{10}$
.01	Read one hundredth	$= \frac{1}{100}$
.001	Read one thousandth	$= \frac{1}{1000}$
.0001	Read one ten thousandth	$= \frac{1}{10,000}$
.00001	Read one hundred thousandth	$= \frac{1}{100,000}$
.000001	Read one millionth	$= \frac{1}{1,000,000}$

* Notice that the number of decimal places to the right of the decimal point agrees with the number of zeros in ten, one hundred, one thousand, etc. For example, one place is used for tenths (one zero in 10), two places are used for hundredths (two zeros in 100), five places are used for hundred thousandths (five zeros in 100,000).

In reading decimals, the last decimal place tells you which unit to use. The numbers tell you how many.

.013 is read thirteen thousandths $= \frac{13}{1000}$
↑
thousandths

.3468 is read three thousand, four hundred, sixty-eight ten thousandths $= \frac{3,468}{10,000}$
↑
ten thousandths

.10 is read ten hundredths $= \frac{10}{100}$
↑
hundredths

In the last example, you might notice that $\frac{10}{100}$ can be reduced to $\frac{1}{10}$. The decimal form would be .1. Indeed, .1 and .10 are equal just as $\frac{1}{10}$ and $\frac{10}{100}$ are equal. In fact, .1, .10, .100, .1000 are all equivalent. The zeros following the one do not change the value of the decimal, they only change the way the decimal is read.

Mixed decimals (a whole number and a decimal) may be compared to money. $5.75 would be read "five dollars and seventy-five cents". So, 5.75 would be read five and seventy-five hundredths. The decimal point is read as "and" in both cases.

Exercise 1

Write these decimals in words.

1. .3 ______________________________

2. .017 ______________________________

3. .48 ______________________________

4. .601 ______________________________

5. 5.839 ______________________________

6. 6.008 ______________________________

II. Writing Decimals

Writing decimals is easy if you remember to compare them to common fractions. Remember that the number of zeros in the fraction's denominator tells you how many decimal places to use.

First: Write the number as a regular fraction.

twenty-five thousandths = $\frac{25}{1000}$

NOTE - 3 zeros

Second: Write the numerator on your paper.

25

Third: Since you had three zeros, you must have three decimal places. This means you must add a zero in front of the 25.

.025

3 decimal places

Examples:

$$\text{Ninety-five hundredths} = \frac{95}{\underset{\underset{\text{2 zeros}}{\nearrow}}{100}} = \underset{\underset{\text{2 decimal places}}{\uparrow}}{.95}$$

$$\text{Fifteen millionths} = \frac{15}{\underset{\underset{\text{6 zeros}}{\uparrow}}{1,000,000}} = \underset{\underset{\text{6 decimal places}}{\nwarrow}}{.000015}$$

$$\text{Five thousandths} = \frac{5}{\underset{\underset{\text{3 zeros}}{\nearrow}}{1000}} = \underset{\underset{\text{3 decimal places}}{\uparrow}}{.005}$$

Exercise 2

Write these decimal as numbers.

1. Five tenths = $\frac{5}{10}$ = .5
2. Thirteen hundredths = $\frac{13}{100}$ = ____________________
3. One hundred five thousandths = $\frac{105}{1000}$ = ____________________
4. Two ten thousandths = $\frac{2}{10,000}$ = ____________________
5. Seventeen ten thousandths = ____________ = ____________________
6. One tenth = ____________ = ____________________
7. Fifty three hundredths = ____________ = ____________________
8. One thousand, four hundred fifty-four millionths = $\frac{1,454}{1,000,000}$ = ____________________
9. Forty hundredths = ____________ = ____________________
10. Ninety-five thousandths = ____________ = ____________________

III. Addition of Decimals

Rule - Line up the decimal points regardless of the number of numerals you have.

Adding decimals is much like adding money. To add money, we always line up the dollars and cents.

```
$    5.75
    10.63
   114.48
 1,210.03
---------
$1,340.89
```

Decimal are worked in the same manner.

```
    3.31
   12.76
  114.38
1,717.17
--------
1,847.62
```

However, with decimals it is also possible to have more than two numbers following the decimal point.

The decimal points, though, are still put in line.

```
 14.6            .7
  5.14          2.08
   .006         3.5
   .8           7.364
 ------        ------
 21.546        13.644
```

Exercise 3
Add the following:

```
1.  32.15     2.  .8       3.  .3        4.  34.        5.  12.45
      .174        .22          .07             .16            .5
      .5          .356         .007            .178          4.
     3.9          .4309        .0007           .9              .008
```

6. .083 + .2267 + 1.347 + .87 =

7. 9.4 + .09 + 14.2 =

8. 319.2 + 21.68 + .032 =

9. 8.3 + 27.04 + 9.28 + 6.387 =

IV. Subtraction of decimals

The rule for addition applies here, too. You must line up the decimals.

However, in order to solve some problem, you will have to add zeros. Any number of zeros may be added to the right of a decimal fraction. They do not change the value of the fraction.

For Example:

.2 = .20 = .200 = .2000

.03 = .030 = .0300 = .03000

Example 1:

$$6.5 - 4.352 = \begin{array}{r} 6.5 \\ \underline{4.352} \end{array} = \begin{array}{r} 6.500 \\ \underline{4.352} \\ 2.148 \end{array}$$

Two zeros were added. Then we borrowed because the subtrahend (bottom number) was larger.

Example 2:

$$6.234 - 4.1 = \begin{array}{r} 6.234 \\ \underline{4.1} \\ 2.134 \end{array}$$

No zeros or borrowing was necessary.

Exercise 4

1. $\begin{array}{r} 6.2 \\ \underline{-4.1} \end{array}$
2. $\begin{array}{r} 7.8 \\ \underline{-4.3} \end{array}$
3. $\begin{array}{r} 12.001 \\ \underline{-\ 3.352} \end{array}$
4. $\begin{array}{r} 62.4 \\ \underline{-\ 8.03} \end{array}$
5. $\begin{array}{r} 14.2 \\ \underline{-11.05} \end{array}$
6. $\begin{array}{r} 187.3 \\ \underline{-\ 15.875} \end{array}$

7. 15.8 - 11.372

8. 183.42 - 171.36

9. 19.71 - 4.581

10. 218.112 - 15.443

V. Comparing Decimals

Which is the larger of each pair? Circle the one that is more. *

.25 and .3	.035 and .35	.5 and .500
.15 and .115	.125 and .13	3.5 and 3.50
.6 and .65	.26 and .3	.625 and .6250

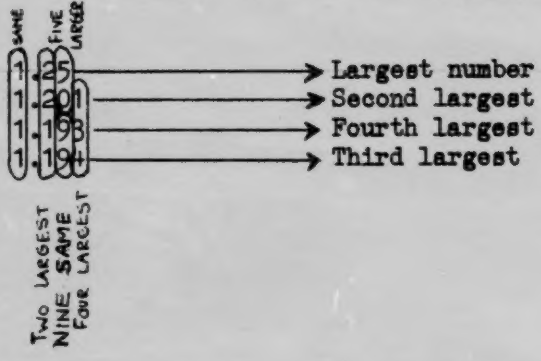
The easiest way to compare decimals is to give them an equal number of digits. (Remember that you can add zeros at the end without changing the value.)

For Example: .25 and .3
1. Add a zero to .3 .30
.30 - thirty hundredths
is larger than .25 - twenty-five hundredths

2. .035 and .35
Add a zero to .35 .350
.350 - three hundred fifty thousandths
is larger than .035 - thirty-five thousandths

Another way to compare decimals is to line up the decimals. Then check them number by number to see which is larger.

Example:


SAME
FIVE LARGER
1.25
1.201
1.193
1.194
Largest number
Second largest
Fourth largest
Third largest
TWO LARGEST
NINE SAME
FOUR LARGEST

* If you circled .3, .15, .65, .35, .13, and .3 go ahead to the next page. (The last three pairs are equal.) If not, finish this page and see your instructor.

Exercise 5

Write these numbers in order starting with the largest.

27, 2.8, .37, 1.35, 1.28, .128, 1.20, 1.1, 1.028

DECIMALS I - ANSWER KEY

EXERCISE 1

1. THREE TENTHS
2. SEVENTEEN THOUSANDTHS
3. FORTY-EIGHT HUNDREDTHS
4. SIX HUNDRED ONE THOUSANDTHS
5. FIVE AND EIGHT HUNDRED THIRTY-NINE THOUSANDTHS
6. SIX AND EIGHT THOUSANDTHS

EXERCISE 2

1. ------
2. .13
3. .105
4. .0002
5. 17/10,000 = .0017
6. 1/10 = .1
7. 53/100 = .53
8. .001454
9. 40/100 = .40
10. 95/1000 = .095

EXERCISE 3

1. 36.724
2. 1.8069
3. .3777
4. 35.238
5. 16.958
6. 2.5267
7. 23.69
8. 340.912
9. 51.007

EXERCISE 4

1. 2.1
2. 3.5
3. 8.649
4. 54.37
5. 3.15
6. 171.425
7. 4.428
8. 12.05
9. 15.129
10. 202.669

EXERCISE 5

27
2.8
1.35
1.28
1.20
1.1
1.028
.37
.128

THE END

INTRODUCTION TO DECIMALS II

(For use with Pentagon p 104-115)

A. Changing Decimals to Fractions and Changing Fractions to Decimals

Because it is sometimes easier to work with fractions and sometimes easier to work with decimals, it is important to be able to use both.

If you can read decimals, you can change decimals to fractions.

Example 1:

.1 is read one-tenth

$\frac{1}{10}$ is also read one-tenth

so .1 is changed to $\frac{1}{10}$

↑ decimal fraction

↑ regular fraction

Example 2:

.2 is read two-tenths

$\frac{2}{10}$ is also read two-tenths

so .2 is changed to $\frac{2}{10}$

↑ decimal fraction

↑ regular fraction

However $\frac{2}{10}$ is not in lowest terms

$$\frac{2}{10} = \frac{1}{5}$$

Example 3:

.96 is read ninety-six hundredths

$\frac{96}{100}$ is also read ninety-six hundredths

so $.96 = \frac{96}{100}$

$\frac{96}{100}$ can be reduced to $\frac{24}{25}$

Exercise 1:
Change these decimals to fractions in lowest terms.

1. .7 = ____________________
2. .50 = ____________________
3. .20 = ____________________
4. .75 = ____________________
5. .56 = ____________________
6. .85 = ____________________
7. .376 = ____________________
8. .450 = ____________________
9. .135 = ____________________
10. .40 = ____________________

B. Changing Fractions to Decimals

Changing fractions to decimals is a little more difficult. You will become familiar with some fractions.

$\frac{1}{4} = .25$ $\frac{2}{3} = .66\frac{2}{3}$ $\frac{1}{8} = .12\frac{1}{2}$ $\frac{7}{8} = .87\frac{1}{2}$

$\frac{1}{2} = .50$ $\frac{3}{4} = .75$ $\frac{3}{8} = .37\frac{1}{2}$

$\frac{1}{3} = .33\frac{1}{3}$ $\frac{1}{5} = .20$ $\frac{5}{8} = .62\frac{1}{2}$

However, for most fractions you need to know how to change fractions to decimals.

Example:

$\frac{1}{4}$ - The line in the middle of a fraction indicates division

Therefore, $\frac{1}{4}$ also means 1 divided by 4

or $4\overline{)1}$

Of course, 4 will not divide into 1. Therefore, you must put in the decimal point and add a zero.

Example 1:

$$\begin{array}{r|l} & \underline{\ .25} \\ 4 & 1.00 \\ & \underline{\ \ 8\ \ } \\ & \ \ 20 \\ & \ \ \underline{20} \end{array}$$

Now, add another zero and take 4 into 20.

This .25 is the decimal equivalent for $\frac{1}{4}$.

Example 2:

$\frac{3}{8}$

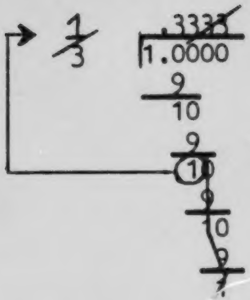
$$\begin{array}{r|l} & \underline{\ .375} \\ 8 & 3.000 \\ & \underline{2\,4\ \ \ \ } \\ & \ \ 60 \\ & \ \ \underline{56} \\ & \ \ \ \ 40 \\ & \ \ \ \ \underline{40} \end{array}$$

You may add additional zeros as you need them.

.375 is the decimal equivalent for $\frac{3}{8}$

Example 3:

$\frac{1}{3}$



$\frac{1}{3}$
.3333
1.0000
9
10
9
10
9
10
9
1

This fraction never comes out even.

If a fraction will not come out even, stop after 2 decimal places and make a fraction of the remainder.

```
       .33 1/3
 3    1.00
       9
       10
        9
        1
```

$\frac{1}{3} = .33\frac{1}{3}$

Example 4:

$\frac{1}{16}$

```
       .0625
16    1.0000
       96
        40
        32
         80
         80
```

16 will not go into 10 so we must put a zero in the answer before adding another zero.

Additional examples:

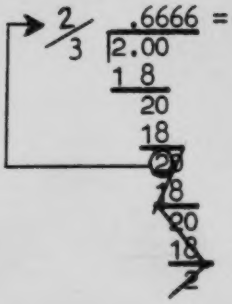
$\frac{1}{2}$

```
     .5
2   1.0
    1 0
```

$\frac{5}{8}$

```
     .625
8   5.000
    4 8
     20
     16
      40
      40
```

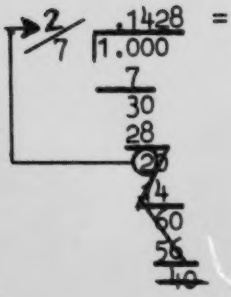
$\frac{2}{3}$



$\frac{2}{3}$
.6666
2.00
1 8
20
18
20
18
20
18
2

$= .66\frac{2}{3}$

$\frac{1}{7}$



$\frac{2}{7}$
.1428
1.000
7
30
28
20
14
60
56
40

$= .14\frac{2}{7}$

Example 5: $3\frac{1}{8}$

For mixed numbers, the whole number remains the same.

Find the decimal equivalent of $\frac{1}{8}$

$$\begin{array}{r} .125 \\ 8\,\overline{)\,1.000} \\ \underline{8} \\ 20 \\ \underline{16} \\ 40 \\ \underline{40} \end{array}$$

Write it with the whole number.

$3\frac{1}{8} = 3.125$

Exercises - Change these fractions to decimals.

1. $\frac{3}{4}$

2. $\frac{1}{8}$

3. $\frac{2}{7}$

4. $\frac{3}{5}$

5. $\frac{5}{6}$

6. $\frac{17}{20}$

7. $\frac{1}{12}$

8. $\frac{7}{8}$

9. $\frac{2}{9}$

10. $\frac{28}{105}$

11. $\frac{1}{11}$

12. $\frac{1}{20}$

13. $4\frac{1}{4}$

14. $1\frac{7}{10}$

15. $1\frac{1}{3}$

Add or Subtract - You must change one or the other to a fraction or a decimal.

1. $1\frac{1}{2} + 1.4$

$\frac{1}{2} = 2\overset{.5}{\overline{)1.0}}$

$\underline{1\ 0}$

so $1\frac{1}{2} = 1.5$

$$\begin{array}{r} 1.5 \\ \underline{1.4} \\ 2.9 \end{array}$$

$1.4 = 1\frac{4}{10} = 1\frac{2}{5}$

$$\begin{array}{rl} 1\frac{1}{2} = & \frac{5}{10} \\ \underline{1\frac{2}{5}} = & \underline{\frac{4}{10}} \\ 2 & \frac{9}{10} \end{array}$$

2. $8\frac{1}{3} + 8.4$

3. $.2 - \frac{1}{16}$

4. $14\frac{1}{4} - 13.7$

5. $2.6 + 1\frac{1}{4}$

6. $3 - \frac{1}{4}$

7. $13.7 + 5\frac{3}{5}$

8. $\frac{1}{4} + .78 + .31$

9. $1.5 + \frac{1}{6}$

10. $12.7 + 15\frac{1}{4}$

DECIMALS II - ANSWER KEY

EXERCISE 1

1. 7/10
2. 50/100 = 1/2
3. 20/100 = 1/5
4. 75/100 = 3/4
5. 56/100 = 14/25
6. 85/100 = 17/20
7. 376/1000 = 47/125
8. 450/1000 = 9/20
9. 135/1000 = 27/200
10. 40/100 = 2/5

NOTE: WHEN A DECIMAL DOESN'T COME OUT EVEN, THE DIVISION ANSWER WILL OFTEN REPEAT NUMBERS. THE BAR IS USED TO INDICATE THE PART THAT REPEATS. (SEE EXERCISE P.5 ANSWERS 3,7,9,10,11,15.)

EXERCISE - PAGE 5

1. .75
2. .125
3. .28 4/7 OR $.\overline{285714}$
4. .6
5. .83 1/3 OR $.8\overline{3}$
6. .85
7. .083 1/3, .08 1/3, OR $.08\overline{3}$
8. .875
9. .22 2/9 OR $.\overline{22}$
10. .26 2/3 OR $.\overline{26}$
11. .09 1/11 OR $.\overline{09}$
12. .05
13. 4.25
14. 1.7
15. 1.33 1/3 OR $1.\overline{33}$

EXERCISE - PAGE 6

2. 16.73 1/3 OR 16 11/15
3. .1375 OR 11/80
4. .55 OR 11/20
5. 3.85 OR 3 17/20
6. 2.75 OR 2 3/4
7. 19.3
8. 1.34 OR 1 17/50
9. 1.66 2/3 OR 1 2/3
10. 27.95 OR 27 19/20

THE END

MULTIPLYING DECIMAL NUMBERS

(For use with Hexagon pp. 71-76)

The decimal points do not have to be lined up to multiply decimal numbers.

In addition and subtraction of decimals, the decimal point is brought down to the answer. In multiplication of decimals the number of decimal places in the answer must equal the number of decimal places in the problem. Look at the example below:

Addition:

.40
.40
+ .40
1.20

Multiplication:

.40 two decimal places
X 3 no decimal places
1.20 two decimal places

Count the number of decimal places in the multiplication problem:
2 + 0 = 2.
Since there are 2 decimal places in the problem, count off 2 decimal places from the right in the answer.

Section One

1. .12 X 4
2. .34 X 2
3. .24 X 4
4. .32 X 3
5. .44 X 2
6. .67 X .2
7. .84 X .3
8. .26 X .6
9. .78 X .4
10. .92 X .2
11. 3.2 X .4
12. 1.4 X .2
13. 5.3 X 3
14. 2.1 X 5
15. 8.4 X 2
16. 5.94 X 2
17. 3.87 X 3
18. 15.6 X .6
19. 10.07 X .5
20. 307.9 X 2

DECIMAL POINTS IN THE FINAL ANSWER

A decimal point is needed only in the final answer of a multiplication problem. The final answer has as many decimal places as there are in the problem. Look at the example below:

$$\begin{array}{rl} 4.84 & \text{two decimal places} \\ \underline{\text{X}\ \ 4.3} & \text{one decimal place} \\ 1452 & \\ \underline{1936} & \\ 20.812 & \text{three decimal places} \end{array}$$

Count off the number of decimal places: 2 + 1 = 3. Since there are 3 decimal places in the problem, count off 3 decimal places from the right in the answer.

Section Two

1. $\begin{array}{r} 63.2 \\ \underline{\text{X}\ .6\ 4} \end{array}$
2. $\begin{array}{r} 1.24 \\ \underline{\text{X}\ \ .12} \end{array}$
3. $\begin{array}{r} 424 \\ \underline{\text{X}\ .26} \end{array}$
4. $\begin{array}{r} .47 \\ \underline{\text{X}\ .85} \end{array}$
5. $\begin{array}{r} 341.3 \\ \underline{\text{X}\ \ .0\ 8} \end{array}$
6. $\begin{array}{r} 48.3 \\ \underline{\text{X}\ \ 7\ 6} \end{array}$
7. $\begin{array}{r} 5.4\ 7 \\ \underline{\text{X}\ \ 2.5} \end{array}$
8. $\begin{array}{r} .279 \\ \underline{\text{X}\ \ .36} \end{array}$
9. $\begin{array}{r} 86.5 \\ \underline{\text{X}\ .0\ 7} \end{array}$
10. $\begin{array}{r} 1.49 \\ \underline{\text{X}\ \ .48} \end{array}$

ZERO AS A PLACE HOLDER

Where would you put the decimal point in this problem's answer?

$$\begin{array}{r} .4 \\ \underline{\text{X}\ .2} \end{array}$$

There are 2 decimal places in the problem. There must be 2 decimal places in the answer. Use a zero to hold a place. Two decimal places can then be counted off:

$$\begin{array}{r} .4 \\ \underline{\text{X}\ .2} \\ .08 \end{array}$$

Section Three

1. $\begin{array}{r} .9 \\ \underline{\text{X}\ .1} \end{array}$
2. $\begin{array}{r} .3 \\ \underline{\text{X}\ .2} \end{array}$
3. $\begin{array}{r} .3 \\ \underline{\text{X}\ .3} \end{array}$
4. $\begin{array}{r} .4 \\ \underline{\text{X}\ .4} \end{array}$
5. $\begin{array}{r} .2 \\ \underline{\text{X}\ .4} \end{array}$

6. .16 X .4
7. .43 X .2
8. .23 X .3
9. .11 X .9
10. .22 X .2

11. .24 X .04
12. .06 X .03
13. .45 X .03
14. .39 X .07
15. .25 X .05

16. .35 X .23
17. .69 X .12
18. .25 X .31
19. .26 X .32
20. .106 X .24

MULTIPLYING MONEY

Amounts of money are multiplied the same way other decimal numbers are multiplied. The number of decimal places in the answer must equal the number of decimal places in the problem. Look at the example below:

$ 5.12
X 2
$ 10.24

Section Four

Multiply. Put a dollar sign and a decimal point in each answer.

1. $46.98 X 2
2. $1.49 X 3
3. $9.99 X 7
4. $21.06 X 5
5. $69.99 X 8

6. $.56 X 72
7. $34.75 X 43
8. $99.75 X 34
9. $.97 X 33
10. $27.82 X 56

WORKING WITH WORD PROBLEMS

Section Five

Solve the following problems. Write the number of the answer on the line before the problem.

____ 1. Chicken costs $.39 a pound. How much do four pounds of chicken cost?
(1) $.96 (2) $1.26 (3) $1.56 (4) $1.60
(5) none of these

____ 2. Record albums are on sale for $2.49 each. You buy 3 albums. How much change do you get from a $10.00 bill?
(1) $2.49 (2) $2.53 (3) $7.51 (4) $7.47
(5) none of these

____ 3. The horse "Lucked Out" paid $3.20 for every dollar bet on him. If you made a $5.00 bet, how much money would you win?
(1) $15.00 (2) $16.00 (3) $150.00 (4) $160.00
(5) none of these

____ 4. You earn $1.65 an hour at a part-time job. Monday you work 4 hours. Tuesday you work 5 hours. Wednesday you don't work. Thursday you work 6 hours. Friday you work 4 hours. How much money did you earn for the week?
(1) $17.40 (2) $24.75 (3) $32.35 (4) $32.30
(5) none of these

____ 5. Some people get paid by the hour. The money they earn is sometimes called "straight time." Overtime pay is often called "time and a half." This means that for one hour's overtime work, you will get paid your regular hourly pay plus an additional half of that. ("Time and a half" can be written with deciamls as 1.5.) You make $1.92 an hour straight time. How much is your hourly overtime pay?
(1) $.288 (2) $2.88 (3) $28.800 (4) $288.00
(5) none of these

____ 6. A mother receives $12.92 each week from public assistance for each of her children. She has 7 children. How much money does she receive each week?
(1) $90.44 (2) $96.00 (3) $100.00 (4) $91.44
(5) none of these

____ 7. One bus ticket costs $.55. A 20-trip book of commuter tickets costs $9.50. If you took 20 trips, how much did you save by buying the book of commuter tickets?
(1) $.55 (2) $1.50 (3) $1.55 (4) $2.50
(5) none of these

_____ 8. The bus fare is raised from $.20 to $.45. You take the bus twice a day, Monday through Friday. How much more per week do you now spend on bus fare?
(1) $1.25 (2) $1.50 (3) $2.50 (4) $2.25
(5) none of these

_____ 9. You get 16.5 miles on one gallon of gas. How many miles can you travel on 12.5 gallons of gas?
(1) 165.5 (2) 206.25 (3) 2062.5 (4) 577.5
(5) none of these

_____ 10. A plant employs 563 men, each of whom earns $137.42 a week. If the men get a cost of living wage increase of $1.24 a week, what is the total cost to management of the men's weekly wages?
(1) $78,065.58 (2) $77,367,46 (3) $17,040.08
(4) $698.12 (5) none of these

ANSWER KEY

SECTION ONE

1. .48 2. .68 3. .96 4. .96 5. .88
6. .134 7. .252 8. .156 9. .312 10. .184
11. 1.28 12. .28 13. 15.9 14. 10.5 15. 16.8
16. 11.88 17. 11.61 18. 9.36 19. 5.035 20. 257.25

SECTION TWO

1. 40.448 2. .1488 3. 110.24 4. .3995 5. 27.304
6. 3670.8 7. 14.175 8. .10044 9. 6.055 10. .7152

SECTION THREE

1. .09 2. .06 3. .09 4. .16 5. .08
6. .064 7. .086 8. .069 9. .099 10. .044
11. .0096 12. .0018 13. .0135 14. .0273 15. .0125
16. .0805 17. .0828 18. .0775 19. .0832 20. .02544

SECTION FOUR

1. $93.96 2. $4.47 3. $69.93 4. $105.30 5. $559.92
6. $40.32 7. $1,494.25 8. $3,391.50
9. $32.01 10. 1,557.92

ANSWER KEY (CONTINUED)

SECTION FIVE

1. (3) $\begin{array}{r} \$\ .39 \\ \times\ \ 4 \\ \hline \$1.56 \end{array}$

2. (2) $\begin{array}{r} \$2.49 \\ \times\ \ 3 \\ \hline \$7.47 \end{array}$ $\begin{array}{r} \$10.00 \\ -\ 7.47 \\ \hline \$\ 2.53 \end{array}$

3. (2) $\begin{array}{r} \$3.20 \\ \times\ 5.00 \\ \hline \$16.00\ 00 \end{array}$

4. (5) $\begin{array}{r} 4 \\ 5 \\ 6 \\ +\ 4 \\ \hline 19 \end{array}$ $\begin{array}{r} \$1.65 \\ \times\ \ 19 \\ 14\ 85 \\ 16\ 50 \\ \hline 31.35 \end{array}$

5. (2) $\begin{array}{r} \$1.92 \\ \times\ 1.\ 5 \\ 960 \\ 1920 \\ \hline \$2.880 \end{array}$

6. (1) $\begin{array}{r} \$12.92 \\ \times\ \ 7 \\ \hline \$90.44 \end{array}$

7. (2) $\begin{array}{r} \$\ .55 \\ \times\ \ 20 \\ \hline \$11.00 \end{array}$ $\begin{array}{r} \$11.00 \\ -\ 9.50 \\ \hline \$\ 1.50 \end{array}$

8. (3) $\begin{array}{r} \$\ .45 \\ -\ .20 \\ \hline \$\ .25 \end{array}$ $\begin{array}{r} \$\ .25 \\ \times\ \ 10 \\ \hline \$\ 2.50 \end{array}$

9. (2) $\begin{array}{r} 1\ 6.5 \\ \times\ 1\ 2.5 \\ 8\ 2\ 5 \\ 33\ 0\ 0 \\ 165\ 0\ 0 \\ \hline 206.2\ 5 \end{array}$

10. (1) $\begin{array}{r} \$137.42 \\ +\ 1.24 \\ \hline \$138.66 \end{array}$ $\begin{array}{r} \$138.66 \\ \times\ 5\ 63 \\ 415\ 98 \\ 8319\ 60 \\ 69330\ 00 \\ \hline \$78,065.58 \end{array}$

THE END

DIVIDING DECIMAL NUMBERS

(For use with Hexagon pp. 80-84)

What is the difference between these two division problems?

$$3\overline{)9} \qquad \begin{array}{r} .3 \\ 3\overline{).9} \end{array}$$

In the first problem a whole number is being divided.

In the second problem a decimal number is being divided. Notice where the decimal point is placed in the answer. The decimal point in the answer always goes above the decimal point in the problem.

Section One

Divide. Remember to put a decimal point in the answer.

1. $2\overline{).8}$ 2. $3\overline{).6}$ 3. $6\overline{).66}$ 4. $4\overline{).84}$ 5. $3\overline{).93}$

6. $3\overline{)3.9}$ 7. $2\overline{)8.4}$ 8. $4\overline{)4.4}$ 9. $6\overline{)12.6}$ 10. $5\overline{)15.5}$

11. $8\overline{)7.28}$ 12. $6\overline{)4.86}$ 13. $9\overline{)5.49}$ 14. $7\overline{)6.37}$ 15. $4\overline{)2.48}$

ZEROS AS PLACE HOLDERS

Zeros are used to hold decimal places when you cannot divide. Look at the example below:

$$\begin{array}{r} .083 \\ 5\overline{).415} \end{array}$$

You cannot divide 5 into 4. Put a zero above the 4. It acts as a place holder. Then divide 5 into 41.

Section Two

1. $6\overline{).366}$ 2. $3\overline{).219}$ 3. $8\overline{).568}$ 4. $7\overline{).287}$ 5. $2\overline{).164}$

6. $4\overline{).028}$ 7. $2\overline{).012}$ 8. $6\overline{).024}$ 9. $9\overline{).063}$ 10. $7\overline{).021}$

ZEROS HELP YOU DIVIDE

Which of these numbers is larger?

3 or 3.0

Both numbers have the same value. Adding a decimal and a zero after a whole number will not change the value of that number. Look at the example below:

3.0 or 3.00

Both of these numbers also have the same value. You can add any number of zeros you wish after the decimal point, and the value of the whole number will be the same.

Sometimes adding zeros helps in division. Look at the example below:

	Step 1	Step 2
$5\overline{)3}$	$5\overline{)3.0}$	$\begin{array}{r} .6 \\ 5\overline{)3.0} \end{array}$

Step 1 - To divide this problem, add a decimal point and a zero.

Step 2 - Divide: 3.0   5 = .6. The decimal point in the answer goes directly above the decimal point in the problem. Notice that the 6 is placed after the decimal point. It is a decimal number.

Section Three

1. $6\overline{)3}$ 2. $5\overline{)4}$ 3. $15\overline{)3}$ 4. $10\overline{)6}$ 5. $12\overline{)6}$

GETTING RID OF REMAINDERS

Do not write remainders with decimal problems. Instead, add zeros and keep dividing until the problem works out evenly. Look at the example below:

Incorrect	Correct
$\begin{array}{r} .9 \text{ r } 6 \\ 8\overline{)7.8} \\ \underline{7\,2} \\ 6 \end{array}$	$\begin{array}{r} .975 \\ 8\overline{)7.800} \\ \underline{7\,2} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 00 \end{array}$

Section Four

1. $8\overline{)2.8}$ 2. $5\overline{).38}$ 3. $4\overline{)26.}$ 4. $6\overline{).51}$ 5. $7\overline{)1.407}$

ROUNDING OFF DECIMAL PLACES

Sometimes you will find that you have a remainder no matter how many zeros you add and divide into. In this case, round off your answer after three decimal places.

To round off a decimal number to the third place, look at the fourth-place number. If the fourth-place number is 4 or less, leave the third-place number as it is. Drop all numbers to the right of the third-place number. Look at the examples below:

.4932 rounded off becomes .493
.6744 rounded off becomes .674

If the fourth-place number is 5 or more, add 1 to the third-place number. Drop all numbers to the right of the third-place number. Look at the examples below:

.3645 rounded off becomes .365
.1286 rounded off becomes .129

Section Five

Divide. Round off your answers to three places.

1. $7\overline{)8.1}$ 2. $6\overline{)49.3}$ 3. $7\overline{).58}$

4. $6\overline{)2.}$ 5. $3\overline{)11}$

DIVIDING BY DECIMAL NUMBERS

You cannot divide by a decimal number. You must change the decimal number to a whole number. Look at the example below:

	Step 1	Step 2	Step 3
$.8\overline{).48}$	$\underset{\curvearrowright}{.8.}\overline{).48}$	$8.\overline{)\underset{\curvearrowright}{.4.}8}$	$8.\overset{.6}{\overline{)4.8}}$

Step 1-Change .8 to the whole number 8. To do this, move the decimal point to the __right__: .8 → 8.

Step 2-Because the decimal point is moved to the right __outside__ the division box, move the decimal point the same number of places to the right __inside__ the division box: .48 ⟶ 4.8

Step 3-Divide: $4.8 \div 8. = .6$

Section Six

1. $.2\overline{).8}$ 2. $.6\overline{)4.2}$ 3. $.9\overline{).81}$ 4. $.7\overline{)5.6}$

5. $.12\overline{).48}$ 6. $1.8\overline{).72}$ 7. $1.6\overline{)9.6}$ 8. $.43\overline{).344}$

9. $.18\overline{).3708}$ 10. $2.3\overline{)1.656}$ 11. $6.7\overline{)40.2}$ 12. $3.9\overline{)138.84}$

ADDING ZEROS TO DIVIDE

Sometimes you must add zeros to the number inside the division box before you divide. Look at the example below:

	Step 1	Step 2	Step 3
$3.45\overline{)6.9}$	$3\underset{\curvearrowright}{.45}.\overline{)6.9}$	$345.\overline{)6\underset{\curvearrowright}{.90}.}$	$345.\overline{)\overset{2.}{690.}}$

Step 1-Change 3.45 to the whole number 345 by moving the decimal point.

Step 2-Move the decimal point the same number of places, or 2 places, inside the division box. Add a zero so that you can move the decimal point 2 places: 6.9 ⟶ 690.

Step 3-Divide: $690 \div 345 = 2.$

Sometimes you may need to add more than 1 zero. Look at the examples below:

$\underset{\curvearrowright}{.134}.\overline{)6\underset{\curvearrowright}{.700}.}$ 2 zeros added

$\underset{\curvearrowright}{.125}.\overline{)79\underset{\curvearrowright}{.000}.}$ 3 zeros added

Section Seven

1. $3.45\overline{)6.9}$ 2. $1.34\overline{)6.7}$ 3. $.125\overline{)79.}$ 4. $.75\overline{)1.}$

5. $.013\overline{)3.9}$ 6. $.22\overline{)1.1}$ 7. $7.5\overline{)60.}$ 8. $1.86\overline{)9.3}$

9. $.04\overline{)5.}$ 10. $4.2\overline{)861.}$ 11. $5.64\overline{)84.6}$ 12. $.722\overline{)25.27}$

DIVIDING MONEY

Amounts of money are divided the same way other decimal numbers are divided. The decimal point in the answer always goes above the decimal point in the problem. Look at the example below:

$$\begin{array}{r} \underline{\$\ .30} \\ 4\overline{)\$1.20} \end{array}$$

Section Eight

1. $25\overline{)\$1.25}$ 2. $16\overline{)\$17.60}$ 3. $32\overline{)\$80.00}$ 4. $12\overline{)\$202.20}$

5. $10\overline{)\$1,200.00}$ 6. $17\overline{)\$212.50}$ 7. $5\overline{)\$65.55}$ 8. $4\overline{)\$128.96}$

9. $312\overline{)\$780.00}$ 10. $3\overline{)\$9,126.66}$

WORKING WITH WORD PROBLEMS

Section Nine

Solve the following problems. Write the number of your answer on the line before the problem.

_____ 1. **You traveled 204.8 miles in 4 hours. How many miles did you travel each hour?**
(1) 51.2 (2) 52.1 (3) 521 (4) 51.1 (5) **none of these**

____ 2. You can buy 5 cans of tomato soup for $1.00. How much does one can cost?
(1) $.19 (2) $.20 (3) $.21 (4) $.22
(5) none of these

____ 3. Subway tokens are $.30 each. How many tokens can you get for $5.10?
(1) 15 (2) 16 (3) 17 (4) 18 (5) none of these

____ 4. A phone bill has a charge of $.78 for additional units. Additional units are charged at $.03 per minute. How many extra minutes did you talk?
(1) 24 (2) 25 (3) 26 (4) 27 (5) none of these

____ 5. Gasoline is $.34 and 9/10 of a cent a gallon ($.349). You spend $6.98 to fill your gas tank. How many gallons of gas did you put in?
(1) 17 (2) 18 (3) 19 (4) 20 (5) none of these

ANSWER KEY

SECTION 1

1. .4 2. .2 3. .11 4. .21 5. .31
6. 1.3 7. 4.2 8. 1.1 9. 2.1 10. 3.1
11. .91 12. .81 13. .61 14. .91 15. .62

SECTION 2

1. .061 2. .073 3. .071 4. .041 5. .082
6. .007 7. .006 8. .004 9. .007 10. .003

SECTION 3

1. .5
2. .8
3. .2
4. .6
5. .5

SECTION 4

1. .35
2. .076
3. 6.5
4. .085
5. .201

SECTION 5

1. 1.157
2. 8.217
3. .0829
4. .333
5. 3.667

SECTION 6

1. 4 2. 7
3. .9 4. 8
5. 4 6. .4
7. 6 8. .8
9. 2.06 10. .72
11. 6 12. 35.6

ANSWER KEY (CONTINUED)

SECTION 7

1. 2 2. 5 3. 632 4. 1.33
5. 300 6. 5 7. 8 8. 5
9. 125 10. 205 11. 15 12. 35

SECTION 8

1. \$.05 2. \$1.10 3. \$2.50 4. \$16.85 5. \$120.00
6. \$12.50 7. \$13.11 8. \$32.24 9. \$2.50 10. \$3,042.22

SECTION 9

1. (1) $\begin{array}{r} 51.2 \\ 4\overline{)204.8} \end{array}$

2. (2) $\begin{array}{r} \$.20 \\ 5\overline{)\$1.00} \end{array}$

3. (3) $\begin{array}{r} 17 \\ \$.30\overline{)\$5.10} \end{array}$

4. (3) $\begin{array}{r} 26 \\ .03\overline{)\$.78} \end{array}$

5. (4) $\begin{array}{r} 20 \\ .349\overline{)6.980} \end{array}$

MEASURING YOUR PROGRESS

Test A

Add or Subtract

1. $\begin{array}{r} .4 \\ .8 \\ \underline{+\ .9} \end{array}$

2. $\begin{array}{r} 4.57 \\ 2.93 \\ \underline{+\ 4.87} \end{array}$

3. $\begin{array}{r} 2.467 \\ 3.56 \\ \underline{+\ \ .987} \end{array}$

4. $\begin{array}{r} 34.98 \\ 2.73 \\ 25.68 \\ \underline{+\ \ \ .98} \end{array}$

5. $\begin{array}{r} 396.1 \\ 426.3 \\ 891.5 \\ \underline{+\ 271.5} \end{array}$

6. $\begin{array}{r} .8 \\ \underline{-\ .4} \end{array}$

7. $\begin{array}{r} 5.4 \\ \underline{-\ 2.9} \end{array}$

8. $\begin{array}{r} .679 \\ \underline{-\ .398} \end{array}$

9. $\begin{array}{r} 8.000 \\ \underline{-\ 5.975} \end{array}$

10. $\begin{array}{r} 176.29 \\ \underline{-\ \ 94.49} \end{array}$

Multiply or Divide

11. $\begin{array}{r} .37 \\ \underline{\times\ \ .3} \end{array}$

12. $\begin{array}{r} 2.53 \\ \underline{\times\ \ \ .6} \end{array}$

13. $\begin{array}{r} 12.63 \\ \underline{\times\ \ \ \ .5} \end{array}$

14. $\begin{array}{r} 469.3 \\ \underline{\times\ \ \ .08} \end{array}$

15. $\begin{array}{r} 468.39 \\ \underline{\times\ \ \ \ .27} \end{array}$

16. $8\overline{)4.88}$

17. $7\overline{).0497}$

18. $.4\overline{)35.6}$

19. $.012\overline{).00060}$

20. $.56\overline{)2.4472}$

21. $.48\overline{)72}$

22. $.005\overline{)3}$

Test B

Put the number of your answer on the line before the problem.

____ 1. The annual rainfall for three cities for a recent year was: New York, 42.94 inches; Chicago, 33.0 inches; and Houston, 45.74 inches. What was the total rainfall for the three cities?
(1) 142.94 (2) 125.74 (3) 121.68 (4) 131.78
(5) none of these

____ 2. Mark has saved $6.39 toward the purchase of a coat that costs $21.00. How much more money does Mark need?
(1) $15.51 (2) 14.61 (3) 13.72 (4) 12.81
(5) none of these

____ 3. Mr. Johnson traveled 792.8 miles in his new car. How much farther should he drive before getting his 1,000 mile check-up?
(1) 108.3 (2) 108.8 (3) 207.2 (4) 209.4
(5) none of these

____ 4. Louise bought a new spring outfit. Her dress cost $19.98; shoes, $22.50; hat, $7.25; and bag, $9.75. How much did she spend?
(1) $59.48 (2) $69.58 (3) $49.78
(4) $39.88 (5) none of these

____ 5. During a recent month Mr. Johns, owner of a neighborhood drug store, made bank deposits (put money in the bank) of $269.43, $179.63, $476.59, and $191.74. He made withdrawals (took money out of the bank) of $121.89, $263.00, $97.99, $139.58, and $6.66. If his previous monthly bank balance was $497.35, find his new bank balance.
(1) $489.54 (2) $985.62 (3) $1,614.74
(4) $1,125.20 (5) none of these

____ 6. How much should you pay for 8.6 gallons of gas at $.33 a gallon?
(1) $2.83 (2) $2.84 (3) $2.93 (4) $3.84
(5) none of these

____ 7. Find the cost of 15 square yards of linoleum at $3.30 per square yard.
(1) $56.43 (2) $59.70 (3) $49.50 (4) $39.50
(5) none of these

____ 8. The school's baseball team bought 12 balls at $2.25 each and 9 bats at $2.45 each. What was the total amount spent?
(1) $50.95 (2) $49.05 (3) $39.00 (4) $49.50
(5) none of these

____ 9. Mr. Wilson earns $7,200.00 annually (yearly). He is paid on a monthly basis. How much does he earn each month?
(1) $600.00 (2) $500.00 (3) $700.00
(4) $650.00 (5) none of these

____ 10. At $.17 per pound, how many pounds of apples can you buy for $1.02?
(1) 5 (2) 7 (3) 6 (4) 8
(5) none of these

MEASURING YOUR PROGRESS

ANSWER KEY

TEST A

ADD OR SUBTRACT:

1,	2,1	2,	12,37	3,	7,014	4,	64,37
5,	1985,4	6,	,4	7,	2,5	8,	,281
9,	2,025	10,	81,80				

MULTIPLY AND DIVIDE

11,	,111	12,	1,518	13,	6,315	14,	37,544
15,	126,4655	16,	,61	17,	,0071	18,	89
19,	,05	20,	4,37	21,	150	22,	600

TEST B

1,	3	2,	2	3,	3	4,	1
5,	2	6,	2	7,	3	8,	2
9,	1	10,	3				

THE END

DECIMALS, FRACTIONS, PERCENTS

(For use in Pentagon pp. 113-115 and Heptagon pp. 26-50)

Lesson #1

Rule 1: Before you can change a decimal to a fraction, you need to be able to read a decimal. If you can read a decimal, then you will have no problem with changing decimals to fractions. Remember

.1 means tenths
.01 means hundredths
.001 means thousandths
.0001 means ten-thousandths
.00001 means hundred thousandths
.000001 means millionths

Step 1. Let's change .24 to a fraction. Do you know what .24 stands for in decimal form? Yes, it stands for 24 hundredths.

Step 2. Since .24 stands for 24 hundredths, we can express it in fraction form like this: .24 = 24/100

Step 3. Now, all we have to do is to reduce the fraction 24/100 to lowest terms.
We think:

(a) 4 will go into 24, 6 times.
(b) 4 will go into 100, 25 times.

therefore; we see that a common multiple of 24 and 100 is 4 and when 24/100 is reduced we get

$$\begin{array}{l} 4 \text{ into} \\ 4 \text{ into} \end{array} \frac{24}{100} = \frac{6}{25}$$

Step 4. We have just changed a decimal into a fraction and have proven that .24 equals 6/25.

Practice Exercises:

1. .05 =
2. .26 =
3. .04 =
4. .96 =
5. .08 =
6. .45 =
7. .125 =
8. .375 =
9. .625 =
10. .875 =
11. .015 =
12. .025 =

Lesson #2

(Changing Fractions to Decimals)

Rule 1: To change a common fraction to a decimal, divide the numerator (top number) by the denominator (bottom number).

Step 1. Let's take the fraction 1/16. You are asked to change the fraction 1/16 to a decimal form. The rule at the top of the page has informed you to divide the numerator by the denominator; therefore, we have to divide 1 by 16.
Example,

$$16\overline{)1.0}$$

Sixteen cannot be divided into 1.0, so we put a decimal and a zero in the quotient (answer).

Step 3. Now you have to add another zero.
Example,

$$\begin{array}{r} .06 \\ 16\overline{)1.00} \\ \underline{.96} \\ .04 \end{array}$$

Now, 16 will go into 1.00, 6 times; therefore, you put 6 in the answer. You take 6 times 16 which gives you 96. Then you subtract .96 from 1.00 which gives you 4.

Step 4. You now add another zero.

$$\begin{array}{r} .062 \\ 16\overline{)1.000} \\ \underline{96} \\ .040 \\ \underline{32} \\ 8 \end{array}$$

Now you take 16 into 40 which will go 2 times. You put your 2 in the quotient. 2 X 16 = 32. You subtract and have 8 left.

Step 5. You add another zero.

$$\begin{array}{r} .0625 \\ 16\overline{)1.0000} \\ \underline{96} \\ .040 \\ \underline{32} \\ 80 \\ \underline{\underline{80}} \end{array}$$

Now you take 16 into 80 which will go 5 times. You put 5 in your quotient. 5 X 16 = 80. You subtract and have nothing left. You now have changed the fraction 1/16 = .0625.

Rule 2: Always carry it out as far as you can; unless, you see it is not going to come out perfect such as the fraction 1/3. Then only carry it out to 2 places.

Example,

$$1/3 \quad = \quad \begin{array}{rl} & .33\ 1/3 \\ 3 & \overline{)\,1.00} \\ & \ \ \underline{9} \\ & \ \ 10 \\ & \ \ \ \underline{9} \\ & \ \ \ 1 \end{array}$$

Note: As you can see when you subtracted 9 from 10 you keep getting 1, therfore, it would be impossible for this to come out even. When you have a problem like this, carry it out two places and put the remainder over the divisor (the number you are dividing by). This is where the 1/3 comes from in .33 1/3.

Practice Exercises:

1. 1/8 =
2. 3/8 =
3. 19/20 =
4. 1/12 =
5. 3/4 =
6. 9/20 =

Lesson #3

(Changing Decimals to Per Cents)

Rule #3 To change a decimal to a per cent move the decimal two places to the right and add the % sign.

Examples,

.45 as a per cent = 45%
.125 as a per cent = 12.5%
2.50 as a per cent = 250%

Exercises: (Write the following decimals as per cents)

1. .06 .125 = 2.50 =

2. .25 = .625 = 3.50 =

3. .50 = .875 = 1.00 =

4. .75 = .175 = 1.50 =

Lesson #4

Rule 4: Any percent (other than a fraction) can be changed to a decimal just by removing the per cent sign and counting off two decimal places to the left.

Example,

50% = .50

If the per cent is already in a decimal form (12.5%), drop your per cent sign and move your decimal two places to the left.

Example,

12.5% = .125

If the per cent contains a fraction (2½%), you drop your per cent sign and move two places to the left. Remember, your fraction is not a whole number, therefore; you do not count it as being a place.

Example,

2½% = .02½

Exercises: (Write the following per cents as decimals)

1. 75% = 1.8% = 70% =

2. 6% = 3.2% = 90% =

3. 5.5% = 99% = 100% =

4. 50% = 6¼% = 150% =

Lesson #5

(Changing a Percent to a Fraction and a Decimal)

Rule 5: Whenever you have to change a per cent, that is in the form of a fraction ($\frac{1}{2}\%$) first change the per cent to a fraction, and then change the fraction to a decimal.

Example of Rule 5,

$$\frac{1}{2}\% = 1/200 = 200\overset{.005}{\overline{)1.00}}$$

Explanation of Rule 5:

Did you ever stop to think why the per cent sign looks like this? (%) Well, the reason for its two little zero's is to help you remember that per cent means hundredths. You also need to remember that you can never work math problems with per cents. You either have to change them to a fraction or you have to change them to a decimal.

Let's take a look at the example of Rule 5. You wee $\frac{1}{2}\%$ equals 1/200. Does that "blow your mind"! Well, don't let it, because all you have to remember is that you have to remove the per cent sign and change the denominator to hundredths. See, 1/4% = 1/400 as a fraction; 3/5% = 3/500 as a fraction. Now, wasn't that easy.

Guess what? You can now find the decimal form of 1/2% since you have changed it into the fraction form 1/200, and you don't have to worry about getting the correct decimal places. All you have to do to get the decimal form is to divide 1 by 200. $200\overline{)1}$ You were taught how to do this in Lesson #2.

Remember: By changing the 1/2% to a fraction form and then to a decimal form, it will be a great deal easier for you to get your correct decimal places.

Let's look at one more example! OK!

Fraction Form- Remove % sign, and change denominator to hundredths.	Decimal Form (Divide numerator by denominator)
3/4% = 3/400	$400\overline{)3.0000} = .0075$

Worksheet for Lesson #5

Change the per cent to a fraction then to a decimal form.

	Fraction Form	Decimal Form
1. 1/2% =		
2. 1/4% =		
3. 3/4% =		
4. 5/6% =		
5. 3/8% =		
6. 2/3% =		
7. 5/8% =		
8. 3/10% =		

Lesson #6

(Changing Mixed Per Cents to Fractions and to Decimals)

Rule 6: Whenever you have a mixed per cent such as (2½%) and you want to change it to a decimal form, first you should change it to an improper fraction (5/2%); then remove the per cent sign and change it to hundredths 5/200; and then divide 5 by 200.

$$200\overline{)5}$$

Example of Rule 6,

$$6\tfrac{1}{4}\% = 25/4\% = 25/400 = 1/16 = 16\overline{)1}$$

Step 1. 6¼% = 24/4%

Explanation of Step 1: To get 25/4%, you take 6 X 4 and add the numerator, which is 1. This is the same process as changing mixed fractions to improper fractions.

Step 2. 25/4% = 25/400

Explanation of Step #2: Remember! You can't work with percents; therefore, you have to change 25/4% to a fraction. This is done by removing the % sign and changing the denominator to hundredths.

Step 3. 25/400 = 1/16

Explanation of Step #3: To get 1/16 you have to reduce 25/400.

25/400 reduced by 25 = 1/16
25 goes into $\frac{25}{400} = \frac{1}{16}$
25 goes into

Step 4. $1/16 = 16\overline{)1}$

Explanation of Step #4: Now that we know the fraction form of 6¼% = 1/16 we can get the decimal form by dividing 1 by 16. (Explained in Lesson #2)

Practice Exercises: Change the mixed per cents to fractions and then to decimals.

Percent	Fraction	Decimal
1. 12½%		
2. 33 1/3%		

Practice Exercises (continued)

Percent	**Fraction**	**Decimal**
3. 1½%		
4. 87½%		
5. 62½%		

Lesson #7

(Finding the Per Cent of a Number)

Note: Remember "of" means "times" (multiply)
Example: 12½% of 1600 **Solution:** 12½% = 25/200 = 1/8
Example: 30% of 50 **Solution:** .30 X 50 = 15.00

Exercises:

1. 50% of 876 =
2. 33 1/3% of 375 =
3. 12½% of 1040 =
4. 40% of 360 =
5. 83 1/3% of 750 =
6. 87½% of 192 =
7. 16 2/3% of 852 =
8. 25% of 2184 =
9. 6¼% of 240 =
10. 4% of 1250 =
11. 62½% of 896 =
12. 66 2/3% of 693 =
13. 37½% of 1136 =
14. 8 1/3% of 288 =
15. 5% of 840 =
16. 2% of 825 =

Lesson #8

(Finding What Per Cent One Number is of Another)

Example: On a spelling test Jane has 18 words right out of 25. What per cent of her words did she have right?

Solution: This is similar to our problems in fractions of finding what part one number is of another. Suppose we ask what part of the words attempted did Jane get correct. Remember "of" means "times"; and "is" means "equals".
Now, our problem can be stated as; What part of 25 is 18?

$$? \times 25 = 18:\ \frac{18}{25} = \begin{array}{r} .72 \\ 25\overline{)18.00} \end{array} \text{ or } 72\%$$

1. What part of 20 is 16? ________ What per cent of 20 is 16? ________
2. What part of 25 is 20? ________ What per cent of 25 is 20? ________
3. What per cent of 72 is 18? ________
4. What per cent of 32 is 24? ________
5. Our team played 16 games and won 12. What per cent of the games did we win? ____________
6. 12 is what per cent of 96? ________
7. $12\frac{1}{2}$ is what per cent of 100? ________
8. What per cent of 100 is 16 2/3? ________
9. $87\frac{1}{2}$ is what per cent of 100? ________
10. Our team won 15 games and lost 5. What per cent of the games played did we win? ________ What per cent did we lose? ________
11. On a test Elsie had 18 problems right out of 24. Janet had 24 right out of 30. Who had the larger percentage right? ________________
12. Jack earned $40 and is going to put $16 in the bank. What per cent of his money is he going to put in the bank? ____________________

Use the Space Below for Figuring

Lesson #9

DIAGNOSTIC TEST IN PERCENTAGE

1. Writing of equivalents:
Underline the correct forms:

1/4 = .4; 4%; .25; 40%; 25%

3/8 = .37 1/2; 37.5%; 37.5; .375

1/2% = .5; .05; .005; .00 1/2

200% = 200; 2.00; .02; .2

2. Change to per cents:

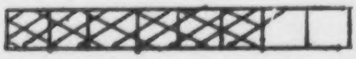
.5 __________; .05 __________; 1.25 __________; 7/10 __________;

3/4 __________; 5/8 __________; 1/3 __________; 3/20 __________

3. Write as both fractions and decimals:

16% ________ ________ 87 1/2% ________ ________ 45% ________ ________

4. What per cent is shaded?



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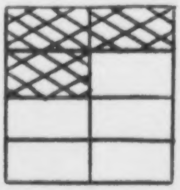

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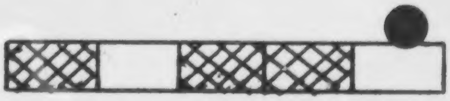

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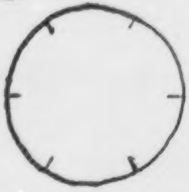
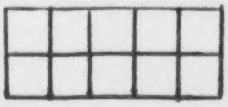
5. Shade:

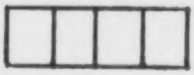
16 2/3%



90%



100%



6. Find:

36% of 45 ____________________ 6% of $75 ____________________

33 1/3% of $90 _______________ 12 1/2% of 48 _______________

4 1/2% of $300 _______________ 3/4% of 2000 ________________

100% of 25 ___________________ 150% of 40 __________________

7. What per cent of $25 is $12? ____________________________

16 is what per cent of 48? ______________________________

What per cent of $10 is $2.50? __________________________

300 is what per cent of 200? ____________________________

ANSWER SHEET FOR PERCENT HANDOUT

Lesson 1

1. 1/20
2. 13/50
3. 1/25
4. 24/25
5. 2/25
6. 9/20
7. 1/8
8. 3/8
9. 5/8
10. 7/8
11. 3/200
12. 1/40

Lesson 2

1. .125
2. .375
3. .95
4. .083 1/3 or .0833
5. .75
6. .45

Lesson 3

1. 6%
2. 25%
3. 50%
4. 75%
5. 12.5%
6. 62.5%
7. 87.5%
8. 17.5%
9. 250%
10. 350%
11. 100%
12. 150%

Lesson 4

1. .75
2. .06
3. .055
4. .50
5. .018
6. .032
7. .99
8. .06 1/4 or .0625
9. .70
10. .90
11. 1.00
12. 1.50

Lesson 5

1.	1/200	.005
2.	1/400	.0025
3.	3/400	.0075
4.	5/600	.0083 1/3
5.	3/800	.00375
6.	2/300	.0066 2/3
7.	5/800	.00625
8.	3/1000	.003

Lesson 6

1.	1/8	.125
2.	1/3	.33 1/3
3.	3/200	.015
4.	7/8	.875
5.	5/8	.625

ANSWER SHEET FOR PERCENT HANDOUT (continued)

Lesson 7

1. 438
2. 125
3. 130
4. 144
5. 625
6. 168
7. 142
8. 546
9. 15
10. 50
11. 560
12. 462
13. 426
14. 24
15. 42
16. 16.50

1. 4/5 80%
2. 4/5 80%
3. 25%
4. 75%
5. 75%
6. 12 1/2%
7. 12 1/2% or 12.5%
8. 16 2/3%
9. 87 1/2%
10. 75% 25%
11. Janet (note: Elsie had 75%, Janet had 80%)
12. 40%

Lesson 9

1. .25 25%

.37 1/2 37.5% .375

.005 .00 1/2

2.00

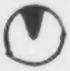
2. 50% 5% 125% 70% 75%

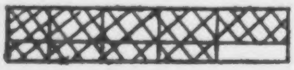
62 1/2% 33 1/3% 15%

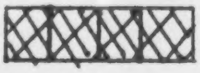
3. 4/25 .16 7/8 .87 1/2 9/20 .45

4. 75% 37 1/2% 60%

No 5







6. 16.20 4.5

$30. 6

13.50 15

25 60

7. 48%

33 1/3%

25%

150%

THE END

INTRODUCTION TO MEASUREMENTS

(For use with Hexagon pp. 94-100)

Units of measures and weights

The letters in parentheses are short ways of writing the unit of measures, weights, and time.

TIME

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
(about) 4 weeks = 1 month (mo.)
12 months = 1 year (yr.)
365 days = 1 year (yr.)
366 days = 1 leap year
(leap year occurs every four years by adding 1 extra day in February.)

LENGTH

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
16 1/2 feet = 1 rod (rd.)
5 1/2 yd. = 1 rod
5,280 feet = 1 mile (mi.)
1,760 yards = 1 mile
320 rods = 1 mile

LIQUID MEASUREMENTS

4 gills = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

DRY MEASUREMENTS

2 pints = 1 quart
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

WEIGHT

16 ounces (oz.) = 1 pound (lb.)
100 lb. = 1 hundredweight (cwt.)
2000 lb. = 1 Ton (T.)
20 cwt. = 1 Ton
2240 lb. = 1 long Ton

Do you know what time it is now? Do you know how long it takes to get to work in the morning? How many hours do you sleep?

When you answer these questions you are measuring time.

$$\begin{array}{lrr} & 10 \text{ min.} & 25 \text{ sec.} \\ + & 15 \text{ min.} & 40 \text{ sec.} \\ \hline & & 65 \text{ sec.} \end{array}$$

Step 1: Add the seconds:
$25 + 40 = 65$

60 sec. = 1 minute

Step 2: Think how many seconds are in a minute

$$\begin{array}{rl} & 1 \text{ min.} \\ 60 & -65 \\ & \underline{60} \\ & 5 \text{ secs.} \end{array}$$

Step 3: Change 65 seconds into 1 min. 5 secs.

$$\begin{array}{lrl} & 1 \text{ min.} & \\ + & 10 \text{ min.} & 25 \text{ sec.} \\ & 15 \text{ min.} & 40 \text{ sec.} \\ \hline & 26 \text{ min.} & 65 \\ & & \underline{-60} = 1 \text{ min.} \\ & & 5 \text{ sec.} \end{array}$$

Step 4: Add the 1 minute to the other minutes

Answer is:

$$\begin{array}{rrl} 1 \text{ min.} & & \\ 10 \text{ min.} & 25 \text{ sec.} & \\ 15 \text{ min.} & 40 \text{ sec.} & \\ \hline 25 \text{ min.} & 65 \text{ sec.} & = 26 \text{ min. } 5 \text{ sec.} \end{array}$$

In adding measures we have dealt with one measure at a time. Now we are going to deal with two or more of the measures at the same time, such as gallons and quarts or minutes and seconds.

Problems such as these are common everyday life.

Example: It takes 1 hour and 40 minutes to go from Washington to Maplewood and 2 hours and 45 minutes to go from Maplewood to Mount Vernon. How long does it take to go from Washington to Mount Vernon?

$$\begin{array}{lllrr} \text{The process:} & \text{Washington to Maplewood} & = & 1 \text{ hr.} & 40 \text{ min.} \\ & \text{Maplewood to Mt. Vernon} & = & \underline{2 \text{ hr.}} & \underline{45 \text{ min.}} \\ \text{(Adding)} & \text{Washington to Mt. Vernon} & = & 3 \text{ hr.} & 85 \text{ min.} = \\ & & & 4 \text{ hr.} & 25 \text{ min.} \end{array}$$

Since there are 60 minutes in one hour, 85 minutes are equal to 1 hour and 25 minutes. This added to the 3 hours gives us 4 hours and 25 minutes.

3 hours 85 minutes

60 min. = 1 hr.   85 min. - 60 min. = 1 hour and 25 minutes

$$\begin{array}{l} {}_{+}3 \text{ hours} \\ \underline{1 + 25 \text{ min.}} \\ 4 \text{ hours } \; 25 \text{ minutes} \end{array}$$

Measuring Length:

Which is longer 4 feet or 1 yard? Again you add and then change to larger units.

$$\begin{array}{r} {}_{+}1 \text{ yard } 2 \text{ feet} \\ \underline{3 \text{ yard } 2 \text{ feet}} \\ 4 \text{ feet} \end{array}$$

Step 1: Add the feet:
2 + 2 = 4

3 feet = 1 yard

Step 2: Think how many feet are in a yard

$$\begin{array}{r|l} & \underline{1 \text{ yd.}} \\ 3 & 4 \text{ ft.} \\ & \underline{3} \\ & 1 \text{ ft.} \end{array}$$

Step 3: Change 4 feet into 1 yard and 1 foot

$$\begin{array}{ll} {}_{+}1 \text{ yard} & 2 \text{ feet} \\ \underline{3 \text{ yard}} & \underline{2 \text{ feet}} \\ 5 \text{ yard} & 4 \text{ feet} \\ & \underline{-3 = 1 \text{ yard}} \\ & 1 \text{ foot} \end{array}$$

Step 4: Add the 1 yard to the other yards

Answer:

$$\begin{array}{ll} {}_{+}1 \text{ yard} & 2 \text{ feet} \\ \underline{3 \text{ yard}} & \underline{2 \text{ feet}} \\ 4 \text{ yard} & 4 \text{ feet} = 5 \text{ yard } 1 \text{ foot} \end{array}$$

Add the following in the same manner:

1. $\begin{array}{r} 5 \text{ gal. } \; 2 \text{ qt.} \\ \underline{2 \text{ gal. } \; 3 \text{ qt.}} \end{array}$

2. $\begin{array}{r} 4 \text{ pk. } 4 \text{ qt.} \\ \underline{2 \text{ pk. } 5 \text{ qt.}} \end{array}$

3. $\begin{array}{r} 4 \text{ hr. } \; 30 \text{ min.} \\ \underline{5 \text{ hr. } \; 50 \text{ min.}} \end{array}$

4. 5 yd. 1 ft.
3 yd. 2 ft.

5. 22 hrs. 30 mins.
10 hrs. 40 mins.

6. 1 yd. 2 ft. 4 in.
2 yd. 1 ft. 9 in.

7. 3 hr. 3 min. 18 sec.
5 hr. 56 min. 42 sec.

8. 2 lb. 6 oz.
3 lb. 12 oz.
4 lb. 14 oz.

Word Problems

1. Yesterday you bought 1 yard 3 inches of wool. Today you bought 2 yards 6 inches of cotton and 2 yards of velvet. How much material have you bought altogether?

2. You buy 2 pieces of lumber to make some shelves. The first piece is 1 foot 5 inches long. The second piece is 1 foot 7 inches long. How much lumber have you bought?

3. In the morning, you spend 15 minutes washing and 10 minutes dressing. It takes you 20 minutes to eat breakfast. You walk 15 minutes to work. How much time do you spend before work?

Subtraction of Measures

Measures are subtracted one unit at a time. You can borrow from a larger unit to subtract a smaller unit.

$$\begin{array}{rr} 3\text{ hr.} & 10\text{ min.}\\ -1\text{ hr.} & 49\text{ min.}\\ \hline \end{array}$$

Step 1: Borrow 1 hour and change it to minutes.
1 hour = 60 minutes

$$\begin{array}{r} 60\\ +10\\ \hline \end{array}$$

Step 2: Add the 60 minutes to 10 minutes you have already.

$$\begin{array}{rr} 2 & 70\\ -\cancel{3}\text{ hr.} & \cancel{10}\text{ min.}\\ 1\text{ hr.} & 49\text{ min.}\\ \hline & 21\text{ min.} \end{array}$$

Step 3: Subtract the minutes.

$$\begin{array}{rr} 2 & 70\\ \cancel{3}\text{ hr.} & \cancel{10}\text{ min.}\\ -1\text{ hr.} & 49\text{ min.}\\ \hline 1\text{ hr.} & 21\text{ min.} \end{array}$$

Step 4: Now subtract the hours.

The answer is 1 hour and 21 minutes.

Example:

Bill is 22 years and 10 months old. Mary is 20 years and 11 months old. How much younger is Mary?

The process:

Since 11 months cannot be subtracted from 10 months, we must change 22 yr. 10 mo. to 21 years 22 mo. by renaming 1 yr.

Bill's age = 22 yr. 10 mo. = 21 yr. 22 mo.
Mary's age = 20 yr. 11 mo. = 20 yr. 11 mo.
Difference 1 yr. 11 mo.

Subtract:

1. $$\begin{array}{rr} 24\text{ lb.} & 8\text{ oz.}\\ 10\text{ lb.} & 12\text{ oz.}\\ \hline \end{array}$$

2. $$\begin{array}{rr} 8\text{ hrs.} & 20\text{ min.}\\ 4\text{ hrs.} & 40\text{ min.}\\ \hline \end{array}$$

3. $$\begin{array}{rrr} 12\text{ bu.} & 2\text{ pk.} & 6\text{ qt.}\\ 8\text{ bu.} & 3\text{ pk.} & 5\text{ qt.}\\ \hline \end{array}$$

4. 6 yd. 2 ft. 9 in.
2 yd. 1 ft. 4 in.

5. 7 qt.
2 qt. 1 pt.

6. 9 ft. 6 in.
2 ft. 8 in.

7. 4 gal. 1 qt.
3 gal. 2 qt.

8. 10 min. 15 sec.
5 min. 40 sec.

1. You have 3 hours to do some shopping. If you spend 1 hr. and 20 min. in one store, how much time do you have left to shop?

2. You have 2 lbs. 2 oz. of ground chuck. A recipe calls for 1 lb. 8 oz. of ground chuck. How much will you have left over?

3. Jim Jackson runs a mile in 7 min. 45 sec. Bob Smith runs a mile in 8 min. 20 sec. How much faster can Jim Jackson run the mile?

Multiplication of Measures

Units of time, length, weight and liquid measures are multiplied separately.

Example:

2 hr. 40 min. 22 sec.
x 3
6 hr. 120 min. 66 sec.

Step 1: Multiply each unit of time separately.
3 x 22 = 66; 3 x 40 = 120;
3 x 2 = 6

2 hr. 40 min. 22 sec.
x 3
66 sec. = 1 min. 6 sec.

Step 2: Is to regroup the measures.
66 sec. - 1 min. 6 sec.

2 hr. 40 min. 22 sec.
x 3
6 hr. 120 min. 66 sec. = 1 min. 6 sec.
+ 1
121 min.

Add the 1 min. to the 120 min.

2 hr. 40 min. 22 sec.
x 3
6 hr. 121 min. 6 sec.
+ 2 hr.-120
8 hr. 1 min. 6 sec.

Regroup again; 121 min. =
2 hr. 1 min.

Think how many minutes are in one hour then find how many hours are in 121 min.

$$\begin{array}{r} \text{2 hrs. 1 min.} \\ 60\overline{)121} \\ \underline{120} \\ 1 \end{array}$$

Now work the problem.

2 hr. 40 min. 22 sec.
x 3
6 hr. 120 min. 66 sec. =
8 hr. 1 min. 6 sec.

Solve these problems by Multiplying:

1. 4 lb. 2 oz.
x 4

2. 8 lb. 6 oz.
x 5

3. 7 ft. 3 in.
x 4

4. 4 mi. 80 rod
x 4

5. 4 yd. 18 in.
x 6

6. 6 qt. 1 pt.
x 4

7. 8 gal. 1 pt.
x 12

8. 6 T. 500 lb.
x 8

9. 3 yd. 2 ft.
x 4

1. It takes 30 minutes to do one load of wash in an automatic washing machine. How long will it take to do 4 loads?

2. You have 3 pieces of wire. Each piece of wire is 2 yd. 1 ft. 4 in. long. What is the total length of the wire you have.

Dividing Measures

Units of time, length, weight, and liquid measure are divided separately.
Look at the example below.

```
    4
3  13 hr.  15 min.
   12
    1 hr.
```

Step 1: Divide 3 into 13 hr.
Then change the 1 hour you have
left over into minutes.
Think: How many minutes in
1 hour

```
    4 hr.
3  13 hr.  15 min.
  -12      +60
    1 hr. =75 min.
```

Step 2: Then add to the 15 minutes.
Now its like a whole new
division problem.

```
    4 hr.   2
3  13 hr.  15 min.
   12      60
    1      75
            6
           15
```

Step 3: Divide 3 into 7, subtract
6 from 7, bring down the 5 along
with the one.

```
    4 hr.  25 min.
3  13 hr.  15 min.
   12      60
    1      75
            6
           15
           15
```

Then divide 3 into 15 to get
your answer.

Example 2:

Gladys lives 8 miles from Martha. She drives this distance in 12 minutes
24 seconds. How long does it take her to go a mile:

Process:

```
     1 min.   33 sec.
8   12 min.   24 sec.
     8       240
     4       264
              24
              24
```

8 into 12 = 1 remainder 4
4 min. = 240 sec.
240 + 24 = 264
8 into 264 = 33

Solve these problems:

1. $5\overline{)\text{11 min. 5 sec.}}$

2. $6\overline{)\text{7 gal. 2 qt.}}$

3. $4\overline{)\text{5 T. 400 lb.}}$

4. $9\overline{)\text{10 da. 21 hr.}}$

5. $3\overline{)\text{5 bu. 1 pk.}}$

6. $7\overline{)\text{9 yd. 1 ft.}}$

7. A new born baby needs to be fed 6 times a day. How many hours are there between feedings?

8. You have 4 lb. 2 oz. of meat. You want to use this meat for 3 meals. How much meat will you serve at each meal.

THE METRIC SYSTEM

The metric system is based on the use of decimals. The principal unit of length is called the meter.

Area is measured in square meters.

Volume is measured in cubic meters.

Capacity is measured in liters.

Weight is expressed in grams.

Smaller units are expressed by adding prefixes:

Deci - 1/10 (one-tenth)
Centi - 1/100 (one-hundredth)
Milli - 1/1000 (one-thousandth)

Larger quantities are expressed by adding prefixes meaning:

Deca - 10 (ten)
Hecto - 100 (hundred)
Kilo - 1000 (thousand)

Instruction: The metric system of measure is based upon units of ten. The basic unit of length in the metric system is the meter (m). Notice that the symbol (m) for meter is written without a period, as are all metric symbols.

Units of length larger and smaller than a meter are formed by adding prefixes to the basic unit.

The table below show the prefixes and their meanings.

Prefixes		Meaning		Example
kilo	-	1,000 times basic unit	-	1 kilometer (km) = 1,000 meter
hecto	-	100 times basic unit	-	1 hectometer (hm) = 100 meter
deka	-	10 times basic unit	-	1 dekameter (dkam) = 10 meter
deci	-	.1 times basic unit	-	1 decimeter (dm) = .1 meter
centi	-	.01 times basic unit	-	1 centimeter (cm) = .01 meter
milli	-	.001 times basic unit	-	1 millimeter (mm) = .001 meter

During gradual conversion from the English system of measurement to the metric system, you may find it necessary to convert measures from one system to the other.

The table on the next page, lists the conversion factors for most common measures in each system.

To use the table, find the unit you know in the left-hand column and multiply it by the given number in the middle column. The answer will be the approximate number of units shown in the right hand column.

(When you know) -	(Multiply by) -	(To find)
Length and Distance		
inches (in.)	2.5	centimeters (cm)
inches (in.)	25	millimeters (mm)
feet (ft.)	30	centimeters
yards (yd.)	0.9	meters (m)
miles (mi.)	1.6	kilometers (km)
millimeters (mm)	0.04	inches
centimeters (cm)	0.4	inches
meters (m)	3.3	feet
meters	1.1	yards
kilometers (km)	0.6	miles
Area		
square inches (sq. in.)	6.5	square centimeters
square feet (sq. ft.)	0.09	square meters
square yards (sq. yd.)	0.8	square meters
square miles (sq. mi.)	2.6	square kilometers
acres	0.4	hectares
square centimeters (cm^2)	0.16	square inches
square meters (m^2)	1.2	square yards
square kilometers (km^2)	0.4	square miles
hectares (ha)	2.5	acres
Capacity (Liquid)		
teaspoon (tsp.)	5	milliliters (ml)
tablespoon (tbsp.)	15	milliliters
fluid ounces (fl. oz.)	30	milliliters
cups (c.)	0.24	liters (l)
pints (pt.)	0.47	liters
quarts (qt.)	0.95	liters
gallons (gal.)	3.8	liters
milliliters (ml)	0.03	fluid oz.
liters (l)	2.1	pints
liters (l)	1.06	quarts
liters (l)	0.26	gallons
Mass (Weight)		
ounces (oz.)	28	grams (g)
pounds (lb.)	0.45	kilograms (kg.)
tons (T)	0.9	metric tons
grams (g)	0.035	ounces
kilograms (kg)	2.2	pounds
metric ton (t)	1.1	tons
Temperature		
degrees Fahrenheit (°F)	5/9 (after subtracting 32)	degrees Celsuis (C)
degrees Celsuis (°C)	9/5 (then add 32)	degrees Fahrenheit

A. WORD PROBLEMS

1. From Chicago to New York is 908.2 miles. From Chicago to Pittsburgh, en route, is 468.4 miles. How far is Pittsburgh from New York in kilometers?

2. Our car has a gas tank which measures 42 ins. by 11 ins. by 15 ins. There are 231 cu. ins. equaling 1 gallon. How many liters will it hold?

3. Allowing 40 cu. ft. to the ton, how many metric tons of coal are there in a bin measuring 60 ft. by 30 ft. by 25 ft.?

4. A train traveling a certain speed must have 0.7 kilometer in which to stop. How far is the stopping distance in meters?

5. Normal body temperature is 98.6° F. What is normal body temperature in Celsuis degrees?

6. A desk is 1.5 meters long. What is the length in centimeters?

7. What is the volume of a box which measures 1 meter by 2 meters by 0.5 meters?

8. A liter contains 1,000 milliliters. How many milliliters would be in three-fourths liter?

9. Dick caught one fish weighing 2 3/4 lb. and another fish weighing 6 2/3 lbs. How much did the two fish weigh in kilograms together? (Round off to nearest hundredths).

10. Carpeting is to completely cover a floor which measures 18 ft. by 39 ft. How many square yards are needed? How many square meters is this?

_______________ _______________

11. Jan jogged 2 kilometers this morning. How many meters is that?____________

12. Find the cost of 4 meters of dress material if it cost $2.20 per meter?__________

B. Convert to metric:

Add: 1. +5 gal. 5 qt.
2 gal. 3 qt.

2. +2 yds. 5 ft.
6 yds. 4 ft.

Multiply: 3. 20 mi.
x 4

4. 22 lbs.
x 14

Subtract: 5. − 88 mi.
33 mi.

6. − 75 mi.
25 mi.

ANSWER SHEET

A.

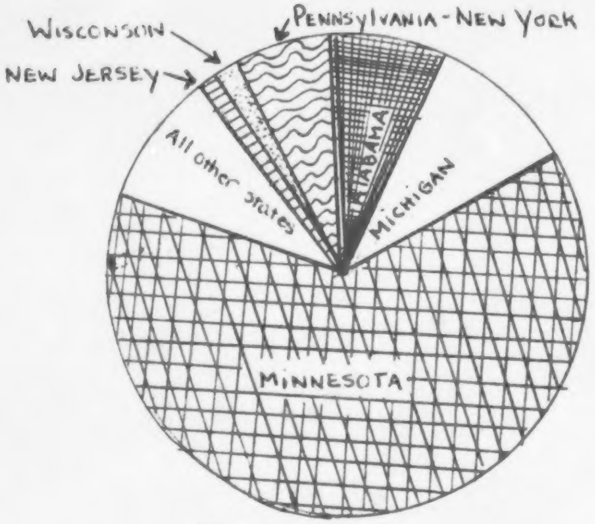
1. Multiply by 1.6 = 703.7 kilometers
2. 114.0 liters
3. 1,012.5 metric tons
4. 700 meters
5. 37° C
6. 150 centimeters
7. 1 m^3 (1 x 2 x .5 = 1 cubic meter)
8. 750 milliliters
9. 4.2375 kilograms or (4.24 kilograms)
10. 1st answer = 78 sq. yds. 2nd answer = 62.4 sq. meters
11. 2,000 meters
12. $8.80

B.

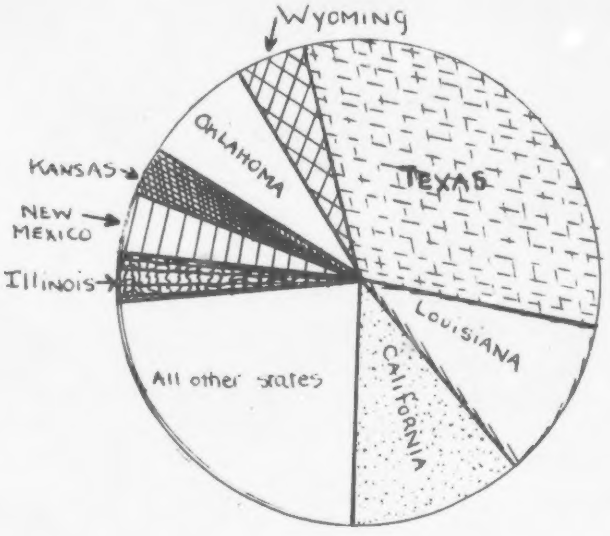
1. 34.2 liters
2. 9.9 meters
3. 128 kilometers
4. 138.6 kilograms
5. 88 kilometers
6. 80 kilometers

USE OF GRAPHS
(For use with Hexagon pp. 113-117)

IRON ORE AND OIL PRODUCTION IN THE UNITED STATES



IRON ORE



Oil

Number your paper from 1 to 20. Read the questions below. After each question are several answers. Look at the graph to see which answer is correct. Then write its letter on your paper after the number of the question. Be sure to read all the answers before you decide which one is correct.

1. The graphs shown are A. bar graphs B. line graphs C. circle graphs. D. picture graphs

2. The kinds of production compared in the graphs are A. iron and steel B. iron and coal C. oil and steel D. iron ore and oil

3. The iron ore and oil shown in these graphs are produced in A. six states B. the United States C. eight states D. fourteen states

4. The kind of production that each graph shows is found in A. the graph at the left B. each part of each graph C. the subtitles D. the graph at the right

5. The whole graph at the left stands for A. seven states in the United States. B. seven kinds of iron ore being produced C. the total iron ore production in the United States D. one million tons of iron ore

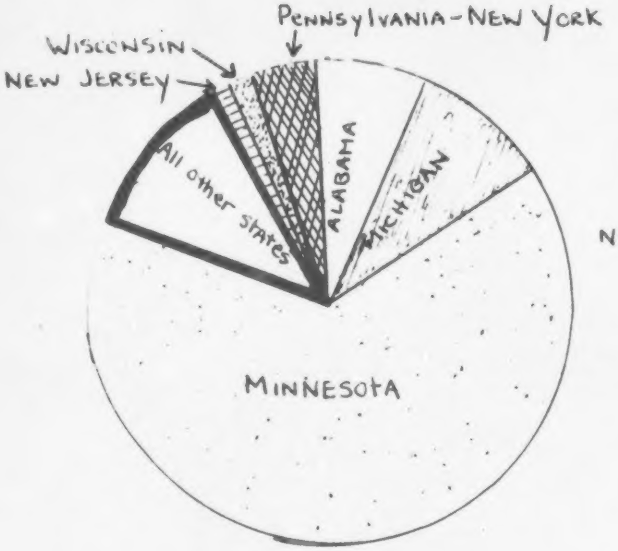
6. How many states produce oil? A. nine B. seven C. fifty D. You can't tell from the graph.

7. The graph that stands for iron ore production shows A. seven equal parts B. seven parts C. nine parts D. nine equal parts.

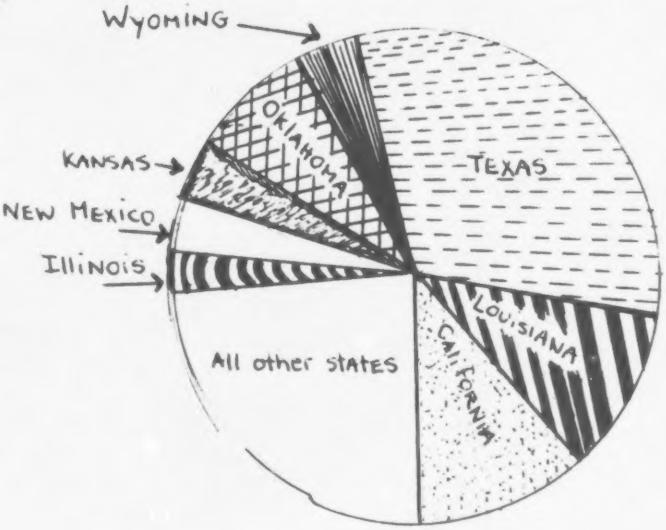
8. Which state had about the same share of both iron ore and oil production? A. Illinois B. New Mexico C. Oklahoma D. You can't tell from the graphs.

9. How many states shown in the graph had the same share of the total iron ore production? A. none B. three C. four C. two

10. The state that produced the most oil was A. Texas B. California C. Louisiana D. Oklahoma



Iron Ore



Oil

11. The state shown in the graph that produced the least iron ore was A. New Jersey B. Alabama C. Michigan D. Wisconsin

12. How many states produced both oil and iron ore? A. fifty B. sixteen C. none D. You can't tell from the graphs.

13. The state that produced about two-thirds of the iron ore in the United States was A. Michigan B. Minnesota C. Alabama D. Wisconsin

14. Almost one-third of the oil produced in the United States came from A. California B. all other states C. Louisiana D. Texas

15. The two states that together produced nearly one-fourth of the oil were A. Texas and Wyoming B. California and Louisiana C. California and Oklahoma D. Texas and Louisiana

16. Alabama produced about the same amount of iron ore as the total amount produced by New Jersey, Pennsylvania-New York, and A. Michigan B. all other states C. Wisconsin D. Minnesota

17. California produced about as much oil as did A. Texas B. Louisiana C. New Mexico D. Wyoming

18. Louisiana produced about the same amount of oil as the total amount produced in Kansas, New Mexico, and A. California B. Illinois C. all other states D. Texas

19. Which state produced the smallest amount of oil? A. Illinois B. New Mexico C. Kansas D. You can't tell from the graph.

20. Michigan produced about the same amount of iron ore as did A. Alabama B. Pennsylvania-New York C. Wisconsin D. all of these together.

POULTRY AND EGG PRODUCTION: 1945 to 1959

Year	Chickens		Eggs		Turkeys	
	Number produced (1,000)	Average price per pound (cents)	Number produced (1,0000,000)	Average price per dozen (cents)	Number produced (1,000)	Average price per pound (cents)
1945	799, 102	26	56,221	38	42,470	34
1946	646, 490	28	55,962	38	39,746	36
1947	635, 729	26	55,384	45	33,693	36
1948	536, 107	30	54,899	47	31,346	47
1949	622, 833	25	56,154	45	41,019	35
1950	535, 266	22	58,954	36	44,134	33
1951	539, 549	25	58,063	48	53,055	38
1952	473, 381	22	58,068	42	62,117	34
1953	463, 891	22	57,891	48	59,626	34
1954	454, 848	17	58,933	37	67,507	29
1955	377, 710	19	59,496	39	65,410	30
1956	392, 524	16	60,877	39	76,527	27
1957	309, 780	14	60,436	36	80,967	23
1958	351, 006	14	60,681	38	78,124	24
1959	318, 103	11	62,401	31	81,860	24

Number your paper from 1 to 20. Read each sentence below. Fill in the blank to make the sentence complete. Look at the table to find your answer.

1. The number of chickens produced in 1945 was ________________.

2. The average price of a pound of chicken in 1958 was __________.

3. The number of turkeys produced in 1955 was ___________________.

4. The average price per pound of turkey in 1952 was __________.

5. The average price of a dozen eggs in 1950 was _________________.

6. From 1945 to 1959 the average price of a pound of chicken reached a low of 11 cents and a high of ________.

7. The number of turkeys produced in 1946 was ___________________.

8. In 1958 the average price of a pound of turkey was ____ cents more than the average price of a pound of chicken.

9. The average price of a pound of chicken in 1953 was __________.

10. The number of eggs produced in 1947 was _____________________.

11. In 1959 the average price of a pound of turkey was _____cents more than the average price of a pound of chicken.

12. The number of turkeys produced in 1952 was __________________.

13. In 1950 the average price of a pound of chicken was ____cents less than the average price of a pound of turkey.

14. The number of eggs produced in 1959 was ___________________.

Year	Chickens		Eggs		Turkeys	
	Number produced (1,000)	Average price per pound (cents)	Number produced (1,0000,000)	Average price per dozen (cents)	Number produced (1,000)	Average price per pound (cents)
1945	799,102	26	56,221	38	42,470	34
1946	646,490	28	55,962	38	39,746	36
1947	635,729	26	55,384	45	33,693	36
1948	536,107	30	54,899	47	31,346	47
1949	622,833	25	56,154	45	41,019	35
1950	535,266	22	58,954	36	44,134	33
1951	539,549	25	58,063	48	53,055	38
1952	473,381	22	58,068	42	62,117	34
1953	463,891	22	57,891	48	59,626	34
1954	454,848	17	58,933	37	67,507	29
1955	377,710	19	59,496	39	65,410	30
1956	392,524	16	60,877	39	76,527	27
1957	309,780	14	60,436	36	80,967	23
1958	351,006	14	60,681	38	78,124	24
1959	318,103	11	62,401	31	81,860	24

15. In 1947 the number of chickens produced was __________ more than the number of turkeys produced.

16. The next year to appear on this table would be ________________.

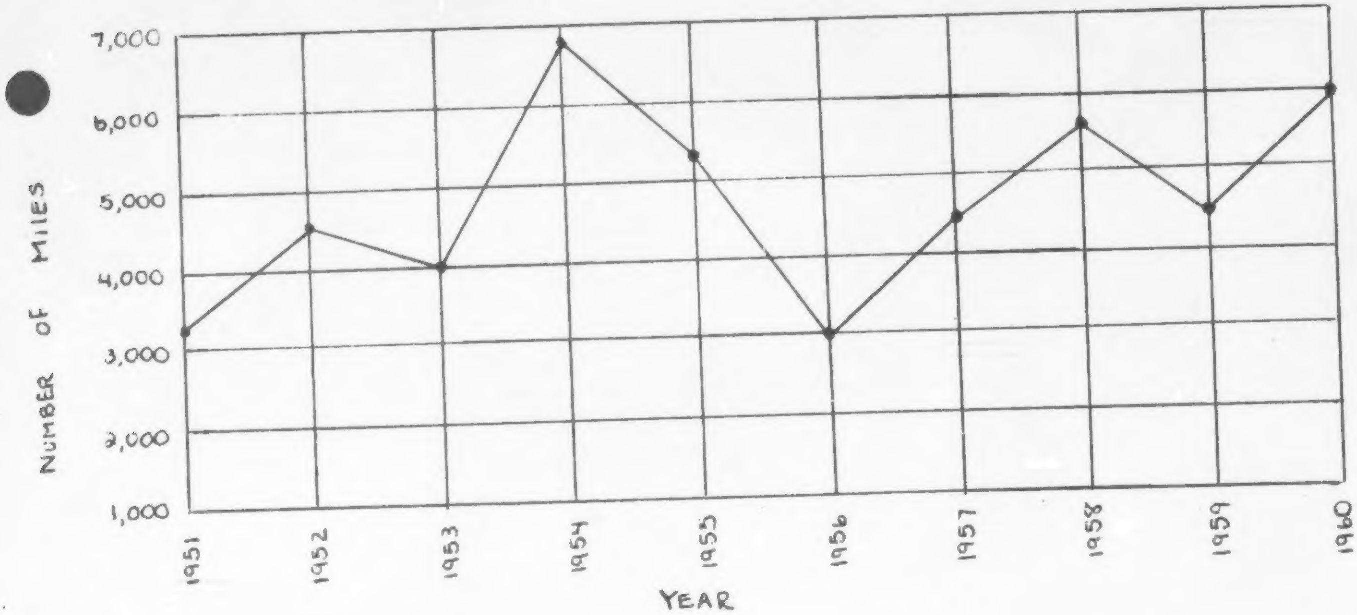
17. Rounded off to the nearest million, the number of turkeys produced in 1956 was ________________.

18. The average price of a pound of chicken dropped below 20 cents for ___________ different years.

19. The year in which the fewest chickens were produces was________.

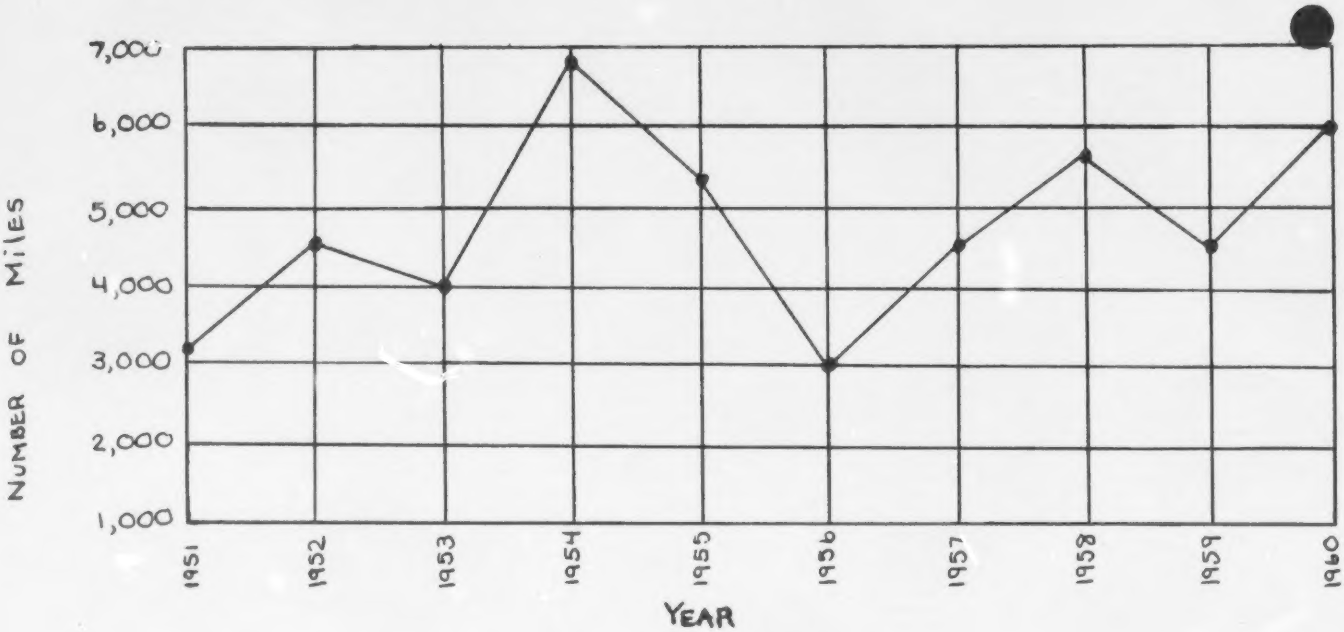
20. The average price of a pound of turkey dropped below 30 cents for ________ different years.

MILES FRED'S FATHER TRAVELS PER YEAR



Number your paper from 1 to 20. Read each sentence below. Fill in the blank to make the sentence complete. Look at the graph to find your answer.

1. You know from the title that the graph will be about the __________ that Fred's father traveled.
2. This graph could show up to _______ miles traveled in one year.
3. The years compared are _______ to __________.
4. The vertical scale is divided into units of ________________.
5. Fred's father traveled over 5,000 miles a year during each of ______ years.
6. During 1957 Fred's father traveled about ___________ miles.
7. During the years shown, Fred's father did not travel less than ___________ miles in any one year.
8. The greatest number of miles traveled in any one year was about __________.
9. Fred's father traveled 4,000 miles in _____________.
10. Fred's father traveled about 5,300 miles in __________.
11. Fred's father traveled about 5,600 miles in ___________.
12. The same number of miles was traveled in the years _______, ______, and ______.
13. In 1954, Fred's father traveled about _________ miles more than in 1956.
14. A distance of about ________ miles was traveled in 1951.
15. The difference in the number of miles traveled in 1960 and the number of miles traveled in ________ is 2,000.



16. Fred's father traveled about _________ more miles in 1954 than in 1959.

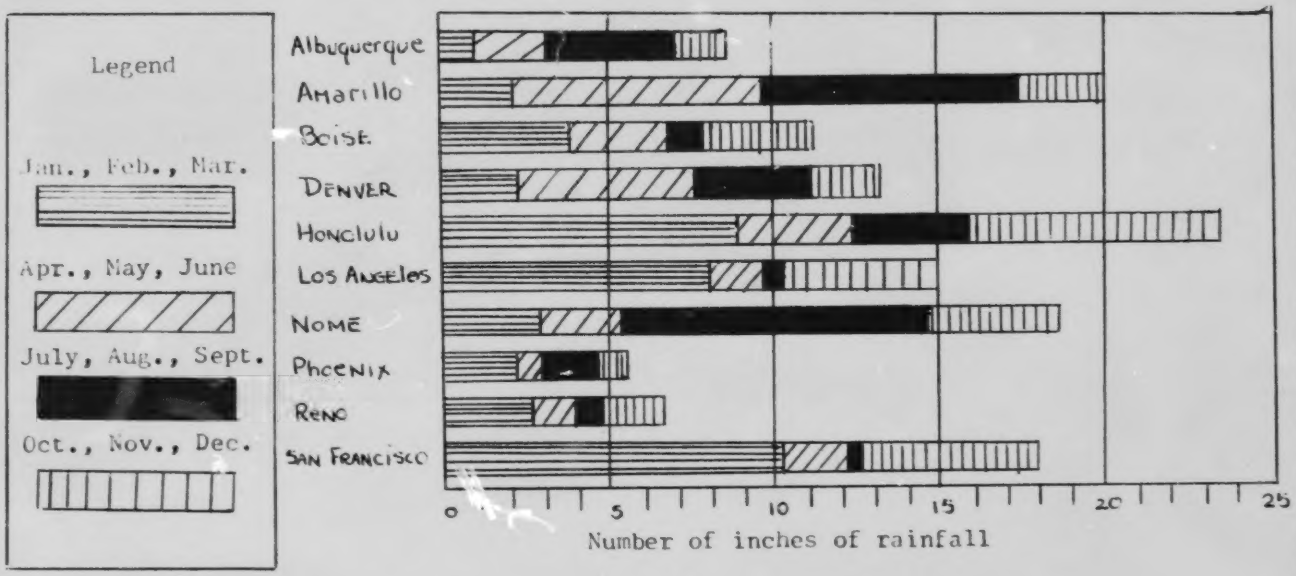
17. The number of miles traveled in ______ was twice the number traveled in 1956.

18. The total number of miles Fred's father traveled in 1951 and 1956 was about ____________________.

19. The number of miles traveled in ________ was more than twice the number traveled in 1951.

20. The difference between the number of miles traveled in 1958 and the number of miles traveled in 1959 was about ____________________.

AVERAGE YEARLY RAINFALL



Number your paper from 1 to 20. Read each sentence below. Fill in the blank to make the sentence complete. Look at the graph to find your answer.

1. The graph shows the average yearly rainfall for ________ cities.

2. The scale line on the horizontal axis is divided into units of ____________.

3. The solid part of each bar shows the average rainfall for each city during the months of _______, _________, and ________________.

4. By reading the legend you know that the part of each bar with horizontal lines stands for the months of __________, ____________, and ______________.

5. The city that gets the most rain during the year is ___________________.

6. _______________ has the least rainfall during the year.

7. During the last three months of the year, the city that gets the most rain is _______________.

8. The city that gets the most rain during the first three months of the year is ____________________________.

9. During the months of July, August, and September, the city with the most rain is _____________________.

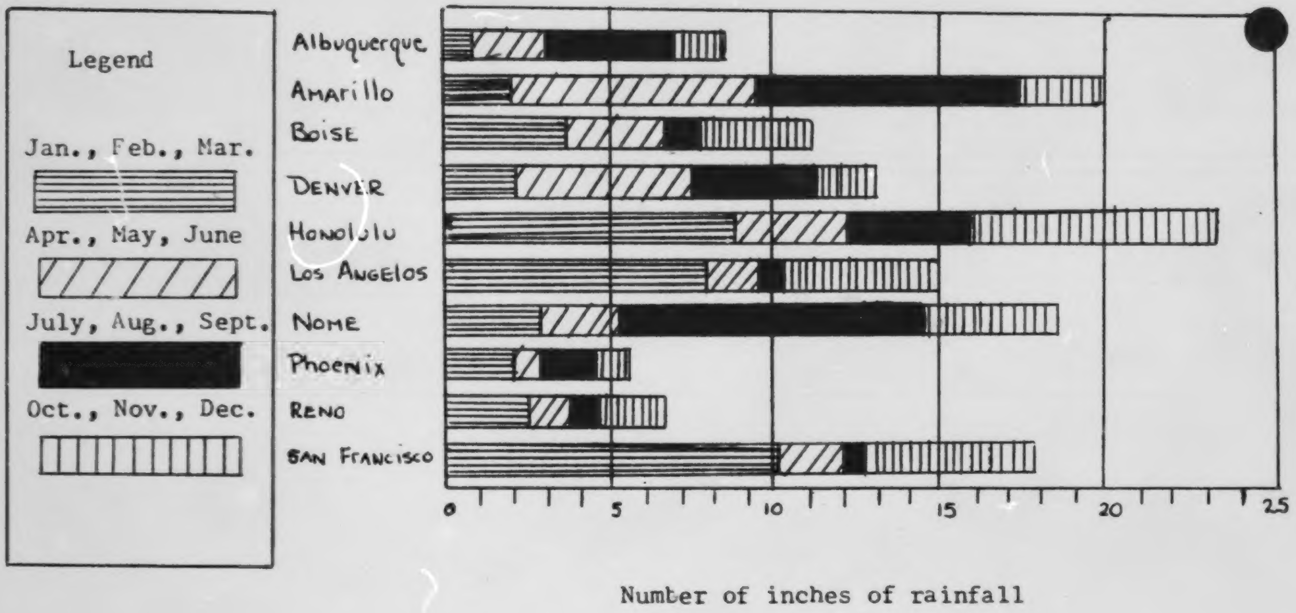
10. Amarillo has about ________ inches of rain during April, May and June.

11. The city with the least rainfall during July, August, and September is ______ ____________________.

12. The rainiest months in Reno are _______, ___________, and ______________.

13. ________ cities have a yearly average of less than 15 inches of rain.

14. The average yearly rainfall in ______ cities is between 10 and 15 inches.



15. The two cities that have most nearly the same amount of rain each year are ________ and ___________________.

16. The city that gets the least rain during January, February, and March is ___________________________.

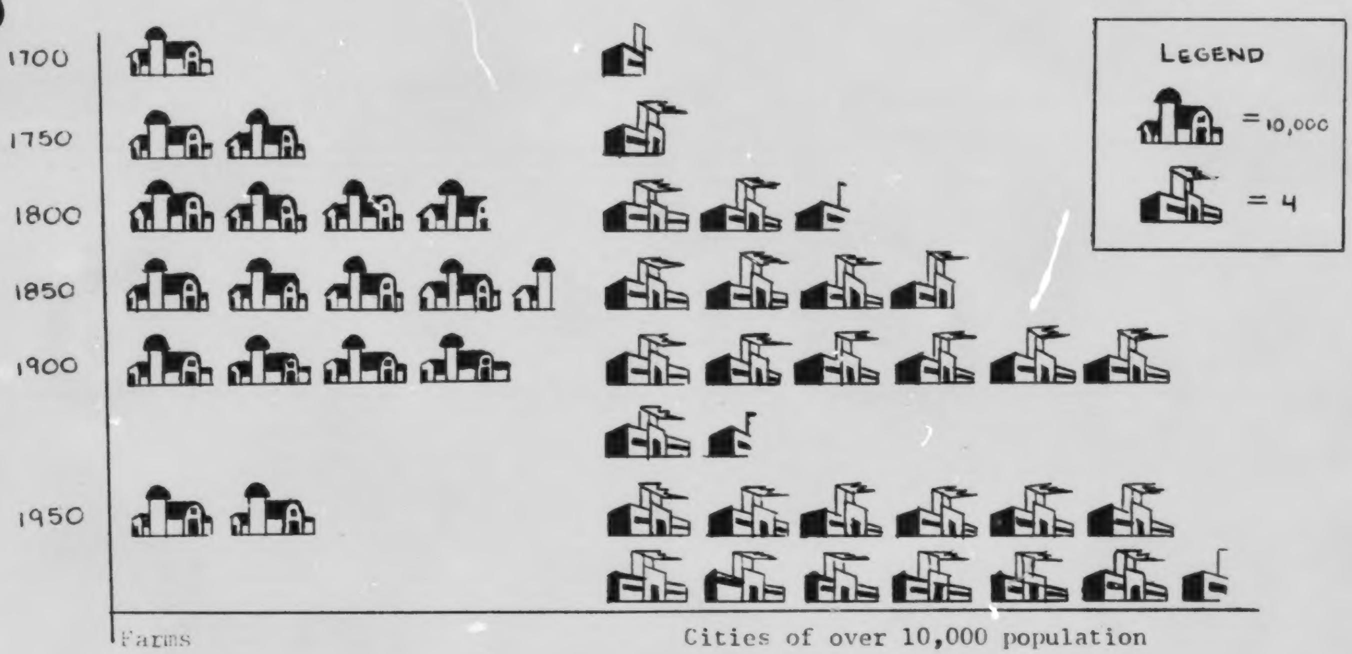
17. The three wettest months for Boise are the first three. Then it rains about ____________ inches during the rest of the year.

18. The average amount of rainfall in Nome during July, August, and September is about the same as for the other nine months together. That amount is about __________________ inches.

19. __________________ gets about one-third as much rain each year as Honolulu.

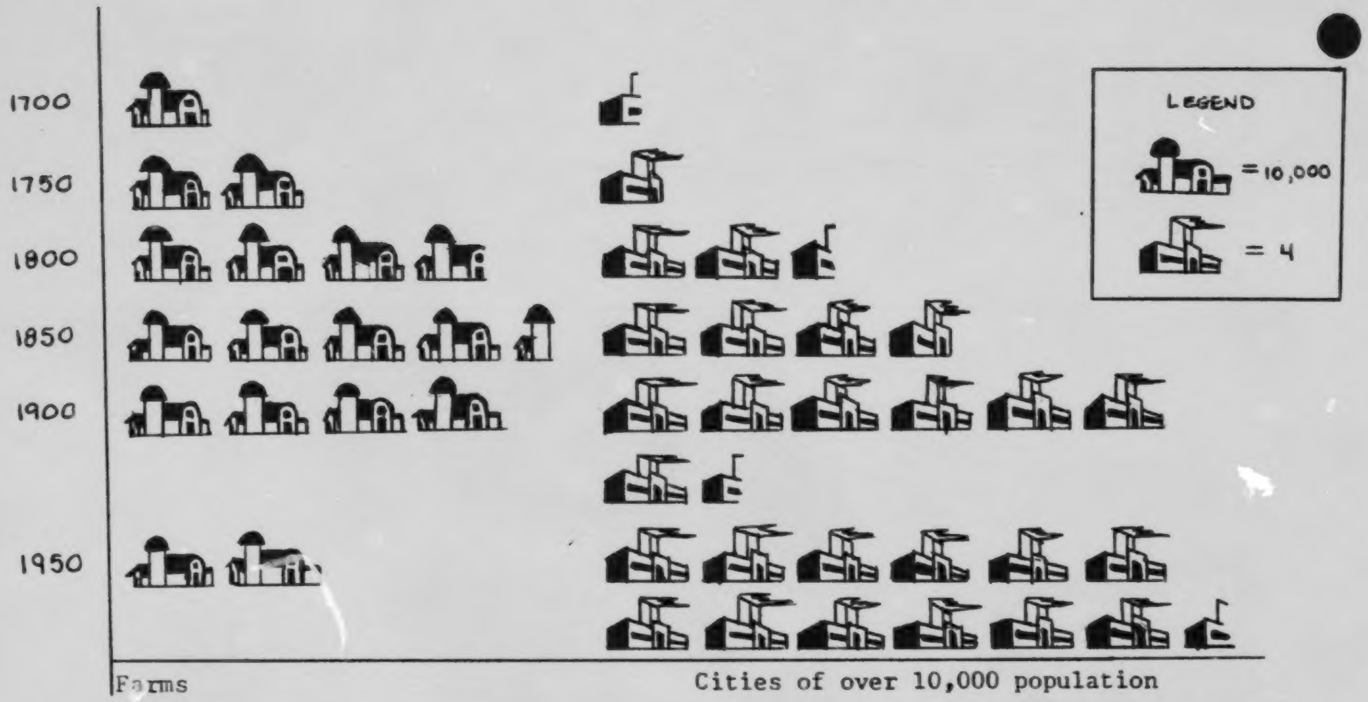
20. It rains about ___________ inches in Reno during April, May and June.

FARMS AND CITIES IN NEW BARRINGTON STATE



Number your paper from 1 to 20. Read each sentence below. Fill in the blank to make the sentence complete. Look at the graph to find your answer.

1. __________ kinds of symbols are shown in the graph.
2. The ____________ shows that each symbol for farms stands for 10,000 farms.
3. Each symbol for cities stands for _____ cities.
4. The number of different years shown in the graph is _________________.
5. The years compared in the graph are listed on the ___________ axis.
6. To find the number of farms in 1950, you must multiply the number of symbols for farms by ________________.
7. Each city shown in the graph has a population of _____________ or more.
8. In 1750 the number of farms in New Barrington was __________________.
9. In 1700 New Barrington had about ______ cities with more than 10,000 people.
10. The number of farms in New Barrington in 1800 was about _____________________.
11. The number of farms in New Barrington doubled between __________ and 1750.
12. The number of cities in New Barrington tripled between 1800 and ____________.
13. New Barrington had the same number of farms in 1750 and in ______________.
14. The graph shows that between 1850 and 1950 New Barrington lost about ________ ____________ farms.



15. The number of farms in 1950 was ________ __________ the number in 1900.

16. The number of cities in 1950 was about ________________.

17. From 1850 to 1950 New Barrington gained about __________ cities of more than 10,000 population.

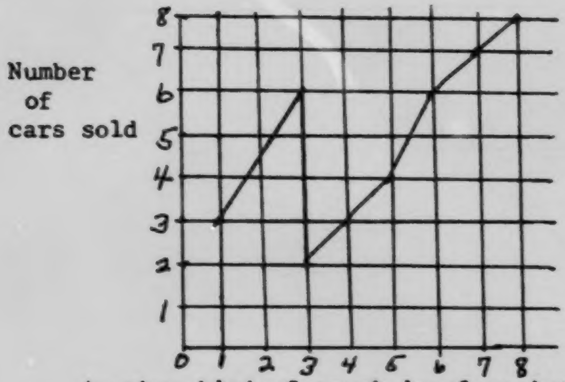
18. New Barrington had one more city in 1750 than in ______________.

19. Between 1850 and 1900 the number of cities doubled, while the number of farms decreased by ____________.

20. From the graph you can guess that the people in New Barrington today are moving from the _______ to the ________________.

RECTANGULAR COORDINATES
Graphs and Graphing Equations
(For use with Geometry)

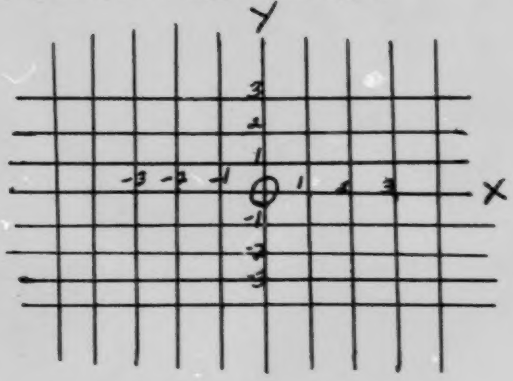
We have seen graphs of many kinds. Most involve a vertical and a horizontal scale presenting some kind of information. For example:



A scale curve may be drawn to show the rate of increase or decrease in sales.

Another kind of graph involves both algebra and geometry.

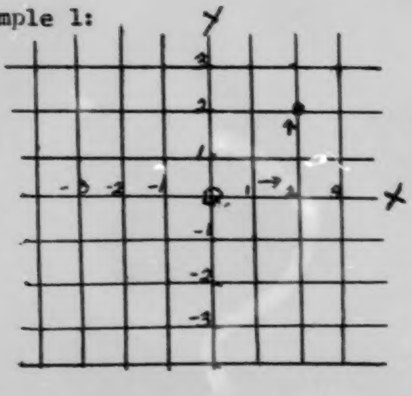
The lines are drawn at right angles to each other. The point at which the lines meet is called the origin (O). The vertical line is called the y-axis (Y). The horizontal line is called the x-axis (X). The origin represents zero. We then mark off equal spaces along the x and y axes. The spaces are then numbered.



Upward and to the right of the origin are the positive numbers. Downward and to the left are the negative numbers. These intersecting, numbered axes are called rectangular coordinates.

If we wanted to locate points on these axes, we would have to say, "so many spaces to the right and so many spaces up; or so many spaces to the left and so many spaces down."

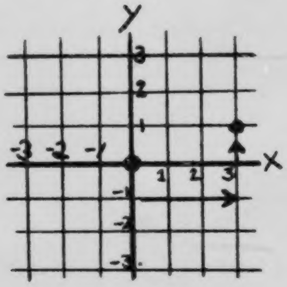
Example 1:



The dot on these axes is located at 2 spaces to the right and 2 spaces up. Another way of stating this is (2,2).

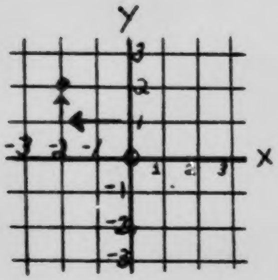
Whenever a point's location is given, the number on the x-axis is given first, followed by a number on the y-axis. The numbers may be positive or negative, indicating the direction to take on the x-or y-axis. These two numbers are called coordinates.

Example 2:
(3,1)



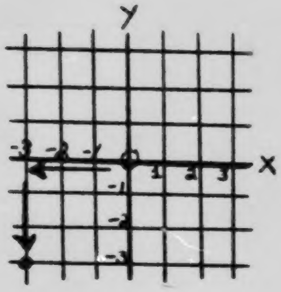
The positive 3 indicates a move 3 spaces to the right on the x-axis. The positive 1 indicates a move 1 space up on the y-axis.

(-2,2)



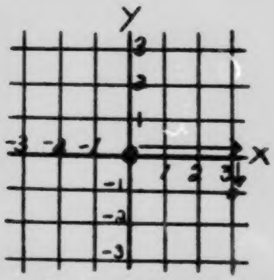
The negative 2 indicates a move 2 spaces to the left on the x-axis. The positive 2 then indicates a move 2 spaces up on the y-axis.

(-3,-3)



The negative 3 indicates a move 3 spaces to the left on the x-axis. The following negative 3 indicates a move 3 spaces down along the y-axis.

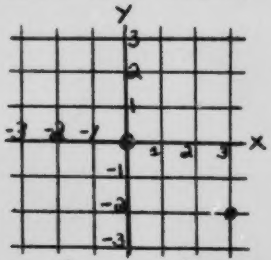
(3,-1)



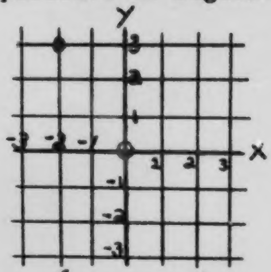
The positive 3 indicates a move 3 spaces to the right along the x-axis. Then the negative 1 indicates a move 1 space down.

Exercise 1:

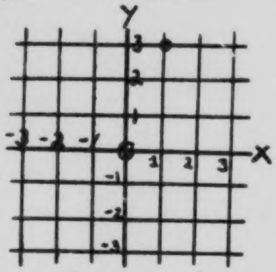
Give the coordinates of the following points. Remember to give the position on the x-axis first. Use positive or negative signs to indicate direction.


Y
X
3
2
1
-3 -2 -1
1 2 3
-1
-2
-3

a. (,)


Y
X
3
2
1
-3 -2 -1
1 2 3
-1
-2
-3

b. (,)

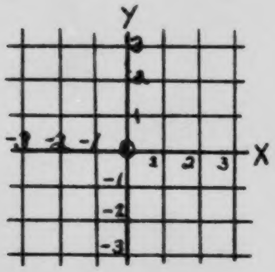

Y
X
3
2
1
-3 -2 -1
1 2 3
-1
-2
-3

c. (,)

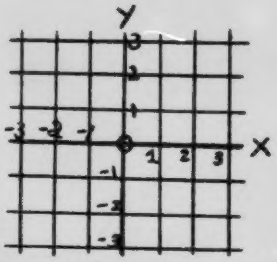
Exercise 2:

Plot these coordinates on the x- and y-axis.

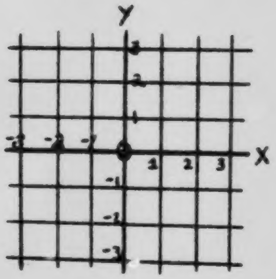
a. (1, 3)


Y
X
3
2
1
-3 -2 -1
1 2 3
-1
-2
-3

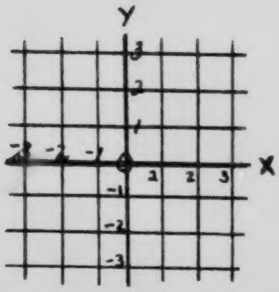
b. (-3,-1)


Y
X
3
2
1
-3 -2 -1
1 2 3
-1
-2
-3

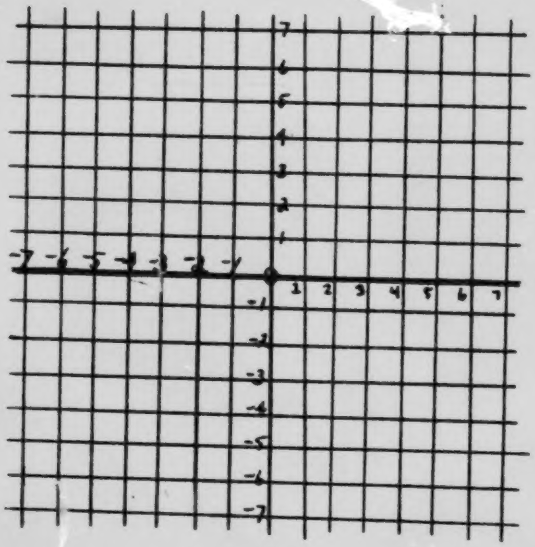
c. (2,-1)


Y
X
3
2
1
-3 -2 -1
1 2 3
-1
-2
-3

d. (-2,3)


Y
X
3
2
1
-3 -2 -1
1 2 3
-1
-2
-3

e. (-6, 7)


7
6
5
4
3
2
1
-7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7
-1
-2
-3
-4
-5
-6
-7

Graphing Equations

We can use our coordinates to graph or draw the picture of algebraic equations. Equations such as $x + y = 3$, $y = 2x$, and $x-y = 2$ are called linear equations. They are so called because solutions for the equations give coordinates which would lie on a straight line. Each linear equation has two unknowns. The value of one unknown changes as the value of the other changes.

Example 3: $x + y = 3$

First: Beginning with zero as a solution for x, find corresponding values for y. Solve for y by using each new value for x.

If x is 0,
$0 + y = 3$
$y = 3 - 0$
$y = 3$

If x is 1,
$1 + y = 3$
$y = 3 - 1$
$y = 2$

If x is 2,
$2 + y = 3$
$y = 3 - 2$
$y = 1$

If x is 3,
$3 + y = 3$
$y = 3 - 3$
$y = 0$

If x is 4,
$4 + y = 3$
$y = 3 - 4$
$y = 1$

If x is 5,
$5 + y = 3$
$y = 3 - 5$
$y = -2$

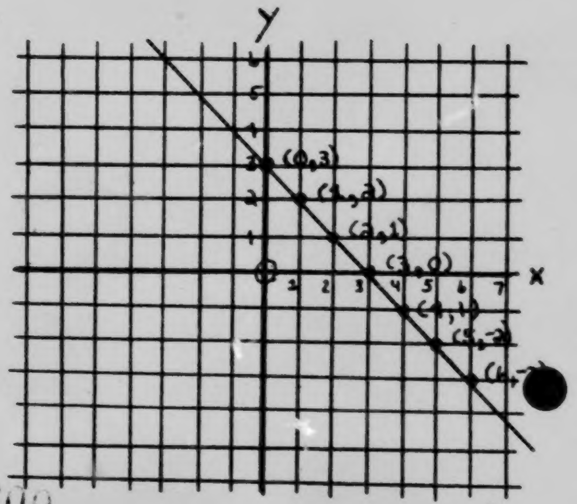
If x is 6,
$6 + y = 3$
$y = 3 - 6$
$y = -3$

Second: Set up a table showing these corresponding values.

x	0	1	2	3	4	5	6
y	3	2	1	0	-1	-2	-3

Now, by using a value for x and a corresponding value for y, we can use these as coordinates to graph the equation.

Third: Graph these points on the x-andy-axes.



Fourth: Draw a line through these points.

Example 4: $x - y = 0$

First: Using zero as the first solution for x, find solutions for y.

If x is 0,
$$0 - y = 0$$
$$-y = 0 - 0$$
$$y = 0$$

If x is 1,
$$1 - y = 0$$
$$-y = 0 - 1$$
$$-y = -1$$
$$y = 1$$

If x is 2,
$$2 - y = 0$$
$$-y = 0 - 2$$
$$y = 2$$

If x is 3,
$$3 - y = 0$$
$$-y = 0 - 3$$
$$y = 3$$

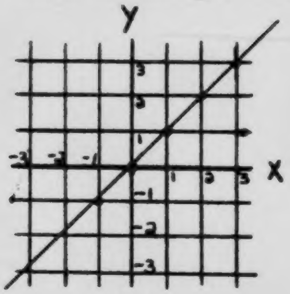
If x is -1,
$$-1 - y = 0$$
$$-y = 0 - 1$$
$$-y = +1$$
$$y = -1$$

Second: Set up a table of coordinates.

x	-1	0	1	2	3
y	-1	0	1	2	3

Third: Plot the points on the x- and y- axes. Draw a straight line through the points.

x	-1	0	1	2	3
y	-1	0	1	2	3



Exercise 3:

Solve these equations in the manner shown and graph them.

a. $x - y = -3$

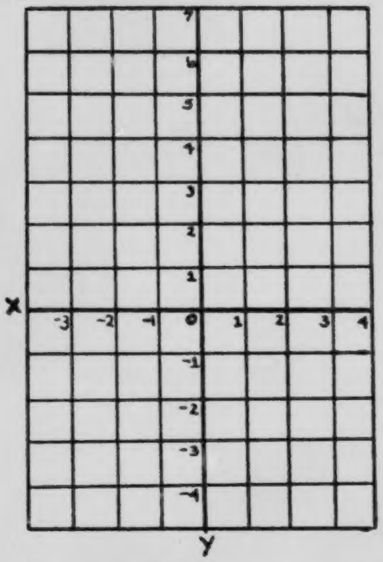
b. $x + y = 5$

c. $x - y = 0$

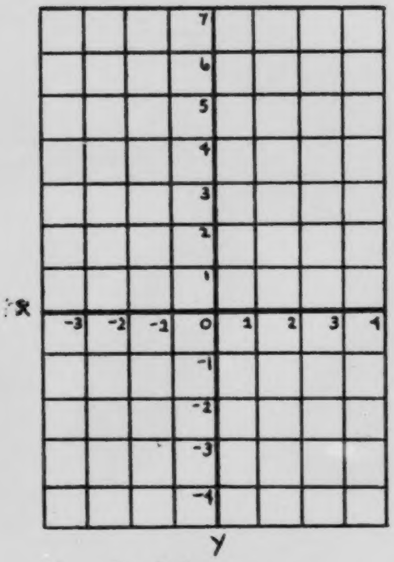
d. $2x + 1 = y$

For Exercise 3

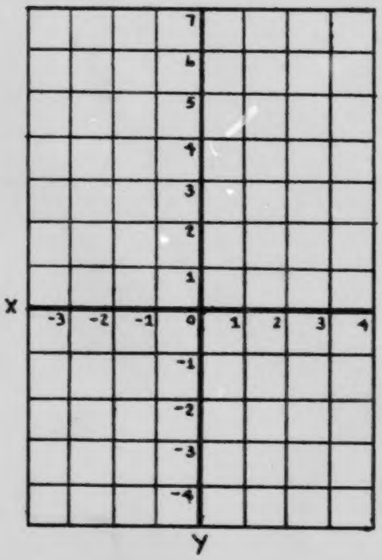
1. $x - y = -3$


X
Y
7
6
5
4
3
2
1
-1
-2
-3
-4
-3
-2
-1
0
1
2
3
4

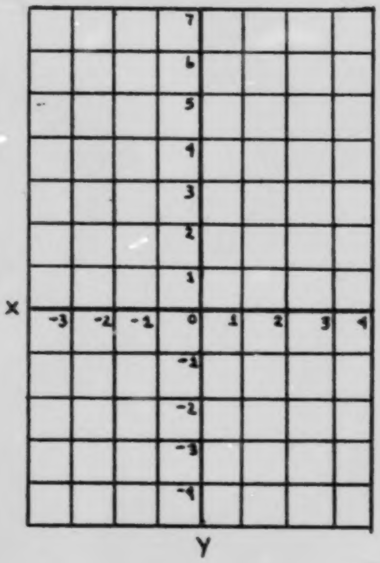
2. $x + y = 5$


X
Y
7
6
5
4
3
2
1
-1
-2
-3
-4
-3
-2
-1
0
1
2
3
4

3. $x - y = 0$


X
Y
7
6
5
4
3
2
1
-1
-2
-3
-4
-3
-2
-1
0
1
2
3
4

4. $2x + 1 = y$


X
Y
7
6
5
4
3
2
1
-1
-2
-3
-4
-3
-2
-1
0
1
2
3
4

Exercise 4:

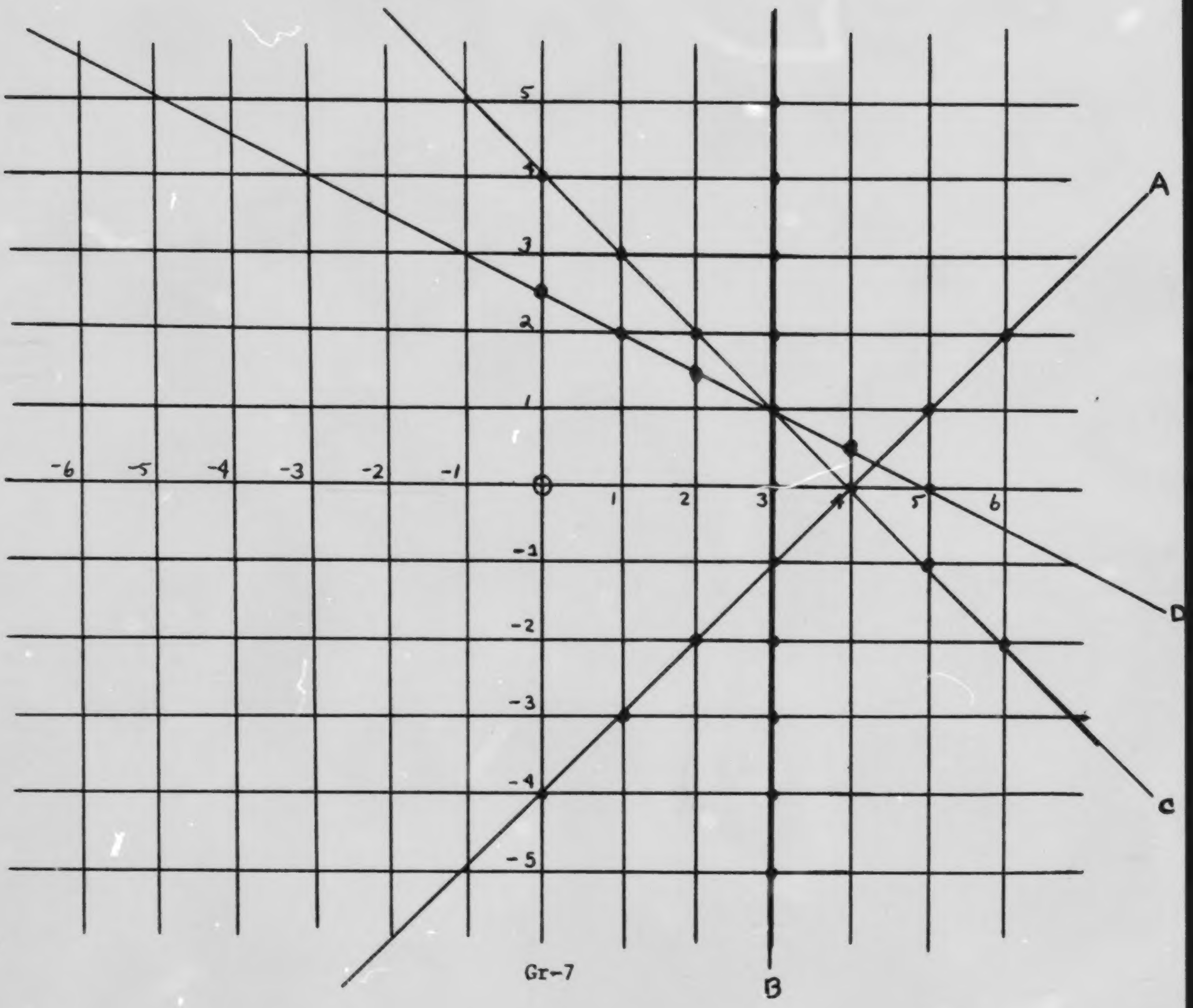
Solve these equations. Then decide which line (A, B, C, or D) represents each equation.

_____1. $x + y = 4$

_____2. $x - y = 4$

_____3. $x + 2y = 5$

_____4. $x = 3$
*Hint: No matter what y equals, x always equals three.



Gr-7

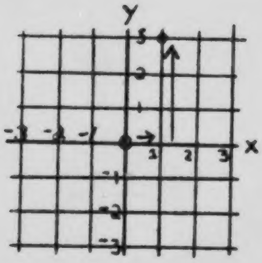
Answer Key

Exercise 1:

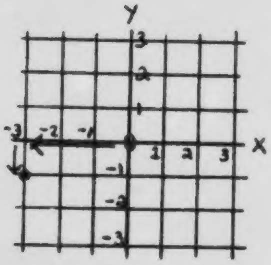
a. (3, -2) b. (-2, 3) c. (1,3)

Exercise 2:

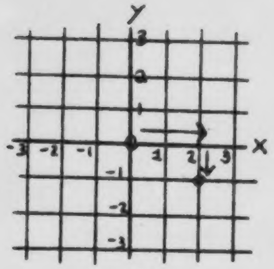
a. (1, 3)



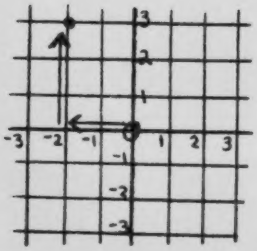
b. (-3, -1)



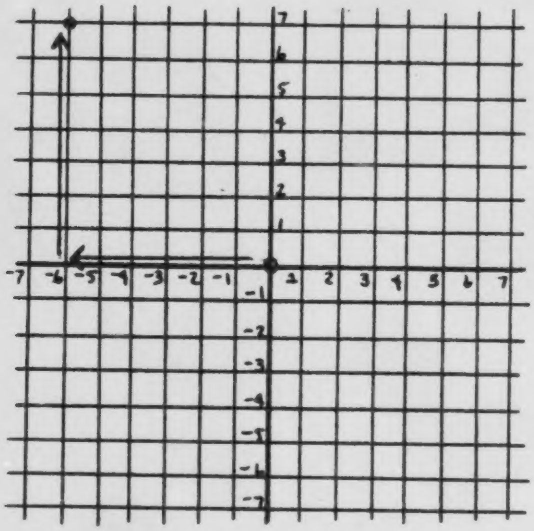
c. (2,-1)



d. (-2, 3)



e. (-6, 7)



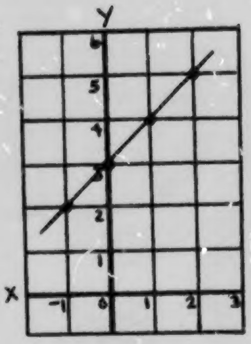
-7
-6
-5
-4
-3
-2
-1
1
2
3
4
5
6
7

Answers:

Exercise 3

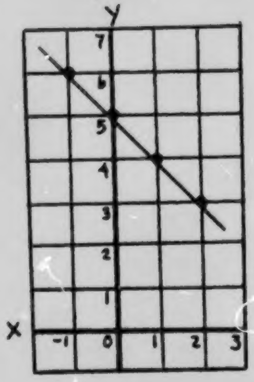
1. $x - y = -3$

x	y
0	3
1	4
2	5
-1	2



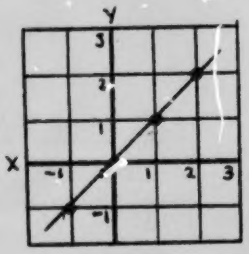
2. $x + y = 5$

x	y
0	5
1	4
2	3
-1	6



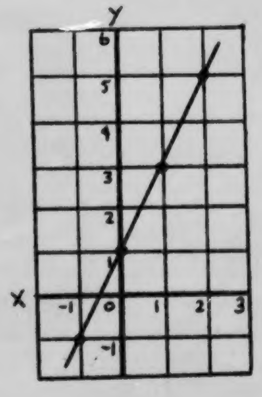
3. $x - y = 0$

x	y
0	0
1	1
2	2
-1	-1



4. $2x + 1 = y$

x	y
-1	-1
0	1
1	3
2	5



Exercise 4:

1. C
2. A
3. D
4. B

THE RECTANGLE AND PERIMETERS
(Hexagon p. 103-111)
(For use after Hexagon Mastery Test)

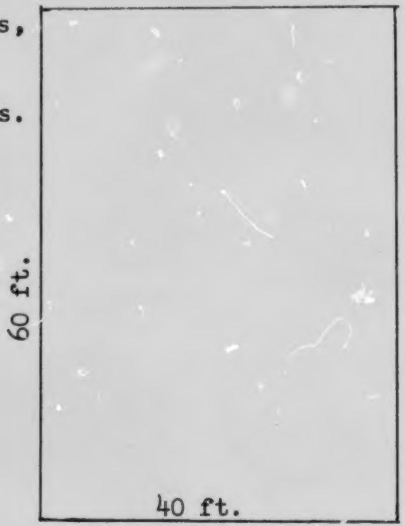


Figure A

Figure A is a rectangle. You have seen many rectangles, such as baseball and softball diamonds, a room floor, most rugs, pictures, etc. The opposite sides of a rectangle are equal, as you observed in the ball diamonds. The opposite sides are also parallel, which means that they are the same distance apart at all points. The angles of a rectangle are also equal, and each angle is a right angle. You will learn more about angles later.

1. With your ruler measure the length and width of this rectangle. What scale has been used in the drawing?

____

2. Since the opposite sides are equal, how many sides need to be measured with your ruler to determine the scale?

____

3. What is the perimeter of the rectangle?

____

4. To get the perimeter of a rectangle, we add the four sides. If we let l represent length, w the width, and P the perimeter, $P = l + w + l + w$. In a shorter form, $P = 2l + 2w$. This is known as a formula. Use the formula to show the perimeter of Figure A like this:

The Process

$l = 60$ ft. $w = 40$ ft. $P = 2l + 2w = 120$ ft. $+ 80$ ft. $= 200$ ft.

Using this formula, work the following problems. It will not only help you with each problem, but it will also help you to develop neatness and orderliness.

5. George is going to fence his yard. How much wire fence will he need to enclose it if the yard measures 20 ft. by 18 ft.?

Answer: __________

6. A farmer wishes to set aside part of a field for pasture. That part is to be 40 rods wide by 60 rods long. How much fencing will be needed?

Answer: __________

7. A fence was erected to enclose the playground at school. The playground is 300 feet by 200 feet. How much fencing was needed?

Answer: __________

8. The scale drawing of a city park measures 25 in. by 30 in. and is marked 1 in.= 24 ft. How much fencing will be needed to enclose the park?

Answer: __________

PERIMETERS OF RECTANGLES

1. Bridget's farm is rectangular in shape. It measures 185 rods by 125 rods. How much fencing would be needed to completely enclose the farm? It makes no difference as to which measurement is the length or width.)

Answer:___________

2. A playground is one-half as wide as it is long. The length is 360 feet. How much fencing would be needed to enclose it?

Answer:___________

3. A four-foot sidewalk encloses one of our parks. The park measures 600 feet long and 300 feet wide. What is the outside perimeter of the sidewalk?

Answer:___________

4. At 50 cents per foot, what would it cost to place picture molding in a classroom measuring 22 ft. by 32 ft.?

Answer:___________

5. Fill in the Perimeter column in the table below.

l	w	P
15	12	
27	32	
2.5	2	
29	33	
18	24	
60	40	
12.4	10.5	
54	46	
22	18	
92	75	
18.6	15.2	
29	27	

Do Your Work Here

USING MIXED UNITS OF MEASUREMENT

In each perimeter problem on the preceding pages both dimensions were given in the same unit. That is, the length and width were both given in feet, or in inches, or in rods, etc. Therefore, finding the perimeter required nothing more than adding 21 to 2w.

When the dimensions are not given in the same unit, however, you must change one of them to agree with the other. Thus, if the length were given in feet and the width were given in inches, both must be changed to feet or both must be changed to inches before they can be added.

1. A teacher's desk measures 5 ft. by 40 inches. What is its perimeter in feet?

The Process

1 = 5 ft.   w = 40 in. = 3 1/3 ft. P = 21 + 2w = 10 ft. + 6 2/3 ft. = 16 2/3 ft.

2. The library table measures 3 yards by 42 inches. Find the perimeter in yards.

Answer: ______________

3. The library measures 24 ft. 6 in. by 70 ft. 6 in. How many feet of picture molding would be needed for this library?

Answer: ______________

4. A farmer walked completely around his farm which was 1 mile long and lacked 80 rods of being a mile in width. What was the shortest distance he could have walked? (320 rods to the mile.)

Answer: ____________

5. A playground is 1/4 mile long and 528 feet deep (wide). Find the perimeter in miles.

Answer: ____________

6. A gymnasium floor measures 30 yards by 140 feet. Find the perimeter in feet.

Answer: _________

7. A park measuring 750 ft. by 200 yds. has a sidewalk around it. What is the shortest distance a person must walk to go around the park once?

Answer: ___________

APPLICATIONS OF MEASUREMENTS

1. The club has decided to furnish its room with pictures, but it must first secure picture molding for the four walls. The room measures 18 ft. by 24 ft. 6 in. Allowing 2 ft. extra for waste in cutting, what will be the cost of the molding at 2 cents a foot?

The Process

1 = 24 ft. 6 in. = 24 1/2 ft. w = 18 ft.
P = 21 + 2w = 49 + 36 = 85 ft.
Waste = 2 ft.
Molding required = 85 + 2 = 87 ft.
Cost = 87 X $.02 = ?

Answer:__________________

2. Brenda bought 54 inches of braid at 60 cents a yard. How much did she pay?

Answer: ______________

3. Our lot is 50 feet wide by 150 feet in depth (length). Andrew wishes to erect a chain link fence along the two sides and the back. How many feet of fence will be needed?

Answer:__________

4. A farmer wished to enclose a field with fencing. The field measured 250 rods by 400 rods. At $5 per rod, what would the fencing cost him?

Answer:__________

5. Ruth's home is on a corner of two streets, and the lot measures 200 feet by 75 feet. How much will it cost to install curbing on one side and the front at $1.10 per foot?

Answer:___________

6. How much baseboard would be needed for a classroom which measures 24 ft. by 28 ft., allowing for two door openings of 42 inches each?

Answer:___________

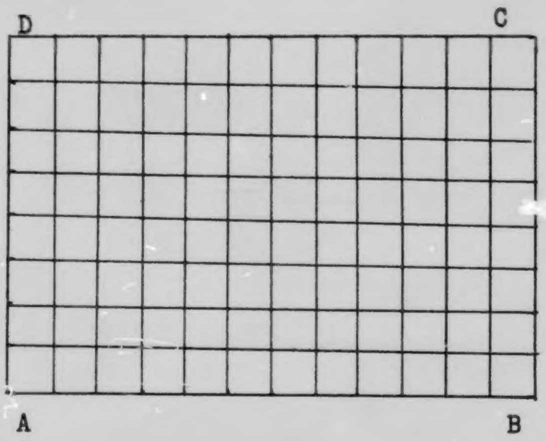
7. A park measures 400 feet by 300 feet. The park has a four-foot walk completely around it and 4 feet of grass between the walk and the curb. How many feet of curbing are there on the four sides of the park?

Answer:___________

8. What would be the cost of a mile of copper cable at $1 per yard?

Answer:__________

THE AREA OF THE RECTANGLE



The space enclosed by the lines of a figure is called the surface or area of the figure. The area of any figure is the number of square units it contains.

1. Figure ABCD is a scale drawing of a rectangle 12 ft. by 8 ft.

   a. What scale has been used?____________

   b. How many units long is the rectangle?________

   c. How many units wide is it?________________

   d. How many units in each row? _______________

   e. How many units in each column?_____________

   f. How many square units in the whole surface?_____________

   g. What relation do you see between the length times the width and the area?

____

Using A for area, 1 for length, and w for width, the rule or formula for finding the area of a rectangle is: A = 1 X w or A = lw

Remember: Both dimensions must be in the same unit before multiplying to find the area.

2. Nell wishes to cover a pillow with silk. The pillow measures 24 in. by 18 in. How many square inches of material (making no allowance for seams or waste) does she need?

The Process

1 = 24 in.
w = 18 in.
A = lw = 24 X 18 =______sq. in.

3. Our living room measures 14 feet by 20 feet. How many square feet of carpeting will be needed to completely cover the floor?

Answer: __________________

4. The playground is to be covered with cement. The area to be covered is 175 feet long and 130 feet wide. What is the area to be covered?

Answer:____________

5. A twin bed is 39 inches wide and 72 inches long. How many square inches of space will it take to place it in the room?

Answer:___________

6. The window in a railroad coach measures 24 in. by 21 in. The window pane contains how many square inches of glass?

Answer:____________

7. Our kitchen measures 9 ft. 6 in. by 16 ft. How many square feet of floor covering are needed for the kitchen?

Answer:________________

PROBLEMS WITH AREAS

1 sq. ft.=12x12=144 sq. i
1 sq. yd. = 3x3=9 sq. ft.
1 sq. rd. = $5\frac{1}{2}$x$5\frac{1}{2}$=$30\frac{1}{4}$ sq. yd.
1 acre (A.)-160 sq. rd.
1 sq. mi.=640 A.

1. Study the table of square measures on the right.
   a. To change square inches to square feet, divide by ____
   b. 432 square inches equals ____ square feet.
   c. To change square feet to square inches, multiply by ____.
   d. Ten square feet equal how many square inches? ____
   e. To change square feet to square yards, divide by ____.
   f. 27 square feet equal how many square yards? ____
   g. To change square yards to square feet, multiply by ____.
   h. Four square yards equal how many square feet? ____
   i. To change square yards to square rods, divide by ____
   j. 121 square yards equal how many square rods? ____

2. A library floor has the dimensions of 70 ft. by 30 ft. How many square yards are there in the floor area?

The Process

l = 70 ft. w = 30 ft. A = lw = 70 X 30 = 2100 sq. ft.
Sq. yd. = 2100 ÷ 9 = 233 1/3 sq. yd.

3. Andrea had the living room rug cleaned at 5 cents a square foot. The rug measures 9 ft. by 12 ft. What did the cleaning cost? . . . . ____

4. The living room (problem 3) where the 9 by 12 rug is placed measures 13 ft. by 16 ft. How many square feet of floor are not covered by the rug? (First find the total floor area and then subtract the rug area.) . . . . ____

5. Find the cost, at $1.50 per square yard, of laying a concrete walk 100 yd. long and 1 1/2 yd. wide. (First find the area and then the cost.) . . . . ____

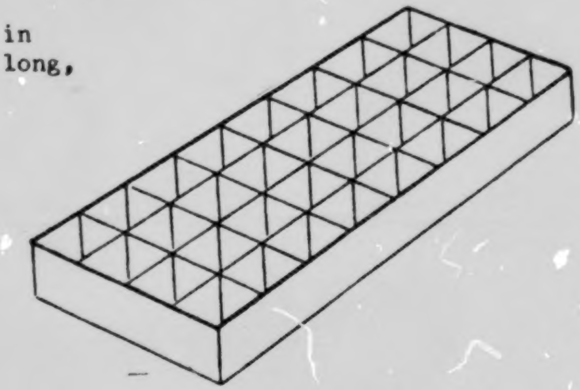
6. We had new linoleum put on the kitchen floor at a cost of $2.50 per sq. yd. The floor measures 18 ft. by 14 ft. What was the cost? . . . . ____

7. The floor of the public library (54 ft. by 36 ft.) was sanded at a cost of 50 cents a square yard. What was the cost? (Sometimes it is easier to change the units before multiplying. Can you work this problem in two ways?) . . . . ____

VOLUMES

We often need to know the volume of a container, such as a bin, a tank, a room,etc. You have had problems dealing with the capacity of a boxcar. Capacity is another term for volume. Capacity however, is sometimes given in pounds or tons.

1. At the right is a drawing of ice cube tray in an electric refrigerator . The cubes are an inch long, an inch wide, and an inch thick (or deep).



a. How many cubes deep is the tray? ______

b. How many inches deep is the tray? ______

c. How many cubes wide is the tray? ______

d. How many inches wide is it? __________

e. How many cubes are in the length?______

f. How many inches are in the length?______

g. How many cubes are in the entire tray?______

2. Multiply the length, width, and depth (or height) of the tray in problem 1. How many cubic inches are in it? ______ Does this agree with the total number of ice cubes found in the problem above? ________________________________

In areas of rectangles, length times width gives square units.
In volumes of rectangular solids, length times width times height gives cubic units.
Our rule for finding the volume of rectangular solids or containers is to multiply together the length, width, and height (depth or thickness). Put into a formula, we have:

Volume = length X width X height, or V = lwh

Remember that all dimensions must be in the same unit before multiplying.

3. Using this formula, fill in the Volume column in the table below.

l	w	h	V
10	20	5	1000
25	24	10	6000
34	28	16	15232
1.5	4	2.5	15
12 1/2	10	20	2500
12	15	8	1440
16	20	6	1920
22	24	12	6336
5.5	8	6.4	281.6
8 1/2	12	10	1020

CAPACITY PROBLEMS

1. The basement of one house required an excavation 30 ft. wide, 40 ft. long, and 10 ft. deep. How many cubic feet of dirt were removed?

The Process

l = 40 ft.
w = 30 ft.
h = 10 ft.
V = 40 X 30 X 10 = 12,000 cu. ft.

2. A "cord" contains 128 cubic feet. How many cords are there in a stack of wood measuring 8 ft. by 8 ft. by 16 ft.?

Answer: ____________________

3. The city park has a new swimming pool. It is 90 feet long, 35 feet wide, and 4 feet deep. How many cubic feet of water will be needed to fill the pool?

Answer: ____________________

4. The inside dimensions of a baggage car are 70 ft. by 9 ft. 6 in. by 10 ft. What is the cubic feet capacity of this car?

Answer: ____________________

5. To pack his shell collection John will need a box which contains at least 120 cubic inches of space. Will a box that is 25 inches long, 4 inches wide, and 3 inches deep be large enough?

Answer: ______________

6. A railroad flat car is 52 feet long and 9 feet wide. If it is loaded 6 feet high, how many cubic feet are in the load?

Answer: ____________________

7. A boxcar 50 feet long and 7 feet wide is loaded with grain to a depth of 6 feet. How many cubic feet of grain are there?

Answer: ______________

8. The city auditorium measures 60 feet by 100 feet and has a height of 40 feet. How many cubic feet of air are there for each of the 500 people who can be seated?

Answer: ______________

PROBLEMS WITH VOLUMES

TABLE OF CUBIC MEASURE

1 cu. ft. = 1728 c. in. (12x12x12)

1 cu. yd. = 27 cu. ft. (3x3x3)

1. How many cubic yards of concrete will Mr. Tom have to buy to build a driveway 6 ft. wide, 108 ft. long, and 4 in. thick?

The Process

l = 108 ft.
w = 6 ft.
h = 4 in. = 1/3 ft.
V = lwh = 108 X 6 X 1/3 = 316 cu. ft.
Cu. yd. = 216 ÷ 27 = 8

2. Mr. Harper lays fresh soil 4 in. deep on his yard. The yard is 60 ft. by 54 ft. How many cubic yards of soil does he need?

Shortcut:

$$\frac{1/\cancel{3} \times \overset{20}{\cancel{60}} \times \overset{2}{\cancel{54}}}{\underset{1}{\cancel{27}}}$$

Answer: ____________

3. Pat Anderson bought a pile of wood 20 ft. long, 16 ft. wide, and 8 ft. high. How many cords did he buy? (1 cord = 128 cu. ft.)

Shortcut:

Answer: ____________

4. How much did Pat pay for the wood (in problem 3) at $3.50 per cord?

Answer: ____________

5. What would be the cost of excavating a basement to measure 54 ft. by 40 ft. by 12ft., at $3 per cubic yard?

Shortcut:

Answer: ____________

6. An office 24 ft. by 22 ft. by 10 ft. has how much air for each of the 6 people working there?

Shortcut:

Answer: ____________

7. There are 231 cu. in. to the gallon. The gas tank of Henry's car measures 33 in. by 14 in 12 in. How many gallons will it hold?

Shortcut:

Answer: ____________

8. In excavating a hole 80 ft. long, 54 ft. wide, and 10 ft. deep, how many loads of dirt had to be moved in trucks that hold 5 cubic yards?

Shortcut:

Answer: ____________

9. How many bushels of corn can be stored in a crib measuring 15 ft. by 6 ft. by 10 ft.? (Count 1.25 cu. ft. to the bushel).

Shortcut:

Answer: ____________

10. A cubic foot of water weighs 62.5 lb. How much does the water weigh in a filled tank that is 10 ft. by 12 ft. by 8 ft.?

Answer: ____________

Page 1

1. 1 in. = 20 ft.
2. 2
3. 200 ft.
4. is answered
5. 76 ft.
6. 200 rd.
7. 1000 ft.
8. 2640 ft.

Page 2

1. 620 Rds.
2. 1080 ft.
3. 1832 ft.
4. $54
5. 54
   118
   9
   124
   84
   200
   45.8
   200
   80
   334
   67.6
   112

Page 3

1. is answered
2. 8 1/3 yds.
3. 190 ft.
4. 1120 rd. or 3.5 mi.
5. 7/10 mile
6. 460 ft.
7. 900 yds. or 2700 ft.

Page 4

1. $1.74
2. $ .90
3. 350 ft.
4. $6500
5. $302.50
6. 97 ft.
7. 1464 ft.
8. $1760.

Page 5

1. a. 1 in. = 4 ft.
   b. 12
   c. 8
   d. 12
   e. 8
   f. 96
   g. equal
2. 432 sq. in.
3. 280 sq. ft.
4. 22750 sq. ft.
5. 2808 sq. in.
6. 504 sq. in.
7. 152 sq. ft.

Page 6

1. a. 144
   b. 3
   c. 144
   d. 1440
   e. 9
   f. 3
   g. 9
   h. 36
   i. 30 1/4
   j. 4
2. is answered
3. $5.40
4. 100 sq. ft.
5. $225.
6. $70.
7. $108

Page 7

1. a. 1
   b. 1
   c. 4
   d. 4
   e. 8
   f. 8
   g. 32
2. 32 YES
3. Volume
   1000
   6000
   15232
   15
   2500
   1440
   1920
   6336
   281.6
   1020

Page 8

1. is answered
2. 8 cards
3. 12,600 cu. ft.
4. 6650 cu. ft.
5. yes
6. 2808 cu. ft.
7. 2100 cu. ft.
8. 480 cu. ft.

Page 9

1. is answered
2. 40 cu. yds.
3. 20 cords
4. $70.
5. $2880.
6. 880 cu. ft.
7. 24 gals.
8. 320 loads
9. 720 bu.
10. 60,000 lbs.

FORMULA WORKSHEET

(For use with Heptagon pp. 76-79)

Find the area and circumference of the circles.

1. $A = \pi r^2$

$\pi = 22/7$

$r = 21$

$A =$

2. $A = \pi r^2$

$\pi = 22/7$

$r = 28$

$A =$

3. $A = \pi r^2$

$\pi = 22/7$

$r = 35$

$A =$

4. $A = \pi r^2$

$\pi = 22/7$

$r = 63$

$A =$

5. $A = \pi r^2$

$\pi = 22/7$

$r = 56$

$A =$

6. $A = \pi r^2$

$\pi = 22/7$

$r = 3.5$

$A =$

7. $C = \pi d$

$\pi = 22/7$

$d = 14$

$C =$

8. $C = \pi d$

$\pi = 22/7$

$d = 7$

$C =$

9. $C = \pi d$

$\pi = 22/7$

$d = 21$

$C =$

10. $C = \pi d$

$\pi = 22/7$

$d = 35$

$C =$

11. $C = \pi d$

$\pi = 22/7$

$d = 175$

$C =$

12. $C = \pi d$

$\pi = 22/7$

$d = 140$

$C =$

Find the perimeter and the area of the rectangles.

L	W	P	A		L	W	P	A
15	12				17	10		
18	24				16	9		
29	27				25	20		
60	40				23	40		

Find the volume of the rectangular prisms.

L	W	h	V
8	9	10	
10	10	10	
6.5	8	10	
6	8	5	

Find the volume of the cylinders. Fill in the blanks.

r	d	h	V
7		10	
	7	10	
10		14	
	12	21	

Find the circumference and area of the circles. Fill in the blanks.

r	d	C	a
	3.5		
3.5			
	21		
7			
	28		

Find the area of the triangles.

a	b	A
8	16	
20	15	
14	5	

Fill in the perimeter of the rectangles.

L	W	P
15	12	
27	32	
2.5	2	
29	33	
18	24	
60	40	
12.4	10.5	
54	46	
22	18	
92	75	
18.6	15.2	
29	27	

Find the volume of the rectangular prisms

L	W	h	V
8	9	10	
10	10	10	
6.5	8	10	
6	8	5	
8	12	10	
20	15	6.5	

Find circumference & area of circles.

r	d	C	A
	3.5		
3.5			
	21		

Find the area of the rectangle

L	W	A
17	10	
16	9	
8.5	6	
9.5	12	
25	20	
23	40	
12	6.5	
18	6 1/2	
15	6	

Find the volume of the cylinders. Fill in blanks.

r	d	h	V
7		10	
	7	10	
10		14	
	12	21	

Find the area of the triangles.

a	b	A
8	16	
20	15	
14	5	

Volume of rectangles.

1. $V = L W h$
$L = 16'$
$W = 20'$
$h = 6'$
$V =$

2. $V = L W h$
$L = 12'$
$W = 15'$
$h = 8'$
$V =$

3. $V = L W h$
$L = 45'$
$W = 30'$
$h = 6'$
$V =$

4. $V = L W h$
$L = 20'$
$W = 10'$
$h = 5'$
$V =$

5. $V = L W h$
$L = 12'$
$W = 12'$
$h = 6'$
$V =$

6. $V = L W h$
$L = 22'$
$W = 10'$
$h = 5'$
$V =$

Volume of cylinders.

1. $V = \pi r^2 h$
$\pi = 22/7$
$r = 7'$
$h = 5'$
$V =$

2. $V = \pi r^2 h$
$\pi = 22/7$
$r = 14'$
$h = 10'$
$V =$

3. $V = \pi r^2 h$
$\pi = 22/7$
$r = 5'$
$h = 7'$
$V =$

4. $V = \pi r^2 h$
$\pi = 22/7$
$r = 6'$
$h = 28'$
$V =$

5. $V = \pi r^2 h$
$\pi = 22/7$
$r = 7'$
$h = 20'$
$V =$

6. $V = \pi r^2 h$
$\pi = 22/7$
$r = 21'$
$h = 60'$
$V =$

Find the perimeter and area of the rectangles.

1. P = 2L + 2W
L = 15'
W = 12'
P =
____

2. P = 2L + 2W
L = 27'
W = 32'
P =
____

3. P = 2L + 2W
L = 2.5'
W = 2'
P =
____

4. P = 2L + 2W
L = 29'
W = 27'
P =
____

5. P = 2L + 2W
L = 12'
W = 8'
P =
____

6. P = 2L + 2W
L = 30'
W = 18'
P =
____

7. A = L W
L = 15'
W = 9'
A =
____

8. A = L W
L = 15'
W = 10'
A =
____

9. A = L W
L = 20'
W = 15'
A =
____

10. A = L W
L = 12'
W = 9
A =
____

11. A = L W
L = 17'
W = 20'
A =
____

12. A = L W
L = 12'
W = 6.5'
A =
____

Find the perimeter and area of the triangles.

1. P = S+S+S

S^1 = 6'

S^2 = 6'

S^3 = 6'

P =

______

2. P = S+S+S

S^1 = 8'

S^2 = 8'

S^3 = 6'

P =

______

3. P = S+S+S

S^1 = 3.5'

S^2 = 3.5'

S^3 = 3.5'

P =

______

4. P = S+S+S

S^1 = 15'

S^2 = 15'

S^3 = 12'

P =

______

5. P = S+S+S

S^1 = 6'5"

S^2 = 6'5"

S^3 = 3'5"

P =

______

6. P = S+S+S

S^1 = 24'

S^2 = 24'

S^3 = 20'

P =

______

7. $A = \frac{ab}{2}$

a = 18'

b = 10'

A =

______

8. $A = \frac{ab}{2}$

a = 14'

b = 22'

A =

______

9. $A = \frac{ab}{2}$

a = 30'

b = 15'

A =

______

10. $A = \frac{ab}{2}$

a = 14'

b = 24'

A =

______

11. $A = \frac{ab}{2}$

a = 15'

b = 8'

A =

______

12. $A = \frac{ab}{2}$

a = 40'

b = 20'

A =

______

FORMULAS

RECTANGLES

Perimeter $P=2L+2W$

Perimeter equals 2 times the length plus 2 times the width

Area $A=LW$

Area equals length times width

Volume $V=LWH$

Volume equals length times width times height

SQUARE

Perimeter $P=4s$

Since all four sides are equal, perimeter equals 4 times a side.

Area $A=s^2$

Area equals side times side.

Volume $V=s^3$

Volume equals side times side times side.

TRIANGLE

Perimeter $P=s_1+s_2+s_3$

Perimeter equals side 1 plus side 2 plus side three.

Area $A=\frac{1}{2}ab$ or $\frac{1}{2}bh$ (Altitude and height are the same)

Area equals ½ x altitude x base or
altitude x base divided by 2

CIRCLES

CIRCUMFERENCE $C=\pi d$

Circumference equals pi (22/7) x the diameter
(Diameter equals 2 times the radius. Radius equals ½ diameter)

Area $A=\pi r^2$

Area equals pi x radius x radius (or, Area equals pi r squared)

VOLUME $V=\pi r^2h$ (cylinders)

Volume equals pi x radius x radius x height

TRAPEZOIDS

Perimeter $P=s_1 + s_2 + b_1 + b_2$

Area $A=\frac{1}{2} h (b_1 + b_2)$

PARALLELOGRAM

PERIMETER $P= 2L + 2W$

Area $A= bh$ Area equals base times height

INTEREST $I= prt$

Interest equals principal x rate x time

DISTANCE $D=rt$

Distance = rate x time

ADDING AND SUBTRACTING

SIGNED NUMBERS

(For use with Algebra pp. 22-24)

A newspaper reported that the temperature at 5 P.M. was +2° (or 2° above zero). The temperature at 6 P.M. was reported at -2° (or 2° below zero). Notice that these two temperatures were on opposite sides of the zero reading on the thermometer.

In math, numbers that are opposites are called positive (+) or negative (-). If you gain $10 it could be written +$10. If you lose $10, it could be written -$10.

A positive number will have a plus (+) sign before it or nothing before it. For example: +2 or 2, +5 or 5, +72 or 72, +103 or 103 are all positive numbers.

A negative number will always have a minus (-) sign before it. For example: -2, -5, -72, -103 are all negative numbers.

Positive and negative numbers are called signed numbers.

1. If +$8 is a profit, what does -$8 mean? ____________________

2. If +40° means north of the equator, what does -40° mean? __________
   ____

3. If +75 is a deposite in the bank, what is -$75? ____________________

4. If +90 means 90 miles west, what does -90 mean? ____________________

Express the following as a signed number:

1. 75° above zero ____________
2. a loss of $75 ____________
3. a rise of 5 feet ____________
4. 5° below zero ____________
5. a gain of $50 ____________

Rules - for adding signed numbers:

A. If the signs of the numbers are all the same just add the numbers.

$$\begin{array}{r} +2 \\ \underline{+2} \end{array} \qquad \begin{array}{r} 2 \\ \underline{2} \end{array} \qquad \begin{array}{r} -2 \\ \underline{-2} \end{array} \qquad \begin{array}{r} +2 \\ +2 \\ \underline{+2} \end{array} \qquad \begin{array}{r} 2 \\ 2 \\ \underline{2} \end{array} \qquad \begin{array}{r} -2 \\ -2 \\ \underline{-2} \end{array}$$

B. If the signs of the numbers are not alike subtract the larger number from the smaller, then use the sign of the larger number in the answer.

The Sign of the Larger Number

$$\begin{array}{r} +5 \\ \underline{-3} \end{array} \qquad \begin{array}{r} -5 \\ \underline{+3} \end{array}$$

The Sign of the Largest Sum of Numbers

$$\begin{array}{r} +5 \\ +1 \\ \underline{-5} \end{array} \qquad \begin{array}{r} -5 \\ -1 \\ \underline{+5} \end{array}$$

C. Two signed numbers that have the same number but opposite signs add up to zero.

$$\begin{array}{r} +5 \\ \underline{-5} \end{array} \qquad \begin{array}{r} +3 \\ \underline{-3} \end{array} \qquad \begin{array}{r} -6 \\ \underline{+6} \end{array} \qquad \begin{array}{r} -10 \\ \underline{+10} \end{array} \qquad \begin{array}{r} +14 \\ \underline{-14} \end{array}$$

ADD:

1. $\begin{array}{r} +15 \\ \underline{+\ 9} \end{array}$ 2. $\begin{array}{r} +\ 8 \\ \underline{+17} \end{array}$ 3. $\begin{array}{r} 5 \\ \underline{2} \end{array}$ 4. $\begin{array}{r} 5 \\ \underline{8} \end{array}$ 5. $\begin{array}{r} +8\ 1/4 \\ \underline{+9\ 3/4} \end{array}$

6. $\begin{array}{r} -17 \\ \underline{-\ 8} \end{array}$ 7. $\begin{array}{r} -28 \\ \underline{-38} \end{array}$ 8. $\begin{array}{r} -8 \\ \underline{-7} \end{array}$ 9. $\begin{array}{r} -5.4 \\ \underline{-2.6} \end{array}$ 10. $\begin{array}{r} -12\ 1/2 \\ \underline{-19\ 3/4} \end{array}$

11. $\begin{array}{r} +70 \\ \underline{-20} \end{array}$ 12. $\begin{array}{r} -55 \\ \underline{+20} \end{array}$ 13. $\begin{array}{r} -8.5 \\ \underline{+6.1} \end{array}$ 14. $\begin{array}{r} -6.9 \\ \underline{+9.4} \end{array}$ 15. $\begin{array}{r} +23\ 5/6 \\ \underline{-9\ 1/6} \end{array}$

16. $\begin{array}{r} -25 \\ \underline{42} \end{array}$ 17. $\begin{array}{r} 18 \\ \underline{-32} \end{array}$ 18. $\begin{array}{r} 8.3 \\ \underline{-8.3} \end{array}$ 19. $\begin{array}{r} 7.1 \\ \underline{-9.4} \end{array}$ 20. $\begin{array}{r} -33\ 1/3 \\ \underline{19\ 2/3} \end{array}$

21. $\begin{array}{r} +27 \\ -\ 9 \\ \underline{-12} \end{array}$ 22. $\begin{array}{r} -45 \\ +12 \\ \underline{+13} \end{array}$ 23. $\begin{array}{r} +20 \\ -12 \\ \underline{-\ 8} \end{array}$ 24. $\begin{array}{r} -1.5 \\ +3.7 \\ \underline{-8.3} \end{array}$ 25. $\begin{array}{r} +8\ 1/2 \\ -4\ 1/4 \\ \underline{+7\ 3/4} \end{array}$

26.	27.	28.	29.	30.
9	-21	-24	-.7	+8 1/6
7	-13	15	+3.1	-13 1/6
-3	17	19	-9.6	+ 3 1/3
-5	10	-12	+ .5	- 9 5/6

RULE: For subtracting signed numbers

A. Change the sign of the second number to the opposite sign and then finish the problem as you would in adding.

	Change				
-2	Sign	-2	-5	Change	-5
-2		+2	-2	Sign	+2

SUBTRACT:

1.	2.	3.	4.	5.
+30	+45	-19	-19	-7
+12	-20	17	- 7	-7

6.	7.	8.	9.	10.
50	18	15	8.7	8 5/6
30	29	15	9.5	3 1/6

11.	12.	13.	14.	15.
+35	+27	+19	+8.3	+7 3/4
-15	- 8	- 9	-6.2	-2 1/4

16.	17.	18.	19.	20.
-30	-45	-6	-6.9	-6 1/3
+15	17	6	+3.7	+3 1/3

21.	22.	23.	24.	25.
-26	-15	-8	-3.6	-8 7/8
-18	-31	-8	-5.2	-3 1/8

Answer Key for Adding and Subtracting Signs

1. loss
2. south of the equator
3. withdrawal
4. east

Signed numbers

1. +75°
2. $75
3. +5 ft.
4. -5°
5. +$50

Add Answers

1) +24 2) +25 3) 7 4) 13 5) +18
6) -25 7) -66 8) -15 9) -8 10) -32 1/4
11) +50 12) -35 13) -2.4 14) +2.5 15) +14 2/3
16) +17 17) -14 18) 0 19) -2.3 20) -13 2/3
21) +6 22) -20 23) 0 24) -6.1 25) +12
26) +8 27) - 7 28) -2 29) -6.7 30) -11 1/2

Subtract Answers

1) +18 2) +65 3) -36 4) -12 5) 0
6) +20 7) -11 8) 0 9) -.8 10) +5 2/3
11) +50 12) +35 13) +28 14) +14.5 15) +10
16) -54 17) -62 18) -12 19) -10.6 20) -9 2/3
21) -8 22) +16 23) 0 24) +1.6 25) -5 3/4

A1-3a

MULTIPLYING AND DIVIDING
SIGNED NUMBERS
(For use with Algebra pp. 37-39)

Rules for Multiplication:

A. When multiplying two numbers that have positive signs, just multiply and keep the same sign.

B. When multiplying two numbers that have unlike signs, just multiply and make the answer negative (-).

C. When multiplying two numbers that have negative signs, the answer will always be positive.

D. When multiplying more than two numbers, just take two at a time and follow rules A, B, or C to determine the answer sign.

Multiply:

1. $\begin{array}{r} 9 \\ \underline{8} \end{array}$ 2. $\begin{array}{r} +23 \\ \underline{+5} \end{array}$ 3. $\begin{array}{r} 17 \\ \underline{9} \end{array}$ 4. $\begin{array}{r} +12 \\ \underline{+7} \end{array}$ 5. $\begin{array}{r} +9 \\ \underline{0} \end{array}$

6. $\begin{array}{r} -11 \\ \underline{-7} \end{array}$ 7. $\begin{array}{r} -15 \\ \underline{-8} \end{array}$ 8. $\begin{array}{r} -24 \\ \underline{-4} \end{array}$ 9. $\begin{array}{r} -75 \\ \underline{-3} \end{array}$ 10. $\begin{array}{r} -19 \\ \underline{-6} \end{array}$

11. $\begin{array}{r} -17 \\ \underline{+3} \end{array}$ 12. $\begin{array}{r} +36 \\ \underline{-2} \end{array}$ 13. $\begin{array}{r} -24 \\ \underline{8} \end{array}$ 14. $\begin{array}{r} 15 \\ \underline{-9} \end{array}$ 15. $\begin{array}{r} -25 \\ \underline{+6} \end{array}$

16. (8) (1/4) = 17. (-3/5) (-20) = 18. (1/2) (-2/3) =

19. (-4 1/2) (2/3) = 20. (-15) (-3 1/5) = 21. (4) (3) (2) =

22. (-1) (-7) (-8) = 23. (-3) (-5) (+4) = 24. (-7) (+2) (-5) =

25. (+8) (-9) (0) = 26. $(+4)^2$ = 27. $(-3)^2$ =

28. $(+5)^3 =$

29. $(-2)^3 =$

30. $(-5)^3 =$

31. $(-1)^2 =$

32. $(+1/2)^2 =$

33. $(-1/3)^2 =$

34. $(+\ 2/3)^3 =$

35. $(-\ 3/5)^3 =$

36. $(-\ 1/4)^3 =$

37. $(-\ 1/5)^2 =$

Rules for Division:

A. If dividing a positive number by a positive number, the answer will be positive.

B. If dividing a negative number by a negative number, the answer will be positive.

C. If dividing a number by a number with a different sign, the answer will be negative.

Divide:

1. $18 \div 6 =$

2. $-36 \div -3 =$

3. $52 \div -4 =$

4. $84 \div -12 =$

5. $-30 \div -6 =$

6. $100 \div 25 =$

7. $55 \div -11 =$

8. $-84 \div -12 =$

9. $-144 \div 9 =$

10. $4 \div -8 =$

11. $-6 \div -9 =$

12. $-15 \div -12 =$

13. $+8.4 \div -4 =$

14. $-9.6 \div -.3 =$

15. $-3.6 \div +12 =$

16. $\frac{75}{15} =$

17. $\frac{-100}{-20} =$

18. $\frac{+144}{-8} =$

19. $\frac{-108}{+9} =$

20. $\frac{-65}{+5} =$

21. $\frac{18}{-4} =$

22. $\frac{-16}{+5} =$

23. $\frac{-34}{+4} =$

24. $\frac{+100}{-8} =$

25. $\frac{-36}{-8} =$

26. $\frac{-4.8}{-4} =$

27. $\frac{+9.6}{-3} =$

28. $\frac{-1.8}{+9} =$

29. $\frac{-2.4}{-.6} =$

30. $\frac{-7.2}{-1.2} =$

Answer Key Multiplying and Dividing Signed Numbers.

Multiplication:

1. +72
2. +115
3. +153
4. +84
5. 0
6. +77
7. +120
8. +96
9. +225
10. +114
11. -51
12. -72
13. -192
14. -135
15. -150
16. +2
17. +20
18. -1/3
19. -3
20. +48
21. +24
22. +56
23. -60
24. -70
25. 0
26. +16
27. +9
28. +125
29. +8
30. -125
31. +1
32. +1/4
33. +1/9
34. -8/27
35. -27/125
36. -1/64
37. +1/25

Dividing:

1. 3
2. +12
3. -13
4. -6
5. +5
6. -4
7. -5
8. +6
9. -16
10. -.5
11. +.66 2/3
12. +1 1/4
13. -2.1
14. 32
15. -.3
16. 5
17. 5
18. -18
19. -12
20. -13
21. -4.5
22. -3.2
23. -8.5
24. -12.5
25. 4.5
26. 1.2
27. 3.2
28. -.2
29. 4
30. 6

ALGEBRA HANDOUT
EQUATIONS
(Octagon, p. 91-111)
(For use with Algebra)

The word equation means a statement of equality. Thus, 2 x 2 = 4 is an equation. While the word equation may be new to us, its use is not at all new. The only important difference between a formula and an equation is that the formula is a rule which applies to any situation for which it is intended, and an equation applies to a particular situation. Thus, A = lw is the formula for finding the area of any rectangle. But when we use that formula to find the area of a rectangle whose length is 15 and whose width is 10 and substitute those values in the formula, the result is the equation: A = 15 X 10.

1. John's father is three times as old as John. The sum of their age is 76 years. How old is each?

THE PROCESS

Let a = John's age
Then, 3a = Father's age.
Then, $a + 3a = 76$
$4a = 76$
$a = 76 \div 4$
$a = 19$ (John's age)
$3a = 57$ (Father's age)
Proof: $19 + 57 = 76$

2. Mary is twice as old as her sister. The sum of their ages is 63 years. What is the age of each?

Mary: ________ Mary's sister: __________

3. Together, Earl and Roy have $4.80. Earl has three times as much as Roy. How much does each have?

Earl: __________ Roy: ____________

4. Frank and Fred together have $7.50. Fred has half as much as Frank. How much does each have? (By changing the problem around, Frank has twice as much as Fred. In this way you can avoid fractions.)

Frank: __________ Fred: ____________

5. Dorothy and Jane have $8.40. Dorothy has only one fifth as much as Jane has. How much does each have?

Dorothy: _________ Jane: _________

6. George and Harry inherited an estate of $1500. George is to receive three times as much as Harry. How much does each receive?

George: ___________ Harry: ____________

7. Mr. Brown has two fields for pasture. The area of one field is 3 times that of the second field. Their combined areas equal 60 acres. How many acres are there in each field?

1st field: _________ 2nd field: _______

Number Right ____________

SOLVING EQUATIONS

We have now learned how to add the terms of the equation and finally to divide for the value of the unknown quantity (letter). Not all equations have addition, or nothing but addition. You will come into contact with equations where all four operations -- addition, subtraction, multiplication, and division--are necessary.

Solve the following for the value of the letter used in the equation. You should be able to do all these mentally.

1. $5a + 4a = 45$ ______________________
2. $3x + 7x = 50$ ______________________
3. $23a + 27a = 100$ ______________________
4. $19a - 7a = 84$ ______________________
5. $16c + 12c = 84$ ______________________
6. $35b - 18b = 68$ ______________________
7. $21b + 9b = 120$ ______________________
8. $17z + 12z = 87$ ______________________
9. $12x + 5x = 51$ ______________________
10. $3b + 4b = 42$ ______________________
11. $15c + 10c = 100$ ______________________
12. $8a - 5a = 60$ ______________________
13. $14b - 6b = 48$ ______________________
14. $5x - 2x = 120$ ______________________
15. $7x - 4x = 30$ ______________________
16. $14m - 6m = 80$ ______________________
17. $23x - 7x = 80$ ______________________
18. $14a + 19a = 165$ ______________________
19. $42c - 19c = 92$ ______________________
20. $25n - 14n = 77$ ______________________
21. $4m + 7m = 66$ ______________________
22. $15a - 3a = 60$ ______________________
23. $6b + 5b = 5.5$ ______________________
24. $14b - 9b = 6.5$ ______________________
25. $6x + 7x = 52$ ______________________
26. $5x - x = 20$ ______________________
27. $3x + 4x = 49$ ______________________
28. $2x + 3x = 20$ ______________________
29. $8x - 6x = 3.8$ ______________________
30. $3x - x = .46$ ______________________
31. $7.5a + 2.5a = 60$ ______________________
32. $3.8a + 2.2a = 60$ ______________________
33. $1.4b + 3.6b = 35$ ______________________
34. $9.6x - 3.6x = 36$ ______________________
35. $12.4z - 4.4z = 64$ ______________________
36. $16.9d - 13.9d = 42$ ______________________

Now, go back and prove each answer by substituting the found value for the letter in each equation. Remember that an equation means equality.

Thus, in problem 1:

$$5a + 4a = 45$$
$$5(5) + 4(5) = 45$$
$$25 + 20 = 45$$
$$45 = 45$$

Number Right ______________

SIGNS OF OPERATION

When working equations we need to know more about how to proceed when there are several terms (or parts) and when there are several operations to be performed. Some equations require addition, subtraction, multiplication, and division, all in the one equation.

1. Frank had $25. He spent $6.50 for a pair of shoes, $2.50 for a shirt, and $1 for socks. How much did he have left?

THE PROCESS

Amount left = $25 - ($6.50 + $2.50 + $1)
Amount left = $25 - $10 = $15

2. A merchant bought 15 dozen ties at $30 per dozen and sold them at $50 per dozen. How much was his profit?

THE PROCESS

Profit = ($50 X 15) - ($30 X 15)
Profit = $750 - $450 = ?

Solve the following equations. (Always perform operations within parenthesis first.)

3. (36 ÷ 6) - (2 X 2) = ________

4. (4 X 2) ÷ (4 ÷ 2) = ________

5. (15 - 3) ÷ (2 X 3) = ________

6. (27 + 3) ÷ (6 + 4) = ________

7. (16 X 3) ÷ (2 X 4) = ________

8. (32 ÷ 8) X (20 ÷ 5) = ________

9. 25 + (8 X 3) = ________

10. (27 ÷ 3) X (4 + 2) = ________

11. (80 X 2) + (40 ÷ 2) = ________

12. (120 ÷ 3) - (3 X 2) = ________

13. 12a + 5b + 2(a + 2b) =
12a + 5b + 2a + 4b = 14a + 9b
Note that 2 (a + 2b) means 2 times (a + 2b)

14. 5x + 3y + 4 (x + 2y) = ________

15. 3x + 5y + 2(3x + y) = ________

16. 5a + 3b + 3(a + b) = ________

17. 6x + 9y + 2(x -y) =
6x + 9y + 2x - 2y = ________

18. 5a + 7b + 3(2a - b) = ________

19. 12a + 4b + 4(a - b) = ________

20. 6x + 3y + 2(x + 2y) = ________

21. 3(x + y) + 2(x + y) = ________

22. 4(x + 2y) + 3(x + 5y) = ________

Number Right ________

COLLECTING TERMS IN EQUATIONS

The form of an equation may be changed by addition, subtraction, multiplication, or division, as long as the same operation is performed to both sides (or members) of the equation. The form of the equation $x = y$ may be changed in any of the following ways.

Addition	Subtraction	Multiplication	Division
$x = y$	$x = y$	$x = y$	$x = y$
$x + 2 = y + 2$	$x - 2 = y - 2$	$2x = 2y$	$\frac{x}{2} - \frac{y}{2}$

To solve an equation in the form $x + 4 = 22 - x$, we must first collect similar terms so all known quantities are on one side of the equation and all unknown quantities are on the other. In this equation we can do this by subtracting 4 from both sides of the equation and adding x. Thus, the equation becomes $x + 4 - 4 + x = 22 - x - 4 + x$, or $x + x = 22 - 4$.

The terms of an equation may also be collected by transposition, or moving a term from one side of an equation to the other by changing the sign of the quantity. Transposition is a short way of collecting terms by addition and subtraction. Thus, $y - 8 = 35$ becomes $y = 35 + 8$.

1. Solve $4x - 5 = 3x + 1$.

The Process

The Long Way

$$4x - 5 = 3x + 1$$
$$4x - 5 - 3x = 3x + 1 - 3x$$
$$4x - 5 - 3x + 5 = 3x + 1 - 3x + 5$$
$$x = 6$$

The Process

The Short Way

$$4x - 5 = 3x + 1$$
$$4x = 3x + 1 + 5$$
$$4x - 3x = 1 + 5$$
$$x = 6$$

Using either of these methods, solve the following equations.

2. $5x + 3 = 18$
3. $3x + 5 = x + 3$
4. $8x - 5 = 5x + 10$
5. $5y - 16 = y$
6. $5(a + 4) = 4(a + 6)$
7. $3(x + 2) = x + 8$
8. $5x - 6 = 2(x + 6)$
9. $3x - 2 = 2(x + 2)$
10. $4(x - 3) = 2(x + 1)$
11. $5x - 2 = 3x + 2$
12. $7x - 4 = 3x$
13. $2x - 9 = x + 1$
14. $5(3x + 1) = 5(2x + 3)$
15. $4x + 5 = 41$
16. $2x + 8 = x + 4$

Number Right ____________

USING SIMPLE EQUATIONS

1. Jack and Bill together have $250. Jack has $10 more than 3 times as much as Bill. How much does each have?

The Process

Let x = Bill's money
Then $3x + \$10$ = Jack's money
$x + 3x + \$10 = \$250$
$4x + \$10 = \$250$
$4x = \$250 - \$10$
$4x = \$240$
$x = \$60$ (Bill's money)
$3x + \$10 = \$190$ (Jack's money)

2. Two fields together have an area of 90 acres. One field is twice as large as the other field. How many acres are there in each field?

1st field: ________ 2nd field: ________

3. The perimeter of a certain field is 160 rods. The length of the field is 3 times the width. What are the dimensions of the field?

Length: ____________ Width: ________

4. The perimeter of another field is 300 rods. The width is one fourth as much as the length. What are the dimensions of the field? (Let x = width. In this way, avoid fractions.)

Length: ____________ Width: ____________

5. Jerry has three times as much money as does Pat. Together they have $12.80. How much does each have?

Jerry: ____________ Pat: ________

6. The perimeter of a field is 60 rods. The length of the field is twice as great as the width. What are the dimensions of the field?

Length: ____________ Width: ____________

7. We have a lot whose perimeter is 360 ft. The length is 3.5 times as great as the width. Find the length and width of the lot.

Length: ____________ Width: ________

8. I am thinking of a number. Five times that number equals 240. What is the number?

Answer: ______________

Number Right __________

PROBLEMS WITH EQUATIONS

1. Jane has five times as much money as does Helen. Together they have $42. How much does each have?

Jane:________________ Helen:__________

2. Ruth has $15 more than Julia. Together they have $55. How much does each have?

Ruth:______________ Julia:_____________

3. One field contains 20 acres more than a second field. Together they contain 80 acres. How many acres does each field contain?

1st field:____________ 2nd field:________

4. One lot contains 500 square feet less than a second lot. Together they have a total area of 15,500 square feet. How many square feet are there in each lot?

1st lot:_____________ 2nd lot:____________

5. Joe's mother is 3 times as old as Joe. The sum of their ages is 84 years. How old is each?

Mother:____________ Joe:______________

6. Frank's father lacks 2 years of being three times as old as Frank. The sum of their ages is 78 years. How old is each?

Father:_____________ Frank:_______________

7. Three numbers, added, total 180. The second number is twice the first, and the third is three times the first. What is each number? (Let x = the first number)

1st no.:________ 2nd no.:____ 3rd no.:____

8. Three numbers, added total 140. The second number is twice the first, and the third number is twice the second. What are the three numbers?

1st no.:_____ 2nd no.:______ 3rd no.:_______

9. If 24 is added to a certain number, the result is 52. What is the number?

Answer:____________

10. Terry has $25 more than twice as much as Tom has. Together they have $145. How much does each have?

Terry:____________ Tom:__________________

Number Right ______________

CHECKING UP

1. From the formula base area x height = V, write the formula for finding h.

2. Find h when V is 240 and base area is 24.

---

3. A television set priced at $300 was purchased for a down payment of $100 and a payment of $20 for each of the following eleven months. What was the rate of interest paid?

4. Fred is 3 years older than twice Frank's age. The sum of their ages is 45 years. How old is each?

Answer: ____________

Fred: ____________ Frank: ____________

---

5. $x + 2 = 8$

6. $3x + 2 = x + 10$

7. $5x + 4 = 4x + 8$

---

8. $4x = x + 9$

9. $3(2x - 4) = 2(2x + 2)$

10. $3(2x - 2) = 4x + 4$

---

11. $3(x - 2) = x + 4$

12. $5(x - 4) = 3x$

13. $2(x + 5) = x + 15$

Number Right ________

THE MEANING OF RATIO

There are two methods of comparing the sizes of two similar quantities--by subtraction and by division. If we subtract, we get their difference. If we divide, we get their ratio. Ratio is defined as a comparison of two similar quantities by division.

For example, Erick Johnston is 20 years old, and his younger brother Nathan is 10. Using subtraction, we say that Erick is 10 years older than Nathan. If we compare their ages by division then Erick is twice as old as Nathan. Thus the ratio of Erick's age to Nathan is 2 to 1. This may also be written as 2: 1, or simply as the fraction 2/1.

Notice, however, that if we compare Nathan's age to Erick's, then the ratio is 1 to 2, which may also be written as 1: 2 or 1/2.

The following problems are designed to help you understand the meaning of ratio. As you read, be sure that you understand which amount is being compared to another; that is, notice which amount is the numerator of your fraction.

1. A tree is 30 ft. tall, and a nearby telephone pole is 25 ft. tall. What is the ratio of the height of the tree to that of the pole?

The Process

Stated in the fractional form, the ratio is $\frac{30}{25}$, or 30 to 25. Reducing this to lowest terms, the ratio is 6: 5, or 6/5. Do you see that this ratio could also be stated as 1 1/5 to 1, or simply 1 1/5?

2. In problem 1, what is the ratio of the height of the pole to that of the tree?

The Process

This time we need to divide 25 by 30, and the ratio of the pole to the tree is 5 to 6. This may also be written as 5: 6 and as the fraction 5/6.

---

3. Fred Aarons is 6 ft. tall, and his wife is 64 in. tall. What is the ratio of Fred's height to that of his wife? (Remember to change to the same units--either feet or inches.)

Answer: ______________

4. The ratio of Jim's age to his father's age is 2 :5. If Jim is 20 years old, how old is his father?

Answer: ______________

---

5. In problem 3, what is the ratio of the wife's height to Fred's?

Answer: __________

6. The diameter of a certain circle is 16 inches. The diameter of a smaller one is 4 inches. What is the ratio of the radius of the larger circle to that of the smaller circle?

Answer: __________

---

7. The total paid attendance at a recent football game was 6,000. However, more adults attended the game than did students. If the ratio of the number of adults to the number of students was 7: 5, how many adults attended the game?

Answer: ______________

8. The ratio of the height of a certain elm tree to that of a nearby oak tree is 3: 2. If the elm tree is 60 ft. tall, how tall is the oak tree?

Answer: ______________ Number Right ________

EQUATIONS INVOLVING RATIO

1. The perimeter of a rectangle is 300 in. The length and width are in the ratio of 2 to 1. Find the length and width.

The Process

Let x = width
2x = length
Then, 2x + 2x + x + x = 300 in.
6x = 300 in.
x = ? (the width)
2x = ? (the length)

2. The perimeter of a rectangle is 400 rods. The length and width are in the ratio of 3 to 2. Find the length and width.

Length:________ Width: ____________

---

3. The length and width of a rectangle are in the ratio of 3 to 1. The perimeter is 80. Find the length and width.

Length:_____________ Width: _________

4. The perimeter of a triangle is 48 in. The sides are in the ratio of 1 to 2 to 3. Find the length of each side.

____

---

5. The perimeter of another triangle is 480 rods. The sides are in the ratio of 5 to 4 to 3. How long is each side?

6. Tom, Dick, and Harry together have $2.40, and the amount each one has is in the ratio of 1 to 2 to 3. How much does each have?

The Process

Let x = Tom's money
2x = Dick's money
3x = Harry's money
Then, 6x = $2.40
x = ? (Tom's money)
2x = ? (Dick's money)
3x = ? (Harry's money)

---

7. Mary, Jane, and Catherine together have $2.70. Their monies are in the ratio of 2 to 3 to 4. How much does each have? (Let 2x = Mary's money)

Mary: ________ Jane:______ Catherine:_____

8. Three men contributed $84 to the Community Chest drive. Their contributions were in the ratio of 3 to 4 to 5. How much did each contribute?

1st man:_____ 2nd man:_____ 3rd man:________

---

9. Three persons gave $48 to the Red Cross. What the three persons gave was in the ratio of 3 to 4 to 5. How much did each give?

1st:______ 2nd:______ 3rd:________

10. The population of a certain town is 3600. The number of men, women, and children is in the ratio of 1 to 1.5 to 3.5. How many are in each of these three groups?

Men:______ Women:_______ Children:_________

Number Right__________

THE MEANING OF PROPORTION

When two ratios are equal, they are said to form a proportion. Thus, the ratios 1 to 2 and 8 to 16 form a proportion. Proportion is usually written in this form: 1 :2 = 8 : 16, and it is read, "1 is to 2 as 8 is to 16." Proportion is also written in fractional form like this: 1/2 = 8/16.

The first and last terms of a proportion are called the extremes, and the second and third terms are called the means: extreme: mean=mean : extreme or $\frac{\text{extreme}}{\text{mean}} = \frac{\text{mean}}{\text{extreme}}$

In the proportion 1 :2 = 8 : 16 notice that 1 X 16 = 2 X 8. That is, the product of the means equals the product of the extremes. This is true in all proportions. This rule may be used to solve a proportion in which three of the members are known and the fourth is unknown.

Use this rule to find the missing term in each of the following proportions.

1. 5 : 10 = 10 : x

The Process

5x = 100

x = 20

Proof: 5: 10 = 10 :20

100 = 100

2. 25 : 3 = 100 : x

3. 5 : 12 = 10 : x

4. 7 : 9 = 21 : x

---

5. 4 : 15 = 8 : x

6. 5: 12 = 10 : x

7. 5 : x = 15 : 9

8. 20 : x = 5: 4

---

9. 100 : x = 25 : 3

10. 12: 4 = x : 7

11. 6: 9 = x : 3

12. 7 : 3 = x : 6

---

13. x : 4 = 5 : 10

14. x : 20 = 4 : 5

15. x : 100 = 3 :25

16. x : 6 = 2 : 3

---

17. x : 3.2 = 5 : 8

18. 6 : 7 = x : 14

19. x : 3 = 10 : 15

20. 7.5 : x = 2 : 4

Number Right ________

FRACTIONAL EQUATIONS

Because proportions are often written in the form of fractions, we should study the solving of fractional equations. The proportion $4 : 5 = 8 : x$ can be written $\frac{4}{5} = \frac{8}{x}$. Using the rule the product of the means equals the product of the extremes, we find that $4x = 40$, or $x = 10$. Thus, the numerator of one fraction is multiplied by the denominator of the other fraction. This procedure is called cross multiplication

Solve each of the following fractional equations to find the unknown value.

1. $\frac{x}{3} = \frac{10}{5}$

The Process
Cross-Multiply.
$5 \times x = 5x$
$3 \times 10 = 30$
$5x = 30$
$x = 6$

2. $\frac{x}{8} = \frac{5}{4}$

3. $\frac{x}{16} = \frac{3}{8}$

4. $\frac{x}{6} = \frac{5}{3}$

---

5. $\frac{x}{9} = \frac{1}{3}$

6. $\frac{x}{3} = \frac{2}{5}$

7. $\frac{x}{12} = \frac{1}{6}$

8. $\frac{x}{4} = \frac{3}{2}$

---

9. $\frac{2x}{3} = \frac{10}{5}$

10. $\frac{x}{5} = \frac{7}{5}$

11. $\frac{8}{x} = \frac{4}{5}$

12. $\frac{5}{x} = \frac{7}{14}$

---

13. $\frac{6}{x} = \frac{3}{5}$

14. $\frac{4}{x} = \frac{2}{3}$

15. $\frac{3}{x} = \frac{5}{2}$

16. $\frac{16}{x} = \frac{8}{3}$

---

17. $\frac{15}{9} = \frac{5}{x}$

18. $\frac{5}{3} = \frac{25}{x}$

19. $\frac{7}{6} = \frac{14}{x}$

20. $\frac{6.5}{5} = \frac{13}{x}$

Number Right ________

MORE FRACTIONAL EQUATIONS

Find the unknown value in each of the following fractional equations.

1. $\frac{3}{8} = \frac{x}{16}$

2. $\frac{3}{5} = \frac{6}{x}$

3. $\frac{2}{3} = \frac{4}{x}$

4. $\frac{2}{5} = \frac{x}{3}$

5. $\frac{3}{4} = \frac{9}{x}$

6. $\frac{2x}{9} = \frac{2}{3}$

7. $\frac{2x}{3} = \frac{4}{5}$

8. $\frac{5}{2x} = \frac{6}{12}$

9. $\frac{2}{3} = \frac{2x}{9}$

10. $\frac{3}{2} = \frac{9}{2x}$

11. $\frac{x}{5} = \frac{2}{3}$

12. $\frac{4}{2x} = \frac{1}{4}$

13. $\frac{5}{4} = \frac{2x}{8}$

14. $\frac{9}{4} = \frac{27}{3a}$

15. $\frac{5}{3} = \frac{4a}{24}$

16. $\frac{7}{3a} = \frac{2}{12}$

17. $\frac{12}{x} = 6$

The Process

Cross multiply.

$$\frac{12}{x} = \frac{6}{1}$$

$$6x = 12$$

$$x = 2$$

18. $\frac{9}{x} = 3$

19. $\frac{16}{x} = 4$

20. $\frac{21}{x} = 7$

Number Right ________

SOLVING PROBLEMS BY PROPORTION

If Bob earns $8 in 5 days, at the same rate how much will he earn in 4 days?

The proportion from which this problem may be solved can be written in a number of different ways.

$$\frac{\text{dollars}}{\text{time}} = \frac{\text{dollars}}{\text{time}} \text{ or } \frac{8}{5} = \frac{x}{4} \qquad \frac{\text{time}}{\text{dollars}} = \frac{\text{time}}{\text{dollars}} \text{ or } \frac{5}{8} = \frac{4}{x}$$

$$\frac{\text{dollars}}{\text{dollars}} = \frac{\text{time}}{\text{time}} \text{ or } \frac{8}{x} = \frac{5}{4} \qquad \frac{\text{dollars}}{\text{dollars}} = \frac{\text{time}}{\text{time}} \text{ or } \frac{x}{8} = \frac{4}{5}$$

It is important to remember that the order of comparison must be the same in both the first and the second ratios. If the first ratio compares dollars to time, then the second ratio must also compare dollars to time in the same order.

2. If Paul can walk 15 miles in 6 hours, how far can he walk, at the same rate, in 8 hours?

The Process

$15 : 6 = x : 8$
$6x = 120$
$x = 120$
He can walk 20 miles in 8 hours.

3. In the 1972 Olympic Games the Soviet Union's Valeri Borzov won first place in the 100-meter dash with a time of 10.1 seconds. If he ran at this same rate, how long would it take him to run 800 meters (about 1/2 mile)?

Answer:

---

4. Tom runs the 50-yard race in 6 seconds. At the same speed, how long will it take him to run 75 yards?

Answer:

5. Mamie saved $5 in 4 weeks. At the same rate how much will she save in 6 weeks?

Answer:

---

6. Joe spent $1.36 for gasoline in driving 68 miles. How much would he spend in driving 85 miles?

Answer:

7. The taxes on a piece of property valued at $4000 were $66. At this same rate, what would be the taxes on property valued at $5500?

Answer:

---

8. A train is traveling at the rate of 75 miles per hour. At the same speed, how far will it travel in 40 minutes? (Change the 1 hour to minutes).

Answer:

9. Frank Budd's best time in running 100 yards is 9.2 seconds. If he ran at this same rate, how many seconds would it take him to run 220 yards?

Answer:

Number Right

PRACTICE IN PROPORTION

Solve each of the following proportions to find the value of the unknown.

1. $\frac{9}{x} = \frac{3}{4}$
2. $\frac{6}{5} = \frac{x}{10}$
3. $\frac{x}{12} = \frac{3}{2}$
4. $\frac{17}{10} = \frac{8.5}{x}$
5. $\frac{6}{11} = \frac{9}{x}$
6. $\frac{9}{2} = \frac{36}{x}$
7. $\frac{15}{40} = \frac{x}{8}$
8. $\frac{35}{14} = \frac{50}{x}$
9. $\frac{20}{19} = \frac{30}{x}$
10. $\frac{27}{x} = \frac{15}{10}$
11. $\frac{3}{9} = \frac{x}{16}$
12. $\frac{21}{75} = \frac{x}{25}$
13. $5 : 1 = 30 : x$
14. $x : 21 = 2 : 3$
15. $12 : x = 16 : 4$
16. $7 : x = 28 : 8$
17. $12 : 6 = 8 : x$
18. $8 : x = 32 : 12$
19. $4 : x = 16 : 12$
20. $35 : 14 = 50 : x$

Number Right_______

PROPORTION IN MEASUREMENT

Remember that a square is a rectangle with all sides equal. This means that the length and width are the same. Therefore, to find the area of a square we need to know only one side, since all sides are equal. Hence, to find the area of a square we multiply a side by itself: s X s, or s^2. Remember that s^2 means s squared or s multiplied by itself. Thus, $2^2 = 2 X 2$, or 4; $3^2 = 3 X 3$, or 9, etc.

1. Notice the figures at the right. Square A has a side of 1", and square B has a side of 2". The area of B is how many times the area of A?

The Process

Using proportion, we have:
$B : A = 2^2 : 1^2$
$B : A = 4 : 1$
The area of B is 4 times that of A.

2. One square has a side of 3" and a second square has a side of 6". The area of the second is how many times that of the first?

Answer: ______________

3. One square has a side of 2 1/2 times the side of a second square. The area of the first is how many times that of the second? (Change 2 1/2 to 2.5.)

Answer: ______________

A cube is a prism having equal length, width, and height. The volume of a cube is s X s X s, or s^3 (read, "s cubed").

4. One cube has a side of 1", and a second cube has a side of 2". The volume of the second cube is how many times that of the first cube?

The Process
$V : V = 1^3 : 2^3$
$V : V = 1 : 8$
The volume of the second cube is 8 times that of the first cube.

5. The side of one cube is 2", and the side of a second cube is 4". The volume of the second cube is how many times that of the first?

Answer: ______________

---

6. The side of one cube is 2", and the side of a second cube is 3". The volume of the second cube is how many times that of the first? (Do not be afraid of fractions in the answer.)

Answer: ______________

7. A picture 7" by 12" is enlarged so that the 7" side will be 14". What will be the other dimention of the enlarged picture?

Answer: ______________

---

8. A mixture of 2 parts sand and 3 parts gravel is used to make 25 cubic yards of concrete. How many cubic yards of each are needed?

The Process
2 parts + 3 = 5 parts
Then 2 : 5 = x : 25 (sand)
And 3 : 5 = x : 25 (gravel)

____

9. In making solder, 2 parts of tin and 3 parts of lead are used. How much of each is used in making 65 pounds of solder?

Answer: ____________________

Number Right ____________

VARIATION

The distance traveled is determined by the speed and the time. The distance varies as the speed or the time varies. Thus, if a car goes at the rate of 50 miles an hour, in 2 hours it will go 100 miles. Here the distance varies as the time varies because the rate of speed is the same. If the speed is changed to 60 miles an hour, the car will go 120 miles in 2 hours.

The circumference of a circle is equal to d. The value of is always the same (or constant). Therefore, the circumference of a circle is said to vary directly as the diameter varies.

1. From the table below we learn that the circumference of a circle whose diameter is 7 is 22. If we doubt the diameter, what is done to the circumference?

____

2. If the diameter is tripled, what happens to the circumference?

____

3. Fill in the table.

Circle	A	B	C	D	E	F
Diameter	7	14	21	28	35	42
Circumference	22	44	66	88	110	132

4. The area of a circle equals . In the areas of two circles, we have this proportion:

$$\frac{A}{A} = \frac{\ }{\ }$$

Since the value of is the same (or constant), we have the proportion:

$$\frac{A}{A} = \frac{r^2}{r^2}$$

Or, the area of a circle varies as the square of the radius. If the radius is doubled, what happens to the area? (Double means times 2. The square of 2 is how much?)

____

5. If the radius is increased to 3 times, how much is the area increased?
Answer: ______________________

Do Your Work Here

6. Fill in the table below. Prove the first one you fill in by the formula method so as to be sure you are on the right track.

Circle	A	B	C	D
Radius	7	14	21	28
Area	154	616	1386	2464

7. Since the area of a circle varies as the square of the radius, the area of a second circle is how many times that of the first circle if the radius of the second is 3 times that of the first?

The Process

$1\text{st } A : 2\text{nd } A = 1\text{st } r^2 : 2\text{nd } r^2$

$1\text{st } A : 2\text{nd } A = 1^2 : 3^2$

$1\text{st } A : 2\text{nd } A = 1 : 9$

The area of the second circle is 9 times that of the first circle. Now go back to the table and check your work to see if you proceeded correctly.

8. If we double the radius of a circle, how many times have we increased the area? ______________

INVERSE VARIATION

We have seen how direct variation works. Thus, if a train travels at a fixed rate of speed, the distance the train travels is dependent upon the number of hours it travels. The distance varies directly as the time varies.

However, if the train doubles its speed, it requires only half as much time to go a fixed distance. The faster the speed, the less the time required to go the fixed distance. Hence, the time required to go the fixed distance varies inversely as the speed. Double the speed, and the time becomes one half; triple the speed, and the time becomes one third, etc.

1. Fill in the table below, showing the time required to go the fixed distance of 600 miles at the different rates of speed shown.

Distance	600	600	600	600	600
Speed	10	20	30	40	80
Time	60	30	20	15	7 1/2

2. Fill in the table below to show the rate of speed required to go the fixed distance at the times shown.

Distance	400	400	400	400	400
Time	40	20	10	5	2.5
Speed	10	20	40	80	160

3. You have probably read the expression "man-hours." The term is used extensively in industry. One man working one hour is said to be a man-hour.

A contractor is engaged on a job that has been estimated to require 6000 man-hours. The time required to finish the job will vary inversely as the number of men working. Fill in the table below to show the number of hours required to finish the job for the different numbers of men employed.

Man-hours	6000	6000	6000	6000	6000
Men	20	40	80	160	
Hours	300	150	75	37.5	18.75

4. When Mr. Allen built his home, he figured on spending $100 for fruit trees. The number of trees he can buy varies inversely as the price increases. Fill in the table below, showing the number of trees that can be bought at the prices shown, the total price being fixed at $100.

Total	$100	$100	$100	$100	$100
Price	$4	$5	$6.25	$10	$12.50
Number	25	20	16	10	8

SIMILAR FIGURES

Similar triangles are triangles of the same shape. Triangles are said to be similar when any two sides of one triangle are in proportion to the corresponding two sides of the other triangle. In the triangles at the right the sides $\frac{AB}{XY} = \frac{BC}{YZ}$. If two sides of one triangle are in proportion to the two corresponding sides of the other triangle, the third sides are likewise in proportion. In the two triangles at the right: $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$. AB = 1", BC = 1.5", AC = 2", and XY = 2", YZ = 3", XZ = 4". Substitute these values in the proportion above to prove that it is a true proportion and the triangles are similar.

You may also note that the corresponding angles of two similar triangles are equal. Thus, in the triangles at the right, A= X, B = Y, and C = Z.

Complete these proportions for the similar triangles at the right.

1. $\frac{AB}{XY} = \frac{BC}{\underline{\quad}}$
2. $\frac{AB}{BC} = \frac{XY}{\underline{\quad}}$
3. $\frac{AB}{XY} = \frac{\underline{\quad}}{XZ}$
4. $\frac{AB}{AC} = \frac{XY}{\underline{\quad}}$
5. $\frac{AC}{XZ} = \frac{\underline{\quad}}{XY}$
6. $\frac{BC}{\underline{\quad}} = \frac{AB}{XY}$

---

7. In the similar triangles ABC and XYZ, AB is 5", XY is 6", and BC is 10". What is the length of YZ? The process is at the right. Study it carefully.

The Process

$\frac{AB}{XY} = \frac{BC}{YZ} \quad \frac{AB}{BC} = \frac{XY}{YZ}$

Then, $\frac{5}{6} = \frac{10}{YZ}$

5YZ = 60 YZ = 12

Do Your Work Here

8. In the same triangles, if AC is 15", how much is XZ? (Use any pair of corresponding sides of problem 7 for the proportion comparing AC and XZ.) . . . . ________

9. Two similar triangles ABC and XYZ have AB = 2", BC = 3", and XY = 3". Find how much YZ is. (AB/XY = BC/YZ) . . . . .________

10. In these same triangles find AC if XZ is 6". (AB/XY =AC/XZ) . . . . .________

11. In similar triangles ABC and XYZ, AB is 4", XY is 3", and BC is 6". Find YZ. . . . . .________

12. In these same triangles (problem 11), if AC is ", find XZ. (AB/XY = AC/XZ) . . . . . .________

USING PROPORTION IN SIMILAR FIGURES

One Saturday afternoon Jack Robertson and Jim Blackwood noticed the shadows cast by a tall oak tree and a nearby fence post.

"Jim," said Jack, "we have always wondered how high that tree is. I believe we can find out by using similar triangles. If we measure the two shadows, that will give the basis of the triangles. Then if we measure the height of the post, we can make a proportion and find the height of the tree."

"But where are the two triangles?" Jim asked. "The shadow and the height make only two sides. Where is the third side?"

"The rays of the sun shining over the top of the tree and the top of the post make the third side," Jack said.

Jack and Jim measured the two shadows and the height of the post and made sketches like those shown.

1. Jack measured the tree's shadow to be 60'. The shadow of the post was 12', and the height of the post was 8'. How high was the tree? (The process is on the right.)

The Process

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{12}{60} = \frac{8}{EF}$$

$$12\ EF = 480$$

$EF = 40$ The tree was 40 ft. tall

Do Your Work Here

2. A flag pole casts a 210' shadow at the same time a pole 20' tall casts a shadow of 30'. How tall is the pole? (It will be easier if you make a sketch.) . . . . .__________

3. A telephone pole casts a shadow 30 ft. long, while a fence post 4 ft. high casts a shadow 3 ft. long. How high is the pole? . . . . .__________

4. The Washington Monument casts a shadow 111 feet long at the same time a 50-foot telephone pole casts a shadow 10 feet long. How high is the Washington Monument? . . . . .__________

5. A smokestack casts a shadow of 40 feet while a fence post nearby casts a shadow of 2 feet. The fence post is 5 feet high. How tall is the smokestack? .__________

6. Frank is 6 feet tall. His shadow was 4 feet long when the shadow of a building was 100 feet. How tall is the building? . . . . .__________

In triangles ABC and DEF, above:

7. $\frac{EF}{DE} = \frac{BC}{\_\_\_}$ 8. $\frac{DF}{\_\_\_} = \frac{DE}{AB}$

9. $\frac{DE}{AB} = \frac{\_\_\_}{AC}$ 10. $\frac{AC}{\_\_\_} = \frac{AB}{DE}$

CHECKING UP

1. Mary has $2.50. Jane has $2. What is the ratio of Mary's money to Jane's?

Answer: ________________

2. What is the ratio of Jane's money to Mary's?

Answer: ______________

---

3. Trenton has a population of 40,000 persons. There are 6,000 children of school age in Trenton. What is the ratio of the school children to the total population?

Answer: ______________

4. The combined ages of Mr. Green and his eldest son is 80 years. The father is three times as old as the son. How old is each?

Mr. Green: __________ Son: ___________

---

5. Mr. Franklin is one-third as old as his father. The sum of their ages is 100 years. How old is each?

Mr. Franklin: ________ His father: ______

6. Ted, Bill, and James together have $8.40. Bill has twice as much as Ted. James has twice as much as Bill. How much does each have?

Ted: _______ Bill: _______ James: __________

---

7. Complete this proportion.

7 : 15 = 21 : x

Answer: ________________

8. Find the missing quantity in this proportion.

x : 10 = 15 : 30

Answer: ________________

---

9. In the similar triangles ABC and DEF, AB is 2", BC is 3", and DE is 4". Find the length of EF.

Answer: _____________

10. In the same similar triangles find AC if DF is 5".

Answer: _______________

---

11. The side of a square is 1". The side of a second square is 2". The area of the second square is how many times that of the first?

Answer: ___________

12. If 5 is added to 2 times a certain number, the result is 55. What is the number?

Answer: ____________

---

13. The sum of three numbers is 180. The three numbers are in the ratio of 1 : 2 : 3. What are the three numbers?

Answer: ________________

14. The ratio of peanuts to pecans to cashews in a certain nut assortment is 5 : 3 : 1. Per pound, peanuts cost $.35, pecans $.46, and cashews $.65. How much would 4.5 pounds of this mixture be worth?

Answer: _____________

PROGRESS TEST

1. Joe has \$5 and Tom has \$4. What is the ratio of Joe's money to Tom's?

Answer: ____________

2. What is the ratio of Tom's money to that of Joe (problem 1)?

Answer: ____________

---

3. Jerry and Pat together have \$15. Jerry's money is to Pat's money as 3 is to 2. How much does each have?

Jerry: ________ Pat: __________

4. June's father is three times as old as she. The sum of their ages is 56 years. How old is each?

June: __________ Father: ___________

---

5. Bill, Frank, and George together have \$24. Frank has twice as much as Bill. George has three times as much as Bill. How much does each have?

Bill: ______ Frank: ______ George: ____

6. The area of a rectangle is twice the area of a second rectangle. The sum of their areas is 600. What is the area of each?

1st rectangle: ______ 2nd rectangle: ______

---

7. Jim earned \$30 in 12 hours. By using proportion, show how much he would earn in 15 hours.

Answer: __________

8. Find the value of x in these proportions

$\frac{10}{20} = \frac{x}{16}$ $x =$ _____ $\frac{x}{15} = \frac{10}{.3}$ $x =$ _____

---

9. The area of a square varies as its side squared. The side of one square is twice the size of another square. The area of the first square is how many times that of the second?

Answer: ______________

10. The Empire State Building casts a shadow 250 feet long, while a nearby 100-foot flag pole casts a shadow 20 feet long. How tall is the Empire State Building?

Answer: ______________

GEOMETRY I

Section A

LINES:

When we think of lines, we think of: Horizontal lines, vertical lines, parallel lines, and intersecting lines.

1. Horizontal lines are lines that extend from left to right as the horizon does. Examples of horizontal lines are lines on writing paper and all level lines which we find in man-made structures.

2. Vertical lines are lines that go straight up and down with respect to the earth. Examples of vertical lines are telephone poles and trees.

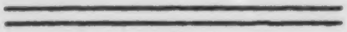
3. Parallel lines are lines that have the same direction and are always the same distance apart. An example of parallel lines are railroad tracks. They are equal distance apart and will never meet no matter how far they extend. They may be vertical, horizontal, or oblique (slanting). The symbol // means " is parallel to."

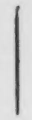
4. Intersecting lines are lines that cross each other. They meet only at one point. An example of intersecting lines would be the lines of roads intersect at corners. Also vertical and horizontals of graph paper intersect.

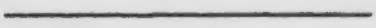
SECTION A
EXERCISES

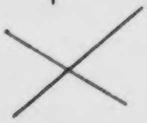
Complete the following exercises. If you miss any questions, discuss those questions and any others you feel unsure about with your instructor. If you do not miss any questions, and if you feel comfortable with the material you have covered, go on to Section B.

1. Label each of the following lines by writing the name under it.

(A) 

(B) 

(C) 

(D) 

2. Parallel lines may be __________, ________, or _______________ .

3. Lines on writing paper are examples of ____________________ lines.

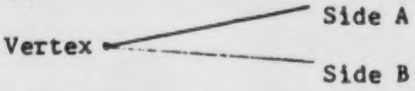
4. Telephone poles are examples of________ lines.

5. Lines that cross each other are called ___________ lines.

Section B
BASIC TYPES OF ANGLES

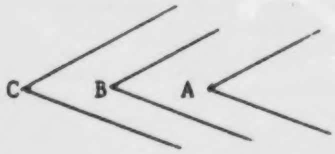
An angle is formed when two straight lines meet at a point. The lines are called the sides of the angle. The point at which the sides meet is called the vertex of the angle.

Example:



The size of an angle depends upon the amount one side has turned away from the other. The length of the sides of an angle does not determine the size.

Example:

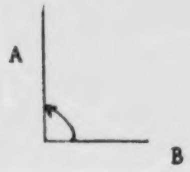


Angles C, B, and A are equal

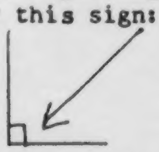
The unit of measure used in measuring an angle is the degree.° The symbol used the represent angle is .

1. Right Angle If one side of an angle turns a quarter of a complete circle away from the other side. The angle that is formed is a right angle.

Example:

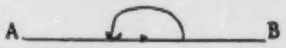


The right angle contains 90° and i represented by this sign:



We also refer to lines that meet to form right angles as being perpendicular lines.

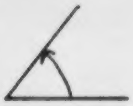
2. Straight Angle If one side of an angle turns half of a complete circle away from the other side, the angle that is formed is a straight angle. The sides of a straight angle lie in the same straight line.
Example:



Notice that a straight angle is twice the size of the right angle since in a straight angle the side has made half a complete turn or two quarter turns. The number of degrees in a straight angle is 180°.

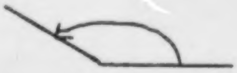
more than
less than

3. ACute Angle If one side of an angle turns less than a quarter of a circle away from the other side, the angle formed is an acute angle. An acute angle, therefore, is smaller than a right angle, or less than 90°. Example:



$<90^\circ$

4. Obtuse Angle If one side of an angle turns more than a quarter circle but less than a half circle, the angle formed is an obtuse angle. Therefore, an obtuse angle is greater than a right angle but smaller than a straight angle. It contains more than 90° but less than 180°. Example:

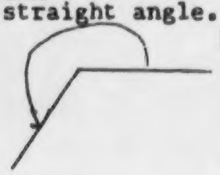


>90

<180

5. Reflex Angle If one side of an angle turns more than half a circle (180°) but less than a complete circle (360°) away from the other side, the angle formed is a reflex angle. Therefore, a reflex angle is greater than a straight angle.

Example:



>180
<360

SECTION B
EXERCISES

Complete the following exercises. If you miss any questions, discuss those questions and any other questions you feel unsure about with your instructor. If you do not miss any questions, and if you feel comfortable with the material you have covered, go on to Section C.

1. How many degrees are there in a right angle? ________________

2. Write the symbol that represents a right angle. ______________

3. A straight angle contains ________________.

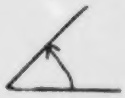
4. An acute angle contains less than __________.

5. An obtuse angle contains more than _______ but less than ___________.

6. An angle that contains more than 180° but less than 360° is called a/an _______________________.

7. In each of these figures in this question, tell what type of an angle it is by writing its name under it.

(A)



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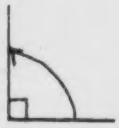
(B)



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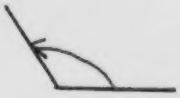
(C)



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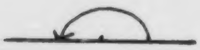
(D)



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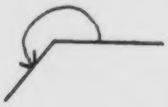


(E) __________________

8. What type of angle is in each corner of a square? _________________

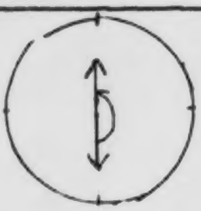
9. What kind of angle is each corner of an octagonal stop sign?

____

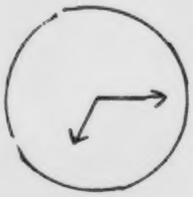
10. Lines that meet to form right angles are called ___________________ lines.

11. Name the type of angle that is formed by the hands of a clock at each of the following hours and write the number of degrees it represents - such as - less than 90°, 180°, or greater than 90°, 180°.

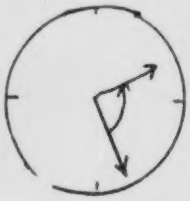
A. 6:00 ______________



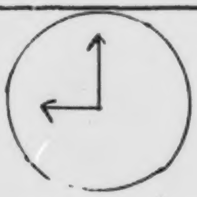
B. 7:15 ∠180° >90° ______________



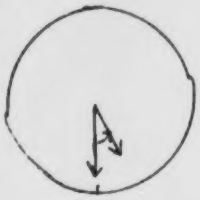
C. 2:25 ______________



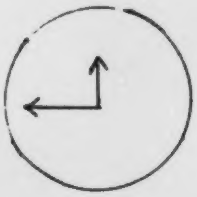
D. 9:00 ______________



E. 4:30 ______________



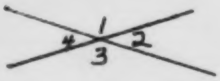
F. 12:45 ______________



Section C
PAIRS OF ANGLES

1. Vertical angles or opposite angles. When two lines cross each other four angles are formed.

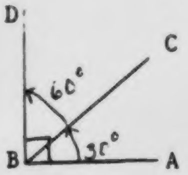
Example:



Since opposite angles have been formed by the same amount of turning, they are equal. Therefore, $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$.

2. Complementary angles. When two angles put together form a right angle, and thus their sum of 90° , the angles are complementary.

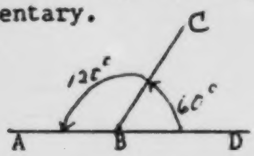
Example:



Angle ABC is the complement of angle CBD since their sum (60° + 30°) equals 90° .

3. Supplementary Angles. When the sum of two angles is 180° , the angles are said to be supplementary.

Example:



Angle ABC is the supplement of angle CBD since their sum (120°+ 60°) is 180° .

It was already mentioned that the basic unit of measure for angles is a degree. However, each degree may be subdivided into 60 minutes and each minute may be further subdivided into 60 seconds. The symbol for minutes is a single quotation mark (') and for seconds is a double quotation mark ("). Therefore, when we think of degrees, we can associate it with time.

Now, lets try breaking down an angle of 90° (complementary) into minutes and seconds.

1st. $90^\circ = 89^\circ\ 60'$

Note: In this process a degree was taken and changed into minutes.

2nd. $90^\circ = 89^\circ\ 59'\ 60''$

Therefore, we see that $89^\circ\ 59'\ 60'' = 90$.

3rd. The same process is used when breaking down an angle of 180° (supplementary) into minutes and seconds.

$180^\circ = 179^\circ\ 60'$

$180^\circ = 179^\circ\ 59'\ 60''$

This is necessary to know when you are to find a complement or supplement of a given angle.

Let's try a couple for practice.

We are asked to find the complement of $28^\circ\ 14'\ 58''$. We have already learned that two complementary angles equal 90° . Therefore we have to subtract $28^\circ\ 14'\ 58''$ from 90°.

Yes, before we can do this we have to break down the 90° into minutes and seconds so we can subtract. Let's do that now.

$90^\circ = 89^\circ\ 59'\ 60''$

Now we can find the complement by subtracting $28^\circ\ 14'\ 58''$ from $89^\circ\ 59'\ 60''$. Let's do that now.

$$\begin{array}{rrrr} & 89^\circ & 59' & 60'' \\ - & 28^\circ & 14' & 58'' \\ \hline & 61^\circ & 45'' & 2'' \end{array}$$

$$\begin{array}{lll} 89^\circ & 59' & \\ \cancel{90}^\circ & \cancel{60} & 60'' \end{array}$$

Now we have found the complement of 28° 14' 58" to be 61° 45' 2". Remember if we add them together we will get 90° because two complementary angles equal 90°.

See:
$$\begin{array}{rrrr} & 28^\circ & 14' & 58'' \\ + & 61^\circ & 45' & 2'' \\ \hline & 89^\circ & 59' & 60'' \end{array} = 90^\circ$$

We use the same process for finding the supplement of an angle. Let's try one:

We are asked to find the supplement of 28° 14' 58". Remember, supplementary angles equal 180°.

1st. We have to change 180° into minutes and seconds.

$$180^\circ = 179^\circ\ 59'\ 60''$$

Now we can find the supplement of 28° 14' 58" by subtracting it from 179° 59' 60". Let's do that:

$$\begin{array}{rrrr} & 179^\circ & 59' & 60'' \\ - & 28^\circ & 14' & 58'' \\ \hline & 151^\circ & 45' & 2'' \end{array}$$

179° 59'
~~180~~ ~~60~~ 60"

Now we have found the supplement of 28° 14' 58" to be 151° 45' 2". Remember, if we add them together we should get 180°, because two supplementary angles equal 180°.

See:
$$\begin{array}{rrrr} & 28^\circ & 14' & 58'' \\ + & 151^\circ & 45' & 2'' \\ \hline & 179^\circ & 59' & 60'' \end{array} = 180^\circ$$

Complete the following exercises. If you miss any questions, discuss those questions and any other questions you feel unsure about with your teacher. If you do not miss any questions and if you feel comfortable with the material you have covered, go on to Section D.

1. When two lines cross each other, four angles are formed and the opposite angles are____________.

2. When two complementary angles are put together, they form a ______ angle which contains________degrees.

3. When two supplementary angles are put together, they form a ________ angle which contains__________degrees.

4. Find the complement of 43 18' 25"

5. Find the supplement of 124 42' 10"

6. Find the complement of 62 25' 12"

7. Find the supplement of 62 25' 12"

8. (') is a symbol that represents________.

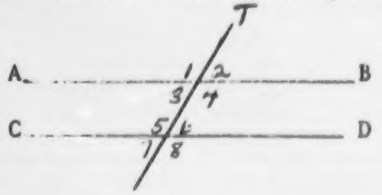
9. (") is a symbol that represents______________.

10. True or False? Supplementary angles are twice as large as the complementary angles.____________

PARALLEL LINES AND THEIR ANGLES

As was stated earlier, two or more lines are parallel if they never meet, no matter how far they are extended, and the symbol $\parallel$ means "is parallel to".

A line drawn cutting across parallel lines is called a transversal. In the diagram below lines AB and CD are parallel, and are cut by the transversal,



When parallel lines are cut by a transversal:

1. The sum of the angles across from each other equals 180°.

   $\angle 1 + \angle 2 = 180^\circ$
   $\angle 3 + \angle 4 = 180^\circ$
   $\angle 5 + \angle 6 = 180^\circ$
   $\angle 7 + \angle 8 = 180^\circ$

2. The corresponding (matching angles) are equal.

   $\angle 2 = \angle 6$
   $\angle 4 = \angle 8$
   $\angle 1 = \angle 5$
   $\angle 3 = \angle 7$

3. The alternate interior angles are equal.

   $\angle 3 = \angle 6$
   $\angle 4 = \angle 5$

4. The alternate exterior angles are equal.

   $\angle 1 = \angle 8$
   $\angle 2 = \angle 7$

Remember: It is the position of the angles that counts.
The numbers are used for labeling.

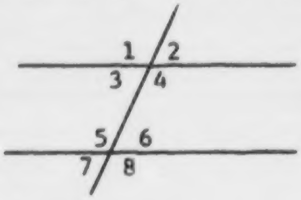
5. The consecutive interior angles are supplementary.

$$\angle 4 + \angle 6 = 180$$
$$\angle 3 + \angle 5 = 180$$

From this information given, if we know the degrees of one angle we can find the degrees of all the angles of parallel lines cut by a transversal.

Let's do one for fun.

In the figure below it was given to us that $\angle 3 = 60$. Let's see what the other angles equal.



1st. We know $\angle 3 + \angle 4 = 180$; therefore, we can subtract 60° (which was given to us) from 180° and get $\angle 4$.

$$\begin{array}{r} 180^\circ \\ -\ 60^\circ \\ \hline 120^\circ \end{array}$$

We see $\angle 4 = 120^\circ$

2nd. Since we know that opposite angles are equal we can figure $\angle 1$ and $\angle 2$. $\angle 1$ is opposite $\angle 4$ so it equals 120° . $\angle 2$ is opposite $\angle 3$ so it equals 60° .

3rd. We also know that corresponding (matching) angles are equal; therefore, we know

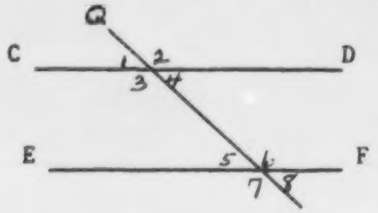
$\angle 2 = \angle 6$, $\angle 6 = 60^\circ$
$\angle 4 = \angle 8$, $\angle 8 = 120^\circ$
$\angle 1 = \angle 5$, $\angle 5 = 120^\circ$
$\angle 3 = \angle 7$, $\angle 7 = 60^\circ$

See: by knowing one angle you can find all the angles of parallel lines cut by a transversal.

EXERCISES

Complete the following exercises. If you miss any questions, discuss those questions and any other questions you feel unsure about with your instructor. If you don't miss any questions and if you feel comfortable with the material you have covered then you may go on to the next section.

In the diagram below, lines CD and EF are parallel and are cut by transversal Q.



Given: ∠ 5 = ∠ 45 Find the degrees for the rest of the angles.

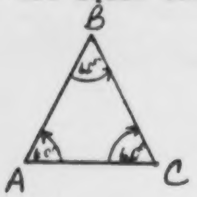
1. ∠ 1 = ________
2. ∠ 2 = ________
3. ∠ 3 = ________
4. ∠ 4 = ________
5. ∠ 6 = ________
6. ∠ 7 = ________
7. ∠ 8 = ________

TYPES OF TRIANGLES

A triangle is a plane figure bounded by three straight lines and containing three angles. The sum of the angles of every triangle is 180°.

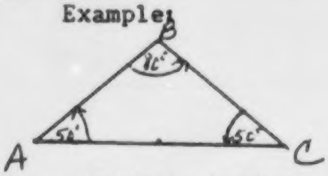
1. An equalateral triangle is a triangle having three equal sides. All three of its angles are equal each containing 60 .

Example:



2. An Isosceles triangle has two equal sides. The equal sides are called the legs (AB & BC). The third side (AC) is called the base. The angles opposite the equal sides called base angles, are also equal.

Example:

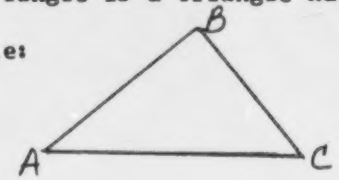


AB = BC

∠BCA = ∠ BAC

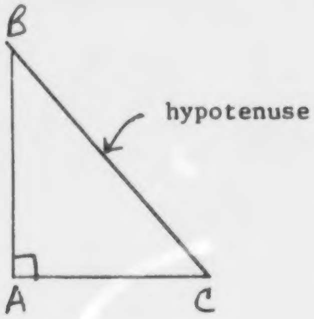
3. A scalene triangle is a triangle having no equal sides.

Example:



4. A right triangle is a triangle which contains one right angle (90). No more than one angle of a triangle can be a right angle. In a right triangle, the side opposite the right angle is called the hypotenuse.

Example:



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Complete the following exercises. If you miss any questions discuss those questions and any other questions you feel unsure about with your instructor. If you don't miss any questions and if you feel comfortable with the material you have covered, then you may go on to the next section.

Match Column II with Column I. There is one definition for each term.

I	II
( ) A. Scalene triangle	1. All three sides and angles are equal.
( ) B. Right triangle	2. The sum of its angles is 180.
( ) C. Isosceles triangle	3. No two sides are equal.
( ) D. Equalateral triangle	4. Two sides only are equal and two angles are equal.
( ) E. Triangle	5. It contains an angle of 90 and has a side called the hypotenuse.

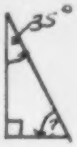
FINDING THE DEGREES OF AN ANGLE IN A TRIANGLE

1st. In order to find the degrees of one angle in a triangle, you need to know the degrees contained in the other two angles. This rule holds true for all triangles except the right triangle and the isosceles triangle.

In a right triangle you already know that one angle contains 90 ; therefore, you just need to know the degrees contained in one other angle in order to find the third angle.

In the isosceles △ , you only need to know the degrees of one base angle in order to find the third angle. Remember, in an isosceles △ , base angles are equal.

Now let's try finding the degrees contained in angle C of the below.



Given: △ABC is a right △ ∠B = 35

1st. It was given to us that △ ABC is a right △ . Therefore, we know ∠ A contains 90°. It was also given to us that ∠ B contains 35° . Now, since we know two angles, we can find the degrees contained in the third angle. Do you know how?

2nd. Since we know a △ contains 180° , we can add the two known angles together and subtract that from 180° which will give us the degrees contained in the unknown angle. (∠C)
Let's look at the process:

$$90^\circ + 35^\circ = 125^\circ$$
$$180^\circ - 125^\circ = 55^\circ$$

3rd. We have found that ∠C contain 55° . Remember all three angles have to equal 180° ; therefore, we can check our answer by adding all three angles to see if our total is 180° .
Let's check our answer.

$$90^\circ + 35^\circ + 55^\circ = 180^\circ$$

See how easy it is to check your answer.

Remember you always need to know two angles of a △ in order to find the third ∠ . Work your practice exercises to make sure you understand the process just presented to you.

EXERCISES

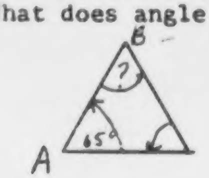
Complete the following exercises. If you miss any questions, discuss those questions and any other questions you feel unsure about with your instructor. If you don't miss any questions, and if you feel comfortable with the material you have covered, then you may go on to the next section.

1. Find the third angle of a triangle if the first two angles contain 14° and 127° .

   ____

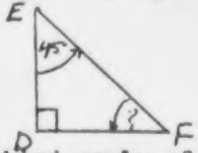
2. △ ABC is an isosceles △ . ∠A which is a base angle and equals 65° . What does angle B equal?

   ____



3. DEF is a right triangle. Angle E equals 45° . Find angle F

   ____



4. Find the third angle of a △ if the first two angles contain 42° and 78°.

   ____

5. Find the third angle of a △ if the first two angles contain 55° and 40°.

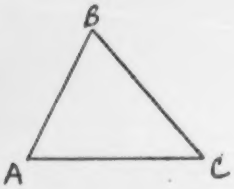
   ____

SIMILAR TRIANGLES AND CONGRUENT TRIANGLES

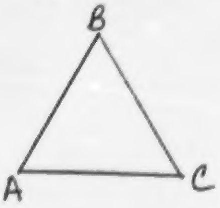
A. Similar Triangles

Two triangles are similar if the three angles of one triangle are equal to the three corresponding angles of another triangle. Similar triangles have identical shapes, but different sizes. Look at the examples below:

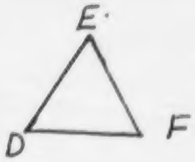
1.



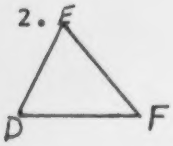
3.



4.



2.



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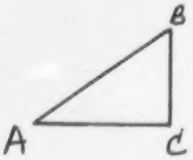

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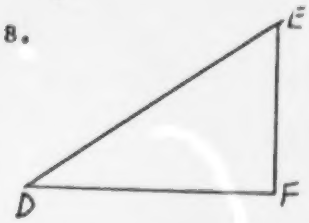
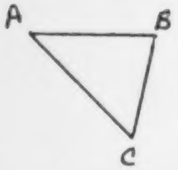
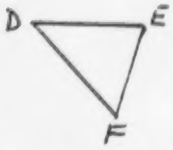


5.



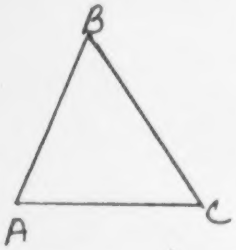
6.

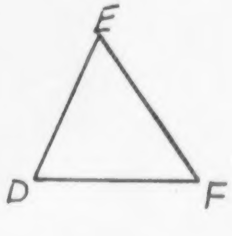


Can you tell that Δ's 1 and 2 are similar; 3 and 4 are similar; 5 and 6 are similar; and 7 and 8 are similar?

B. Congruent Triangles

Δ's are congruent when each side and each angle of one triangle is equal to the corresponding side and the corresponding angle of the other triangle. If a Δ is placed on top of another and they fit exactly in every respect, they are congruent. The sign used to show that two aagles are congruent is $\cong$. Look at the example below:



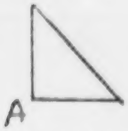


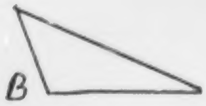
Congruent $\triangle$'s differ from similar $\triangle$'s in the respect that their sides are equal as well as their angles.

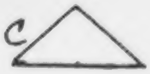
EXERCISES

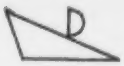
Complete the following exercises. If you miss any questions, discuss those questions and any other questions you feel unsure about with your instructor. If you don't miss any questions and if you feel comfortable with the material you have covered, then you may go on to the next section.

1. Circle the two $\triangle$'s below that are similar.









2. What is the sign used to show that $\triangle$'s are congruent?

3. Two $\triangle$'s are similar (equal) if their corresponding ________ are equal.

4. How do congruent $\triangle$'s differ from similar $\triangle$'s?

____

GEOMETRY I - Answer Key

Page 2

1. A. Parallel
   B. Vertical
   C. Horizontal
   D. Intersecting

2. Vertical, Horizontal, or Oblique (slanting)
3. Parallel or Horizontal
4. Vertical
5. Intersecting

Page 4-6 Section B, Exercises

1. 90°
2. ∟
3. 180°
4. 90°
5. 90°, 180°
6. A Reflex Angle
7. A. Acute
   B. Right
   C. Obtuse
   D. Straight
   E. Reflex
8. Right or 90°
9. Obtuse
10. Perpendicular
11. A. 180° straight
    B. Obtuse ∠ 180°
    C. Acute < 90°
    D. Right 90°
    E. Acute∠90°
    F. Right 90°

Page 10 Exercises

1. Equal
2. Right, 90°
3. Straight, 180°
4. 46° 41' 35"
5. 55° 17' 50"
6. 27° 34' 48"
7. 117° 34' 48"
8. Minutes '
9. Seconds "
10. True

Page 13 Exercises

1. 45°
2. 135°
3. 135°
4. 45°
5. 135°
6. 135°
7. 45°

Page 15 Exercises

A. 3
B. 5
C. 4
D. 1
E. 2

Page 17 Exercises

1. 39°
2. 50°
3. 45°
4. 60°
5. 85°

Page 19 Exercises

1. B & D
2. $\cong$
3. Angles
4. All sides and angles are equal

Geometry II

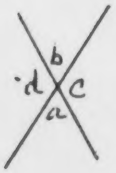
This hand-out is meant to provide additional practice with geometry and algebra. You will be combining skills of both.

Some things you must know:

1. vertical angles
2. supplementary angles
3. complementary angles
4. parallel lines
   - corresponding angles
   - alternate interior and exterior angles

1. and 2.

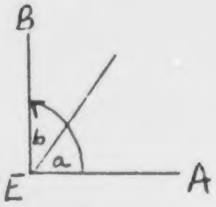
1)



∠a = ∠b
∠c = ∠d

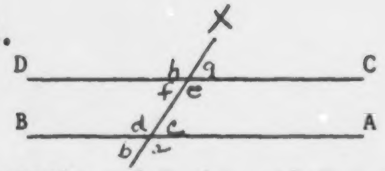
2) ∠a + ∠c = 180
∠c + ∠b = 180
∠b + ∠d = 180
∠d + ∠a = 180

3.



∠AEB is right angle = 90°
∠a + ∠b = 90°

4.

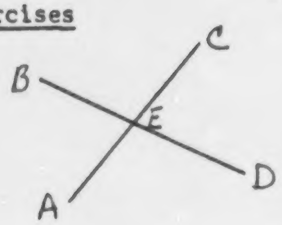


alternate ∠ d = ∠ e   ∠b = ∠ g alternate
interior ∠ c = ∠ f   ∠a = ∠ h exterior

corresponding ∠ a = ∠ e   ∠d = ∠ h
∠ b = ∠ f   ∠c = ∠ g

AB is parallel to CD
X is the transversal

Exercises

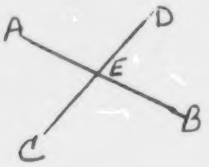


1. Line AC and BD intersect at E
   If $\angle AED = 7x$ and $\angle BEC = 9x - 8^\circ$, find the number of degrees in $\angle BEC$. ____________

2. If $\angle AED = 55^\circ\ 17'\ 5''$, find $\angle DEC$. ____________________

3. If $\angle AEB = 116^\circ\ 34'\ 29''$, find $\angle BEC$. ______ _____________

For problems 4-6, Lines AB and CD intersect at E.



4. $\angle AED = 42^\circ$, find the number of degrees in $\angle CEB$. ________

5. $\angle DEB = (4y + 7)^\circ$ $\angle CEA = (7y - 2)^\circ$, find the degrees in $\angle DEB$. ________

6. $\angle CEA = 123^\circ 30'\ 29''$, find the degrees in $\angle BEC$. ____________

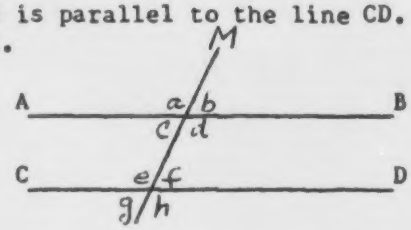
7. Find the complement of $(90-t)^\circ$ _____

8. Find the complement of $82^\circ\ 14'\ 59''$ _________________

9. Find the supplement of $(a + 170)^\circ$. _______________

10. Find the supplement of $108^\circ\ 14'\ 51''$. _______________

For problems 1-7, Line AB is parallel to the line CD. Line M is the transversal.



1. Indicate all pairs of angles which are equal.

a.		e.	=		i.	=	
b.	=	f.	=		j.	=	
c.	=	g.	=		k.	=	
d.	=	h.	=		l.	=	

2. If $\angle f = 47°$, find $\angle d$

3. If $\angle f = 47°$, find $\angle c$

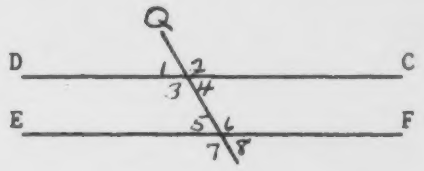
4. $\angle f + \angle h =$ ____________

5. $\angle a + \angle b =$ ____________

6. $\angle c + \angle d =$ ____________

7. $\angle e + \angle g =$ ____________

Problems 1-5



1. If $\angle 2 = X + 30°$, and $\angle 8 = X°$, find the degrees in both angles.

2. If $\angle 8 = 1/3 \angle 2$, find the degrees in both angles.

3. If $\angle 1 = 63°$, how many degrees are in $\angle 8$?

4. If $\angle 2 = 3X + 10$, and $\angle 1 = 6X - 10$ find both angles.

5. If $\angle 6$ is 12° more than two times $\angle 5$, find the degrees in both angles.

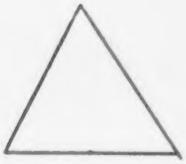
6. One angle is five times larger than another. If the angles are complementary, find the degrees in each angle.

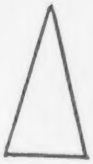
7. The complement of an angle is 6° less than the angle. Find the number of degrees in the angles.

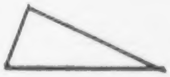
8. The supplement of an angle is 68° more than the angle. Find the number of degrees in the angles.

Triangles

Equilateral triangle	3 equal sides 3 equal angles = 180° Each angle = 60°
Isosceles	2 equal sides angles opposite the equal sides are equal
Scalene	no equal sides no equal angles
Right triangle	has 1 right angle = 90°







* The sum of the angles of any triangle is 180°

Example I

Find the third angle of a triangle if the first 2 angles contain 42° and 78°.

$42^\circ + 78^\circ = 120^\circ$

$180^\circ - 120^\circ = 60^\circ$

in whole

Example II

One angle is 63°. The difference between the other two angles is 13°. How many degrees are there in each of the other two angles?

$\angle 1 = 63^\circ$

$\angle 2 = X = 65^\circ$

$\angle 3 = X - 13 = 52^\circ$

$\angle 1 + \angle 2 + \angle 3 = 180^\circ$

$63^\circ + X^\circ + X - 13^\circ = 180^\circ$

$2X + 50^\circ = 180^\circ$

$2X = 180^\circ - 50^\circ$

$2X = 130^\circ$

$X = 65^\circ$

PRACTICE EXERCISE 1:
Triangles and their Angles

1. Find the third angle of a triangle if the first two angles contain 14° and 127°. ____

2. Find the number of degrees in each angle of a triangle if the angles are in the ratio 4:9:5. ____________

3. One angle of a triangle is 20°. The second angle is 10° larger than the third angle. Find the number of degrees in the second and third angles. ______________

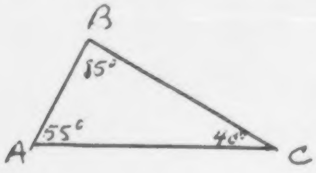
4. In triangle BCD, angle B is 5 times as large as angle D and angle C is 3 times as large as angle D. Find the number of degrees in each angle of triangle.

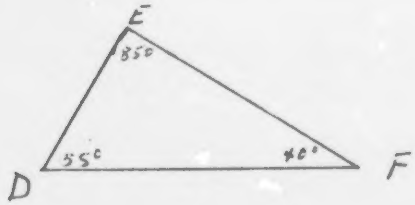
   ____

5. In triangle ABC, angle A is 1/2 as large as angle C, while B is 6 times as large as angle C. Find the number of degrees in each angle of the triangle. ______________________________

SIMILAR TRIANGLES

Two triangles are similar if the three angles of one triangle are equal to the three corresponding angles of another triangle. Similar triangles have identical shapes, but different sizes.



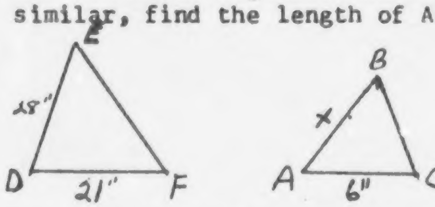


Triangles ABC and DEF are similar since all of their corresponding angles are equal.
The corresponding sides of similar triangles are those pairs of sides which are opposite equal angles. Thus, in the triangles above, AB corresponds to DE, AC corresponds to DF, and BC corresponds to EF.

Corresponding sides of similar triangles are in proportion.

EXAMPLE 1:

If the two triangles below are similar, find the length of AB.



Let AB = x. Since corresponding sides of similar triangles are in proportion:

$$\frac{x}{28} = \frac{6}{21}$$

By cross multiplication we get

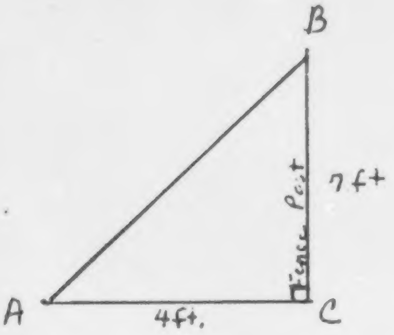
$$21x = 6\ (28)$$

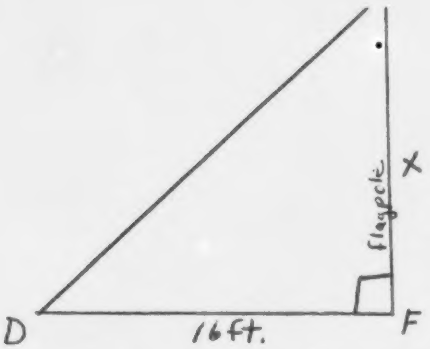
$$\frac{21x}{21} = \frac{168}{21}$$

$$x = 8$$

EXAMPLE 2:

If the sun casts a shadow 4 ft. long from a fence post 7 ft. tall and the shadow from a nearby flagpole is 16 ft. long, how tall is the flagpole? Assume that both fence post and flagpole are at right angles to the ground.





Since corresponding sides of similar triangles are in

$$\frac{7}{x} = \frac{4}{16}$$

By cross-multiplication we get

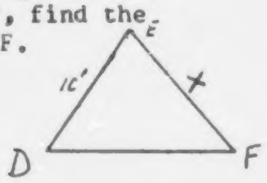
$$4x = 7\ (16)$$
$$\frac{4x}{4} = \frac{112}{4}$$
$$x = 28 \text{ ft.}$$

PRACTICE EXERCISE 2:

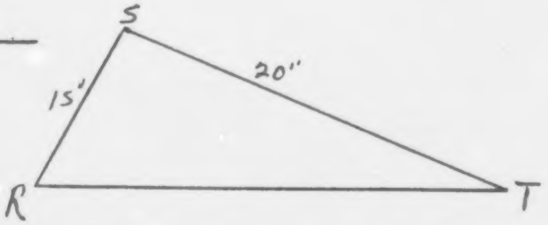
Similar Triangles

Answers to practice exercises appear at the end of the chapter

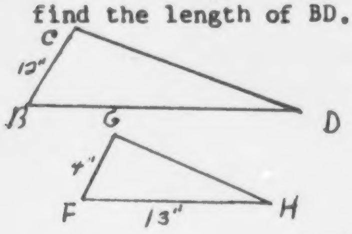
1. If the triangles below are similar, find the length of EF.



Answer:_______



2. If the triangles below are similar, find the length of BD.



Answer:______

3. A 12" ruler when held vertically casts a shadow 18" long. How long will the shadow of a vertically held yardstick be at the same time of day?

Answer: _________

4. The Empire State Building casts a shadow 360 ft. long at the same time that a nearby skyscraper 500 ft. tall casts a shadow 150 ft. long. How tall is the Empire State Building?

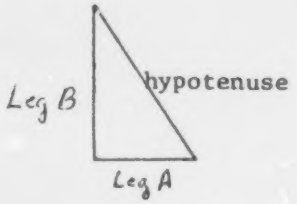
Answer: _________

5. A tree casts a shadow 54 ft. long. At the same time Steven, who is 5'8" tall, casts a shadow 9 ft. long. How tall is the tree?

Answer:________

The Pythagorean Theorem

Pythagoras, a Greek mathematician, discovered a special relationship among the three sides of all right triangles.

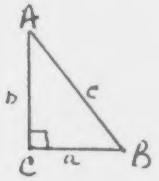


The side opposite the right angle is called the hypotenuse.

The other two sides are called legs.

The theorem states:

In any right triangle, the square of the hypotenuse (C^2) is equal to the sum of the square of the legs ($a^2 + b^2$)



For right triangle ABC,
$a^2 + b^2 = c^2$

By using your knowledge of how to solve equations and how to find square roots, it is possible to solve two basic types of problems involving the use of the Pythagorean theorem:

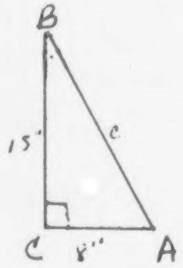
1. To find the length of the hypotenuse when the lengths of the two legs are given.

2. To find the length of one leg when the lengths of the hypotenuse and the other leg are given.

EXAMPLE 1:

Find the hypotenuse of a right triangle whose legs are 8" and 15".

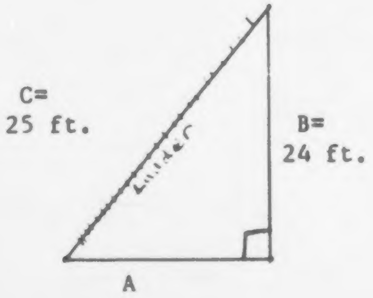
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 8^2 + 15^2 &= c^2 \\ 64 + 225 &= c^2 \\ 289 &= c^2 \\ \sqrt{289} &= c \\ 17 &= c \end{aligned}$$



EXAMPLE 2:

A ladder leaned against a building at a point 24 ft. from the ground. If the ladder is 25 ft. long, how far is the foot of the ladder from the base of the building?

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 24^2 &= 25^2 \\ a^2 + 576 &= 625 \\ -576 &= -576 \\ \hline a^2 &= 49 \\ a &= \sqrt{49} \\ a &= 7 \text{ ft.} \end{aligned}$$



Practice Exercise 3:
The Pythagorean Theorem

1. Find the hypotenuse of a right triangle whose legs are 12" and 16" long.

2. If the hypotenuse of a right triangle is 13" and one of the legs is 5", find the length of the other leg.

3. If Joy travels 18 miles north and then 24 miles west, how far will she be from her starting point?

4. A wire 26 feel long stretches from the top of a pole to a point on the ground 10 feet from the base of the pole. Find the height of the pole.

5. Find the length of the diagonal of a rectangle whose width is 7" and whose length is 24".

6. The diagonal of a children's pool table is 34" and its width is 16". Find its length.

Answer Key For Geometry II

Page 2

1. 28°
2. 124° 42' 55"
3. 63° 25' 31"
4. 42°
5. 19°
6. 56° 29' 31"
7. t°
8. 7° 45' 1"
9. (10-a)°
10. 71° 45' 9"

Page 3

1. a. < a= < d
   b. ∠ e= ∠ h
   c. ∠ b= ∠ c
   d. ∠ f= ∠ g
   e. ∠ d= ∠ e
   f. ∠ f= ∠ c
   g. ∠ h= ∠ a
   h. < b = < g
   i. ∠ a = ∠ e
   j. ∠ h = ∠ d
   k. ∠ b = ∠ f
   l. ∠ g = ∠ c
   also possible
2. 133°
3. 47°
4. 180°
5. 180°
6. 180°
7. 180°

Page 3

1. < 8 = 75°, < 2 = 105°
2. ∠ 8 = 45°, ∠ 2 = 135°
3. ∠ 8 = 63°
4. ∠ 2 = 70°, ∠ 1 = 110°
5. ∠ 5 = 56°, ∠ 6 = 124°
6. 1st = 15°, 2nd = 75°
7. 1st = 48°, 2nd = 42°
8. 1st = 56°, 2nd = 124°

Page 5 - Ex. 1

1. 39°
2. 40°, 90°, 50°
3. 85°, 75°
4. B = 100°, C = 60°, D = 20°
5. A = 12°, B = 144°, C = 24°

Page 6 - Ex. 2

1. 13 1/3"
2. 39"
3. 54"
4. 1200 ft.
5. 34 ft.

Page 9 - Ex. 3

1. 20"
2. 12"
3. 30 miles
4. 24"
5. 25"
6. 30"

STEPS TO MATH I

MATH PLACEMENT INVENTORY

1. Jim works 8 hours a day for 5 days a week. If he is paid \$4.00 an hour, how much does Jim earn each week? \$160.00

2. Bob works from 7:30 a.m. to 4:30 p.m. He takes an hour off for lunch. He earns \$24.88 per day. How much is this per hour? \$3.11

3. Sue has \$8.76. She pays \$2.35 for vegetables, \$1.20 for milk, and \$4.10 for meat. How much money does she have left? \$1.11

ADD

$\begin{array}{r} 7 \\ +3 \\ \hline 10 \end{array}$ $\quad$ $\begin{array}{r} 1 \\ 8 \\ +7 \\ \hline 16 \end{array}$ $\quad$ $\begin{array}{r} 33 \\ 25 \\ +41 \\ \hline 99 \end{array}$ $\quad$ $\begin{array}{r} 408 \\ 281 \\ \hline 689 \end{array}$

SUBTRACT

$\begin{array}{r} 7 \\ -5 \\ \hline 2 \end{array}$ $\quad$ $\begin{array}{r} 12 \\ -7 \\ \hline 5 \end{array}$ $\quad$ $\begin{array}{r} 87 \\ -34 \\ \hline 53 \end{array}$ $\quad$ $\begin{array}{r} 895 \\ -332 \\ \hline 563 \end{array}$

MULTIPLY

$\begin{array}{r} 7 \\ x3 \\ \hline 21 \end{array}$ $\quad$ $\begin{array}{r} 93 \\ x3 \\ \hline 279 \end{array}$ $\quad$ $\begin{array}{r} 301 \\ x\ 5 \\ \hline 1505 \end{array}$

DIVIDE

$\begin{array}{r} 32 \\ 3\overline{)96} \end{array}$ $\quad$ $\begin{array}{r} 21 \\ 7\overline{)147} \end{array}$ $\quad$ $\begin{array}{r} 71 \\ 4\overline{)284} \end{array}$

With a full load, Bob's truck weighs 57,423 pounds. The truck weighs 26,780 pounds. How much does the load weigh? 30,643

Bill earned $417.00 in May, $398.00 in June, and $365.00 in July. How much did he make in these three months? $1,180

STEPS TO MATH II

WATCH THE SIGN

$$\begin{array}{r} \$43.06 \\ 21.34 \\ 17.85 \\ +\ 6.02 \\ \hline \$88.27 \end{array} \qquad \begin{array}{r} \$17.34 \\ -\ 10.38 \\ \hline \$\ 6.96 \end{array} \qquad \begin{array}{r} \$43.89 \\ \times\ \ 4 \\ \hline \$175.56 \end{array}$$

$$\begin{array}{r} 1569 \\ 4\overline{)6276} \end{array} \qquad \begin{array}{r} 547 \\ 5\overline{)2735} \end{array} \qquad \begin{array}{r} \$13.16 \\ 6\overline{)\$78.96} \end{array}$$

Dave paid $87.32 for 2 new tires. How much did each tire cost? $43.66

Tires for Janet's car cost $26.85 each. How much will she pay for 4 new tires? $107.40

ADD

$$\begin{array}{r} 1542 \\ 2617 \\ 1928 \\ 2437 \\ +\ 1658 \\ \hline 10182 \end{array} \qquad \begin{array}{r} .1604 \\ 124.6 \\ 9.307 \\ +\ 17.08 \\ \hline 151.1474 \end{array} \qquad \begin{array}{r} 15\frac{1}{4} \\ 23\frac{1}{3} \\ \hline 38\frac{7}{12} \end{array}$$

SUBTRACT

$$\begin{array}{r} 48\frac{1}{4} \\ -15\frac{3}{4} \\ \hline 32\frac{1}{2} \end{array} \qquad \begin{array}{r} 1462.5 \\ -\ 476.352 \\ \hline 986.148 \end{array} \qquad \begin{array}{r} 706090 \\ -370804 \\ \hline 335286 \end{array}$$

MULTIPLY

$$\begin{array}{r} 738 \\ \times\ 635 \\ \hline 468{,}630 \end{array} \qquad \begin{array}{r} 1095 \\ \times\ 38 \\ \hline 41{,}610 \end{array}$$

DIVIDE

$$\begin{array}{r} 24 \\ 82\overline{)1968} \end{array} \qquad \begin{array}{r} 123 \\ 234\overline{)28782} \end{array}$$

PENTAGON 3-20, 20-43

Change to mixed number: $\frac{16}{3} = 5\frac{1}{3}$ Change to improper fractions: $3\frac{1}{3} = \frac{10}{3}$

Change as indicated: $\frac{1}{2} = \frac{2}{4}$ $\frac{2}{5} = \frac{4}{10}$

Change to decimals: $\frac{1}{10} = .1$ $\frac{7}{10} = .7$ $\frac{1}{2} = .5$ $\frac{3}{4} = .75$

Change to common fractions and reduce:

$.5 = \frac{1}{2}$ $.25 = \frac{1}{4}$ $.75 = \frac{3}{4}$ $.03 = \frac{3}{100}$

PENTAGON P. 82

Michael drove his car 207 miles on 9 gallons of gas. How many miles did he get to the gallon? 23 MILES

$9\overline{)207}$

From St. Louis to Little Rock is 348.9 miles. From St. Louis to San Antonio is 923.6 miles. How far is it from Little Rock to San Antonio? 574.7 MILES

SOLVE:

$\frac{3}{5} \times \frac{15}{16} = \frac{9}{16}$ $\frac{3}{4} \times 6\frac{2}{3} = 5$ $2\frac{4}{5} \times 7\frac{1}{2} = 21$ $64 \times \frac{3}{4} = 48$

$5 \div \frac{1}{5} = 25$ $\frac{1}{4} \div 2 = \frac{1}{8}$ $1\frac{1}{2} \div 1\frac{1}{5} = 1\frac{1}{4}$ $\frac{2}{3} \div \frac{4}{9} = 1\frac{1}{2}$

HEXAGON P. 46

$$\begin{array}{r} 2.604 \\ \underline{\times\ .03} \\ .07812 \end{array}$$

$$\begin{array}{r} 150 \\ .15\overline{)22.5} \end{array}$$

$$\begin{array}{r} 5 \text{ ft. } 9 \text{ in.} \\ \underline{+\ 2 \text{ ft. } 6 \text{ in.}} \\ 8 \text{ FT. } 3 \text{ IN.} \end{array}$$

$$\begin{array}{r} 21 \text{ yr. } 4 \text{ mo.} \\ \underline{-\ 12 \text{ yr. } 6 \text{ mo.}} \\ 8 \text{ YR. } 10 \text{ MO.} \end{array}$$

$$\begin{array}{r} 5 \text{ lb. } 12 \text{ oz.} \\ \underline{\times \qquad 4} \\ 23 \text{ LBS.} \end{array}$$

$$\begin{array}{r} 1 \text{ MIN. } 25 \text{ SEC.} \\ 5\overline{)7 \text{ min. } 5 \text{ sec.}} \end{array}$$

Fill in the missing numbers:

A square foot = 12 x 12 = 144 square inches.
A square yard = 3 x 3 = 9 square feet.
A cubic foot = 12 x 12 x 12 = 1728 cubic inches.
A cubic yard = 3 x 3 x 3 = 27 cubic feet.

A living room measures 18 ft. by 20 ft. How many square yards of carpet would be needed to cover this floor?

40 SQ. YDS.

A cubic foot of water equals 7.5 gallons. How many gallons are there in a full tank 12 feet long, 6 feet wide, and 5 feet deep?

2,700 GALLONS

How much fencing would be needed to enclose a lot measuring 50 ft. by 150 ft.?

400 FT.

HEXAGON

Change to percents

.20 = 20%

Change to decimals

50% = .5

Change to fractions

25% = $\frac{1}{4}$

Change to percents

$\frac{1}{3} = 33\frac{1}{3}\%$

$\frac{3}{8} = 37\frac{1}{2}\%$

Change to percents

$\frac{2}{3} = 66\frac{2}{3}\%$

$\frac{3}{10} = 30\%$

Change to decimals

$1\frac{1}{2}\% = .015$

$\frac{1}{4}\% = .0025$

HEPTAGON

Change to decimals:

$\frac{1}{10}\%$ = .001

$\frac{1}{2}\%$ = .005

Change to percents:

.015 = 1.5% -OR- 1½%

.075 = 7.5% -OR- 7½%

Change to mixed decimals:

210% = 2.1

125% = 1.25

Change to percents:

2.25 = 225%

1.10 = 110%

15% of 60 = 9

HEPTAGON

Using the circumference formula $C = \pi D$, find C when D is 28, ($\pi = \frac{22}{7}$).

C = 88

Using the formula V = LWH, find V when L is 30, W is 20, and H is 10.

V = 6,000

Using the circle area formula $A = \pi R^2$, find A when R is 14.

A = 616

Using the triangle formula A = ½AB, find A when A is 24 and B is 10.

120

Using the area formula A = LW, find A if L is 40 and W is 20.

800

A circle whose diameter is 28 has what radius?

14

In the perimeter formula P = 2L + 2W, find P when L is 300 and W is 200.

1,000

Heptagon, REFRESHER, OCTAGON OR ALGEBRA, DEPENDING ON RESULTS OF PREVIOUS PAGES

Using the cylinder volume formula $V = \pi R^2 H$, find V when R is 7 and H is 10.

1540

Wilbur borrowed $500.00 at 6% for 90 days. How much interest did he owe? $7.50

Agatha bought a hat for $7.50, which was at 25% discount. What was the original price of the hat? $10.00

10 is what percent of 40? 25%

Kathryn bought a $40.00 suit at a discount of 25%. How much did she pay for the suit? $30.00

Dan's bank pays 5½% interest. How much interest will he have in his savings account of $360.00 at the end of a year? $19.80

How much interest will be due on a note made out for $480.00 at 8% for 9 months? $28.80

Mr. Earnest shipped 450 crates of strawberries at $3.50 per crate. The market agent, Mr. James earned a commission of 8%. How much did Mr. Earnest receive? $1449

12½% of 400 = 50

½% of 200 = 1

Steps To Mathematics #1

Add:

1. 4
2
6

2. 8
0
8

3. 3
6
7
16

4. 3
8
4
15

5. 4
3
2
4
13

6. 5
1
3
9
18

7. 2
10
12

8. 53
56
109

9. 841
634
1475

10. 801
12
163
322
1298

11. 502
612
34
351
1499

12. 253
432
214
899

13. 201
128
660
989

14. 298
326
438
525
1587

15. 124 + 421 + 332 + 29 = 906

Subtract:

1. 8
-2
6

2. 9
-5
4

3. 15
-8
7

4. 57
-32
25

5. 65
-32
33

6. 499
-263
236

7. 568
-224
344

8. 735
-621
114

9. 479
-135
344

10. 331
-211
120

Multiply:

1. $\begin{array}{r} 3 \\ x1 \\ \hline 3 \end{array}$

2. $\begin{array}{r} 80 \\ x6 \\ \hline 480 \end{array}$

3. $\begin{array}{r} 11 \\ x9 \\ \hline 99 \end{array}$

4. $\begin{array}{r} 71 \\ x7 \\ \hline 497 \end{array}$

5. $\begin{array}{r} 300 \\ x4 \\ \hline 1200 \end{array}$

6. $\begin{array}{r} 901 \\ x9 \\ \hline 8109 \end{array}$

7. $\begin{array}{r} 911 \\ x8 \\ \hline 7288 \end{array}$

8. $\begin{array}{r} 321 \\ x4 \\ \hline 1284 \end{array}$

9. $\begin{array}{r} 800 \\ x9 \\ \hline 7200 \end{array}$

10. $\begin{array}{r} 201 \\ x6 \\ \hline 1206 \end{array}$

Divide:

1. $\begin{array}{r} 5 \\ 3\overline{)15} \end{array}$

2. $\begin{array}{r} 7 \\ 4\overline{)28} \end{array}$

3. $\begin{array}{r} 21 \\ 4\overline{)84} \end{array}$

4. $\begin{array}{r} 12 \\ 4\overline{)48} \end{array}$

5. $\begin{array}{r} 91 \\ 9\overline{)819} \end{array}$

6. $\begin{array}{r} 42 \\ 4\overline{)168} \end{array}$

7. $\begin{array}{r} 91 \\ 7\overline{)637} \end{array}$

8. $\begin{array}{r} 21 \\ 7\overline{)147} \end{array}$

9. $\begin{array}{r} 43 \\ 3\overline{)129} \end{array}$

10. $\begin{array}{r} 92 \\ 4\overline{)368} \end{array}$

1. Joe drives 300 miles in 5 hours. How many miles did he drive in 1 hour?

60

2. Bob bought 5 shirts. Each shirt cost $2.21. How much does he pay for all 5 shirts?

$11.05

3. Mary has $25.00. She bought groceries for $15.00. How much money does she have left?

$10.00

4. How much money would you have if you had a quarter, one dime, 2 nickles, and 4 pennies?

$.49

5. Tom bought a hammer for $3, a saw for $5, and a drill for $21. How much was his bill?

$29.00

THE END

MASTERY TEST

Steps to Mathematics #2

Add:

1. $$\begin{array}{r} 31\\ 3\\ 71\\ 54\\ \hline 159 \end{array}$$

2. $$\begin{array}{r} \$17.95\\ 21.03\\ 60.00\\ \hline \$98.98 \end{array}$$

3. $$\begin{array}{r} 131\\ 312\\ 342\\ 216\\ \hline 1001 \end{array}$$

4. $$\begin{array}{r} 35.04\\ 12.38\\ .26\\ 32.14\\ \hline 79.82 \end{array}$$

5. $$\begin{array}{r} \$52.43\\ 10.76\\ 3.07\\ 22.32\\ \hline \$88.58 \end{array}$$

Subtract:

1. $$\begin{array}{r} \$47.32\\ -25.89\\ \hline \$21.43 \end{array}$$

2. $$\begin{array}{r} \$17.05\\ -\ 3.98\\ \hline \$13.07 \end{array}$$

3. $$\begin{array}{r} \$46.60\\ -38.81\\ \hline \$\ 7.79 \end{array}$$

4. $$\begin{array}{r} \$27.55\\ -17.36\\ \hline \$10.19 \end{array}$$

5. $$\begin{array}{r} \$6.82\\ -4.75\\ \hline \$2.07 \end{array}$$

Multiply:

1. $$\begin{array}{r} \$43.65\\ 8\\ \hline \$349.20 \end{array}$$

2. $$\begin{array}{r} \$43.89\\ 4\\ \hline \$175.56 \end{array}$$

3. $$\begin{array}{r} \$10.03\\ 7\\ \hline \$70.21 \end{array}$$

4. $$\begin{array}{r} \$12.98\\ 4\\ \hline \$51.92 \end{array}$$

5. $$\begin{array}{r} \$23.14\\ 4\\ \hline \$92.56 \end{array}$$

Divide:

1. $$\begin{array}{r} 234\\ 8\overline{)1872} \end{array}$$

2. $$\begin{array}{r} 1316\\ 6\overline{)7896} \end{array}$$

3. $$\begin{array}{r} 107\\ 7\overline{)749} \end{array}$$

4. $$\begin{array}{r} \$1.41\\ 6\overline{)\$8.46} \end{array}$$

5. $$\begin{array}{r} 19.75\\ 5\overline{)98.75} \end{array}$$

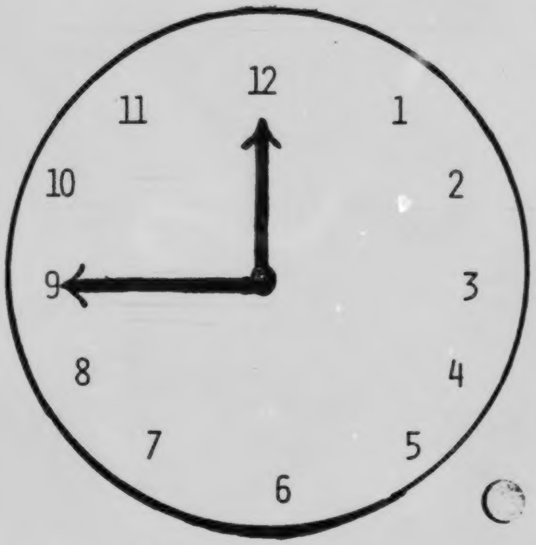
1. 1 cup = 8 ounces
2. 2 cups = 1 pints
3. 1 pint = 16 ounces
4. 1 quart = 2 pints
5. 1 quart = 3 ounces
6. 1 gallon = 4 quarts

7. Beans

Large Can	Small Can
24 ounces	6 ounces
$1.50 each	$.45 each

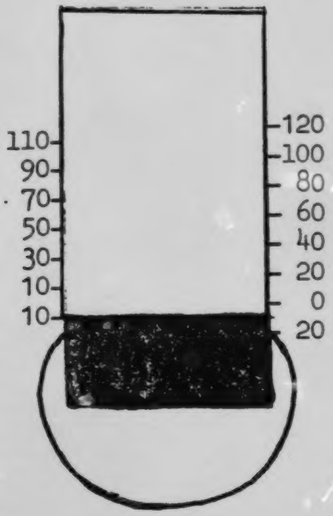
Vi bought 4 small cans of beans. How many ounces did she buy? 24 oz.

How much money could she have saved by buying one large can? $.30


12
1
2
3
4
5
6
7
8
9
10
11

8. What time does this clock show? 11:45

9.


110
90
70
50
30
10
10
120
100
80
60
40
20
0
20

What temperature is shown on this thermometer in degrees Fahrenheit?

10°

10. 1 day = 24 hours

1 year = 365 days

1 year = 52 weeks

THE END

Mastery Review I
WORKING WITH NUMBERS, Pentagon Book Revised

Name .... **Score** ....

Add.

1.
$$\begin{array}{r} \$.47 \\ .25 \\ .17 \\ .35 \\ .29 \\ \hline \$1.53 \end{array}$$

2.
$$\begin{array}{r} 214 \\ 354 \\ 196 \\ 248 \\ 185 \\ \hline 1,197 \end{array}$$

3.
$$\begin{array}{r} 1542 \\ 2617 \\ 1928 \\ 2437 \\ 1658 \\ \hline 10182 \end{array}$$

4.
$$\begin{array}{r} 363245 \\ 275487 \\ 328924 \\ \hline 967,656 \end{array}$$

5. Martha paid $4.35 for meat, $1.50 for flour, $2.45 for potatoes, $1.02 for coffee and $6.34 for canned goods. How much did she spend, in all?

Answer: $15.66

6.
$$\begin{array}{r} \$58.75 \\ -6.34 \\ \hline \$52.41 \end{array}$$

7.
$$\begin{array}{r} 6345 \\ -1469 \\ \hline 4,876 \end{array}$$

8.
$$\begin{array}{r} 56409 \\ -23045 \\ \hline 33,364 \end{array}$$

9.
$$\begin{array}{r} 954673 \\ -729779 \\ \hline 224,894 \end{array}$$

10. A freight car and its load weighed 63,450 pounds. The load alone weighed 51,325 pounds. How much did the car alone weigh?

Answer: 12,125 LBS.

11.
$$\begin{array}{r} \$7.65 \\ \times\ 8 \\ \hline \$61.20 \end{array}$$

12.
$$\begin{array}{r} 326 \\ \times 15 \\ \hline 1630 \\ 326 \\ \hline 4890 \end{array}$$

13.
$$\begin{array}{r} 1234 \\ \times 24 \\ \hline 29,616 \end{array}$$

14.
$$\begin{array}{r} 253 \\ \times 234 \\ \hline 59,202 \end{array}$$

15. $\begin{array}{r} 8 \\ 12\overline{)96} \end{array}$

16. $\begin{array}{r} 15 \\ 16\overline{)240} \end{array}$

17. $\begin{array}{r} 15 \\ 15\overline{)225} \end{array}$

18. $\begin{array}{r} 340 \\ 26\overline{)8840} \end{array}$

19. $\begin{array}{r} 500 \\ 8\overline{)4000} \end{array}$ **20.** $\begin{array}{r} 210 \\ 25\overline{)5250} \end{array}$ **21.** $\begin{array}{r} 311 \\ 26\overline{)8086} \end{array}$ **22.** $\begin{array}{r} 212 \\ 45\overline{)9540} \end{array}$

23. Write the Roman numerals for:

20 XX 50 L 40 XL 100 C

24. Write the numerals for:

X 10 XC 90 XL 40 LXX 70

25. In a six-day bicycle race Eric Peterson rode his bicycle 240 miles in 12 hours. How many miles an hour did he average for the 12 hours?

Answer: 20 MPH

26. Michael drove his car 207 miles on 9 gallons of gas. How many miles did he get to the gallon?

Answer: 23 MPG

27. On a trip last week Steven drove 405 miles in 9 hours. How many miles per hour did he average?

Answer: 45 MPH

28. The baker made 1440 cookies to be sold one day. How many dozen were made?

Answer: 120 DOZEN

29. Karen bought oranges at 2 for 40 cents. How much would she pay for a dozen?

$$\begin{array}{r} .40 \\ \times\ 6 \\ \hline \$2.40 \end{array}$$

Answer: $2.40

30. Pauline Johnson bought a dozen shirts at $12.25 each. How much did she pay for the 12 shirts?

$$\begin{array}{r} \$12.25 \\ \times\quad 12 \\ \hline 24.50 \\ 122\ 5 \\ \hline \$147.00 \end{array}$$

Answer: $147.00

Mastery Review II
WORKING WITH NUMBERS, Pentagon Book Revised

Name .... **Score** ....

Add.

1.
$$\begin{array}{r} 44 \\ 75 \\ 23 \\ 56 \\ 59 \\ \hline 257 \end{array}$$

2.
$$\begin{array}{r} \$12.45 \\ .29 \\ 8.17 \\ 4.78 \\ 12.45 \\ \hline \$38.14 \end{array}$$

3.
$$\begin{array}{r} 10644 \\ 26057 \\ 23179 \\ 47635 \\ 12548 \\ \hline 120,063 \end{array}$$

4.
$$\begin{array}{r} \frac{3}{8} \\ \frac{1}{8} \\ \frac{5}{8} \\ \hline 1\frac{1}{8} \end{array}$$

5.
$$\begin{array}{r} \frac{1}{4} \\ \frac{1}{2} \\ \hline \frac{3}{4} \end{array}$$

6.
$$\begin{array}{r} \frac{1}{2} \\ 12\frac{1}{4} \\ \hline 12\frac{3}{4} \end{array}$$

7.
$$\begin{array}{r} 5\frac{1}{2} \\ 9\frac{1}{6} \\ \hline 14\frac{2}{3} \end{array}$$

8.
$$\begin{array}{r} 15\frac{1}{4} \\ 23\frac{1}{3} \\ \hline 38\frac{7}{12} \end{array}$$

9. Fred can jump $5\frac{1}{2}$ feet. George can jump $4\frac{3}{4}$ feet. Their uncle can jump as far as the two boys together. How many feet can the uncle jump?

Answer: $10\frac{1}{4}$ feet

10.
$$\begin{array}{r} 65312 \\ -32409 \\ \hline 32,903 \end{array}$$

11.
$$\begin{array}{r} 706090 \\ -370804 \\ \hline 335,286 \end{array}$$

12.
$$\begin{array}{r} \frac{5}{8} \\ -\frac{1}{8} \\ \hline \frac{1}{2} \end{array}$$

13.
$$\begin{array}{r} \frac{2}{3} \\ -\frac{1}{6} \\ \hline \frac{1}{2} \end{array}$$

14.
$$\begin{array}{r} 7 \\ -\ \frac{1}{4} \\ \hline 6\frac{3}{4} \end{array}$$

15.
$$\begin{array}{r} 8\frac{1}{3} \\ -3\frac{1}{6} \\ \hline 5\frac{1}{6} \end{array}$$

16.
$$\begin{array}{r} 48\frac{1}{4} \\ -15\frac{3}{4} \\ \hline 32\frac{1}{2} \end{array}$$

17.
$$\begin{array}{r} 9\frac{1}{8} \\ -4\frac{1}{2} \\ \hline 4\frac{5}{8} \end{array}$$

18. See problem Number 9 above. Fred can jump how much farther than George?

Answer: $\frac{3}{4}$

19.

$$\begin{array}{r} 802 \\ \times 145 \\ \hline 116{,}290 \end{array}$$

20.

$$\begin{array}{r} 23045 \\ \times \quad 32 \\ \hline 737{,}440 \end{array}$$

21.

$$\begin{array}{r} 111 \\ 63\overline{)6993} \end{array}$$

22.

$$\begin{array}{r} 121 \\ 234\overline{)28314} \end{array}$$

23. Change as indicated.

$\frac{1}{2} = \frac{2}{4}$ $\frac{2}{5} = \frac{4}{10}$ $\frac{2}{3} = \frac{4}{6}$

Reduce to lowest terms.

$\frac{8}{12} = \frac{2}{3}$ $\frac{6}{10} = \frac{3}{5}$ $\frac{12}{16} = \frac{3}{4}$

24. Change to mixed numbers.

$\frac{7}{2} = 3\frac{1}{2}$ $\frac{12}{8} = 1\frac{1}{2}$ $\frac{16}{3} = 5\frac{1}{3}$

Change to improper fractions.

$3\frac{1}{3} = \frac{10}{3}$ $4\frac{1}{5} = \frac{21}{5}$ $4\frac{3}{4} = \frac{19}{4}$

25. Add.

$$\begin{array}{r} 12.06 \\ .12 \\ 2.27 \\ 4.24 \\ \hline 18.69 \end{array} \qquad \begin{array}{r} .017 \\ .3042 \\ 12.78 \\ .9 \\ \hline 14.0012 \end{array} \qquad \begin{array}{r} .1604 \\ 124.6 \\ 9.307 \\ 17.08 \\ \hline 151.1474 \end{array}$$

26. Subtract.

$$\begin{array}{r} .48 \\ -.3 \\ \hline .18 \end{array} \qquad \begin{array}{r} 7.26 \\ -.47 \\ \hline 6.79 \end{array} \qquad \begin{array}{r} 1462.5 \\ -476.352 \\ \hline 986.148 \end{array}$$

27. Change to decimals.

$\frac{1}{10} = .1$ $\frac{7}{10} = .7$ $\frac{1}{2} = .5$ $\frac{3}{4} = .75$

Change to common fractions.

$.5 = \frac{1}{2}$ $.25 = \frac{1}{4}$ $.75 = \frac{3}{4}$ $.03 = \frac{3}{100}$

28. Jack drove 576.8 miles to attend the football game. Bill drove 254.9 miles. How much farther was Jack's trip?

Answer: 321.9 miles

29. From Planta to Otis Junction is 17.4 miles. From Otis Junction to Villita is 14.5 miles farther. How far is it from Planta to Villita?

Answer: 31.9 mi.

30. Jamestown is 1205.45 feet above sea level. Jonesboro is 205.25 feet higher. What is the "above-sea-level" elevation of Jonesboro?

Answer: 1410.70 feet

Mastery Review I
WORKING WITH NUMBERS, Hexagon Book, Revised

Name .... **Score** ....

Perform the operation indicated. Watch the signs.

1.
$$\begin{array}{r} 459 \\ 765 \\ 19 \\ +1284 \\ \hline 2{,}527 \end{array}$$

2.
$$\begin{array}{r} 48103 \\ 57920 \\ 80507 \\ +36070 \\ \hline 222{,}600 \end{array}$$

3.
$$\begin{array}{r} 86490 \\ -37984 \\ \hline 48{,}506 \end{array}$$

4.
$$\begin{array}{r} 916543 \\ -419798 \\ \hline 496{,}745 \end{array}$$

5.
$$\begin{array}{r} \frac{2}{3} \\ \frac{3}{4} \\ +\frac{1}{2} \\ \hline 1\frac{11}{12} \end{array}$$

6.
$$\begin{array}{r} 5\frac{3}{10} \\ 6\frac{1}{2} \\ +8\frac{3}{5} \\ \hline 20\frac{2}{5} \end{array}$$

7.
$$\begin{array}{r} 15\frac{1}{2} \\ -\ 9\frac{2}{5} \\ \hline 6\frac{1}{10} \end{array}$$

8.
$$\begin{array}{r} 34 \\ -\ 5\frac{2}{3} \\ \hline 28\frac{1}{3} \end{array}$$

9. $\frac{2}{3}\times\frac{3}{4}= \frac{1}{2}$

10. $\frac{3}{5}\times\frac{15}{16}=\frac{9}{16}$

11. $\frac{3}{4}\times 5\frac{1}{3}= 4$

12. $2\frac{4}{5}\times 7\frac{1}{2}= 21$

13. $\frac{1}{5}\div 5= \frac{1}{25}$

14. $8\div\frac{1}{3}= 24$

15. $5\frac{1}{3}\div\frac{4}{9}= 12$

16. $3\frac{1}{3}\div 6\frac{2}{3}= \frac{1}{2}$

17.
$$\begin{array}{r} 209.4 \\ 17.06 \\ .907 \\ +\ \ 4. \\ \hline 231.367 \end{array}$$

18.
$$\begin{array}{r} .0708 \\ .9 \\ 5.206 \\ +\ .32 \\ \hline 6.4968 \end{array}$$

19.
$$\begin{array}{r} 7.906 \\ -3.46 \\ \hline 4.446 \end{array}$$

20.
$$\begin{array}{r} 14.4 \\ -\ 5.506 \\ \hline 8.894 \end{array}$$

21. Change to decimals.

$\frac{1}{2}=$ .5 $\quad \frac{1}{4}=$ .25 $\quad \frac{2}{5}=$ .4 $\quad \frac{7}{10}=$ .7 $\quad 3\frac{1}{2}=$ 3.5

22. Change to fractions.

$.5=\frac{1}{2} \quad .75=\frac{3}{4} \quad .30=\frac{3}{10} \quad .9=\frac{9}{10} \quad .80=\frac{4}{5}$

MT-10a

23. Write as decimals.

a. One-half .5 Seventy-five hundredths .75 One-fourth .25

b. Three-tenths .3 One-thousandth .001 Fifteen-thousandths .015

24. Place the decimal point where it belongs.

a. Seven and seven tenths: 7.7

b. Six hundred three and fifty-nine hundredths: 603.59

25. Six children divided three-fourths of a pie equally among themselves. What part of the pie did each receive?

Answer: $\frac{1}{8}$

26. At $2.40 per yard, what did Kenneth pay for 2½ yards of material?

Answer: $6.00

27. The Red Cross bought 10½ yards of material to cover 14 pillows. How much was used for each cover?

Answer: $\frac{3}{4}$ yd. or 27 mi.

28. Susan paid $5.25 for 1½ meters of material. How much was this per meter?

Answer: $3.50

29. Over the week end we drove 378.2 miles and 248.9 miles. How many miles, in all, did we drive?

Answer: 627.1 mi.

30. From St. Louis to Los Angeles is 2315.8 miles. In between is Salt Lake City, which is 1531.9 miles from St. Louis. How far is it from Salt Lake City to Los Angeles?

Answer: 783.9 mi.

THE END

Mastery Review II
WORKING WITH NUMBERS, Hexagon Book, Revised

Name .... **Score** ....

Perform the operation indicated. Watch the signs.

1.
13673
40092
39634
+25877
119,276

2.
$\frac{7}{12}$
$\frac{5}{6}$
$\frac{3}{4}$
$+\frac{1}{2}$
$2\frac{2}{3}$

3.
$2\frac{3}{4}$
$7\frac{3}{8}$
$6\frac{1}{2}$
$+4$
$20\frac{5}{8}$

4.
12.008
6.34
143.092
+ 19.8
181.240

5.
869073
−450978
418,095

6.
$\frac{1}{2}$
$-\frac{2}{5}$
$\frac{1}{10}$

7.
12.006
− 5.07
6.936

8.
29.6
−17.002
12.598

9.
24653
× 32
788,896

10. $\frac{3}{5}\times\frac{5}{8}=\frac{3}{8}$

11. $\frac{3}{4}\times 6\frac{2}{3}=5$

12.
2.604
×.03
.07812

13. $630\overline{)147220}$ 233 r 430

14. $5\div\frac{1}{5}=25$

15. $1\frac{1}{2}\div 1\frac{1}{5}=1\frac{1}{4}$

16. $.15\overline{)22.5}$ 150

17.
5 ft. 9 in.
+2 ft. 6 in.
8 ft. 3 in.

18.
21 yr. 4 mo.
−12 yr. 6 mo.
8 yr. 10 mo.

19.
5 lb. 12 oz.
× 4
23 lbs.

20. 5) 7 min. 5 sec.
1 min. 25 sec.

21. At two for a nickel, how much would a dozen pencils cost?

Answer: 30¢

22. At the store, lettuce sells at 2 heads for 98 cents. At the farmers' market, lettuce sells for $5.00 per dozen heads. How much cheaper is the farmers' market price?

Answer: 88¢

23. At $.08 per pound, how much would 1000 lbs. of cabbage cost?

Answer: $80.00

24. From St. Louis to Kansas City is 278.97 miles. One train makes the trip in 5 hours and 6 minutes. What is the average speed? (Change time to decimal form.)

Answer: 54.7

25. A bricklayer worked 7½ hours per day for five days. She was paid $9.75 per hour. How much did she earn?

Answer: $365.625

26. A wheat field measures 70 rods by 90 rods. How many rods of fence would be needed to completely enclose it?

Answer: 320 rods

27. At 12 cents per square foot, what would be the cost of sanding a gymnasium floor which measures 100 feet by 60 feet?

Answer: $720.00

28. A living room floor measures 18 feet by 20 feet. How many square yards of carpet would be needed to cover this floor?

Answer: 40 sq. yds.

29. How many millimeters are in 75 meters?

Answer: 75,000 cu. ft.

30. Fill in the missing numbers.

A square foot = 12 × 12 square inches.

A square yard = 3 × 3 square feet.

A cubic foot = 12 × 12 × 12 cubic in.

A cubic yard = 3 × 3 × 3 cubic feet.

MT-13a
THE END

Mastery Review 1

WORKING WITH NUMBERS, Heptagon Book, Revised

Name ______________________________ **Score** ____________

1. Add.

$$\begin{array}{r} \frac{7}{16} \\ 5\frac{1}{2} \\ 3\frac{1}{4} \\ \frac{5}{8} \\ \hline 9\frac{13}{16} \end{array}$$

2. Add.

$$\begin{array}{r} .9 \\ 108.064 \\ 7.42 \\ 25.001 \\ \hline 141.385 \end{array}$$

3. Subtract.

$$\begin{array}{r} 15\frac{3}{8} \\ 9\frac{7}{16} \\ \hline 5\frac{15}{16} \end{array}$$

4. Subtract.

$$\begin{array}{r} 156.059 \\ 79.48 \\ \hline 76.579 \end{array}$$

5.

$$\begin{array}{r} 205.16 \\ \times \quad 2.9 \\ \hline 594.964 \end{array}$$

6. $3\frac{1}{8} \times 1\frac{3}{5} = 5$

7. $6\frac{3}{4} \div 1\frac{1}{2} = 4\frac{1}{2}$

8. $.125\overline{)\,5.}$ 40

9. Change to percents.

.20 = 20%

.6 = 60%

.01 = 1%

.025 = $2\frac{1}{2}$%

10. Change to decimals.

50% = .5

1½% = .015

62.5% = .625

40% = .4

11. Change to fractions.

25% = $\frac{1}{4}$

20% = $\frac{1}{5}$

12½% = $\frac{1}{8}$

75% = $\frac{3}{4}$

12. Change to percents.

$\frac{1}{3}$ = 33%

$\frac{3}{8}$ = $37\frac{1}{2}$%

13. Change to percents.

$\frac{2}{3}$ = $66\frac{2}{3}$%

$\frac{3}{10}$ = 30%

14. Change to decimals.

$1\frac{1}{2}$ % = .015

$\frac{1}{4}$ % = .0025

15. Change to decimals.

$\frac{1}{10}$ % = .001

$\frac{1}{2}$ % = .005

16. Change to percents.

.015 = $1\frac{1}{2}$%

.075 = $7\frac{1}{2}$%

17. Change to percents.

.001 = $\frac{1}{10}$ %

.105 = $10\frac{1}{2}$%

18. **Change to mixed decimals.**

210% = 2.1

125% = 1.25

19. **Change to mixed decimals.**

240% = 2.40

501% = 5.01

20. **Change to percents.**

2.25 = 225%

1.10 = 110%

21. **Change to mixed decimals.**

540% = 5.4

275% = 2.75

22. 40% of 250 = 100 15% of 60 = 9 225% of 100 = 225

23. Joanne bought a coat at a 25%-discount sale. The original price of the coat was $32. How much did she save?

Answer: $8.00

24. Tommy bought a $16.75 shirt at 20% discount. How much did he save?

Answer: $3.35

25. How much interest will be due on $600 at 12% for 2 years?

Answer: $144.00

26. Steven borrowed $750 at 12% for 8 months. How much interest will he pay?

Answer: $60.00

27. How much interest will be due on $450 at 14% for 90 days?

Answer: $15.75

28. Mrs. Cook works on a commission of 25%. She sold $5000 worth of merchandise last month. What was her commission?

Answer: $1,250.00

29. A commission house sold 1200 bushels of potatoes at $2.50 per bushel. The commission charged was 5%. How much was the commission?

Answer: $150.00

30. If the freight charges were $130, how much would the person who sent the potatoes net after the commission and freight charges were deducted?

Answer: $2,720

MT-15a
THE END

Mastery Review 2
WORKING WITH NUMBERS, Heptagon Book, Revised

Name ______________________________ **Score** __________

1. Add.
$5\frac{1}{8}$
$\frac{5}{16}$
$2\frac{1}{4}$
$9\frac{1}{2}$
$17\frac{3}{16}$

2. Add.
3096.5
2578.79
4847.408
3648.02
14170.718

3. Subtract.
$26\frac{1}{2}$
$17\frac{5}{8}$
$8\frac{7}{8}$

4. Subtract.
2864.7
978.253
1886.447

5.
125.04
× 3.7
462.648

6. $3\frac{1}{3} \times 2\frac{1}{4} = 7\frac{1}{2}$

7. $4\frac{1}{2} \div \frac{3}{8} = 12$

8. $100\overline{)125.4}$ 1.254

9. In the perimeter formula $P = 2l + 2w$, find P when l is 300 and w is 200.

Answer: 1,000

10. Using the same formula, find the cost of enclosing with fence at $2 per meter a lot measuring 50 meters by 20 meters.

Answer: $2,000

11. Using the area formula $A = lw$, find A if l is 40 and w is 20.

Answer: 800

12. Using the same area formula, what is the cost of a 9′-by-12′ rug at $9.50 per sq. yd.?

Answer: $114.00

13. Using the formula $V = lwh$, find V when l is 30, w is 20, and h is 10.

Answer: 6,000

14. Using the triangle formula $A = \frac{1}{2}ab$, find A when a is 24 and b is 10.

Answer: 120

15. Using the circumference formula $C = \pi d$, find C when d is 28. ($\pi = 22/7$)

Answer: 88

16. A circle whose diameter is 28 has what radius?

Answer: 14

17. Using the circle area formula $A = \pi r^2$, find A when r is 14.

Answer: 616

18. Using the cylinder volume formula $V = \pi r^2 h$, find V when r is 7 and h is 10.

Answer: 1,540

19. There are 128 cubic feet in a cord. How many cords are in a pile of wood 16′ by 8′ by 8′?

Answer: 8

20. There are 1.25 cubic feet in a bushel. How many bushels will a bin hold which has measurements of 10′ by 20′ by 10′?

Answer: 1,600

21. Wilbur borrowed $500 at 12% for 90 days. How much interest did he owe?

Answer: $15.00

22. I borrowed $500 on the 17th of March and repaid it on June 26th. For how many days do I owe interest?

Answer: 101

23. Tim borrowed $60 from his parents. He repaid 66⅔% of it. How much did he repay?

Answer: $40.00

24. After working during the summer vacation, Dan put $100 in the savings bank. This was ⅖ of what he had earned. How much had he earned during the summer vacation?

Answer: $250.00

25. 10 is what percent of 40?

Answer: 25%

26. 40 is what percent of 10?

Answer: 400%

27. Last month's meter-reading was 1247. This month the reading was 1643. At 6 cents per K.W.H., what was the bill?

Answer: $23.76

28. William pays $.80 per thousand cubic feet of gas. What was his bill for the month if he used 2500 cubic feet?

Answer: $2.00

29. A lawyer collected an old debt of $450. The commission was 10%. How much was the commission?

Answer: $45.00

30. Agatha bought a hat for $27, which was at 25% discount. What was the original price of the hat?

Answer: $36.00

MT-17a
THE END

Mastery Test I (Through page 75) — ALGEBRA

Name ______________________ Score ____________

Add:

1. $\begin{array}{r} x \\ 3x \\ 4x \\ x \\ \hline 9x \end{array}$

2. $\begin{array}{r} -2b \\ -3b \\ -4b \\ -\ b \\ \hline -10b \end{array}$

3. $\begin{array}{r} 3x^2y \\ -\ x^2y \\ -2x^2y \\ x^2y \\ \hline x^2y \end{array}$

4. $\begin{array}{r} 4a + 2b \\ -\ a -\ b \\ -2a +\ b \\ \hline a + 2b \end{array}$

Subtract:

5. $\begin{array}{r} 7x \\ 3x \\ \hline 4x \end{array}$

6. $\begin{array}{r} -7y \\ -\ y \\ \hline -6y \end{array}$

7. $\begin{array}{r} -2x^2 \\ 4x^2 \\ \hline -6x^2 \end{array}$

8. $\begin{array}{r} 5a - 2b \\ -2a - 3b \\ \hline 7a + b \end{array}$

Multiply:

9. $\begin{array}{r} x \\ 3x \\ \hline 3x^2 \end{array}$

10. $\begin{array}{r} a \\ 4a \\ \hline 4a^2 \end{array}$

11. $\begin{array}{r} -2b^2 \\ -3b \\ \hline 6b^3 \end{array}$

12. $\begin{array}{r} 3d^3e^2f \\ -4d\ e^2f^3 \\ \hline 12d^4e^4f^4 \end{array}$

Divide:

13. $x\,\overline{)\,3x}$

3

14. $a^3\,\overline{)\,4a^4}$

4a

15. $x\,\overline{)\,-5x^3}$

$-5x^2$

16. $\dfrac{-9a^3b^6c^5}{-3a\ b^2c^4} = 3a^2b^4c$

17. Using the formula $V = lwh$, find V when l is 20, w is 10 and h is 5.

V = 1,000

18. From the formula $V = lwh$, write the formula for finding h.

$h = \dfrac{V}{LW}$

Solve the following equations:

19. $5a + 2a = 14$

$7a = 14$
$a = 2$

20. $4a^2 = 64$

$a = 4$

21. $a^3 - 2 = 6$

$a = 2$

22. $4x - 3(x + 1) = 2$

$4x - 3x - 3 = 2$
$x - 3 = 2$
$x = 5$

23. $3(x - 4) = 2(x - 2)$

$x = 8$

24. $\frac{3a}{8} = 6$

$a = 16$

25. $\begin{cases} x + y = 7 \\ x - y = 3 \end{cases}$

$x = 5$
$y = 2$

Write the equations and solve:

26. June is three times as old as Mary. The sum of their ages is 56 years. How old is each?

$x + 3x = 56$

Mary = 14 years June = 42 years

27. Five times a number increased by 6 is 51. Find the number.

$5x + 6 = 51$
$5x = 45$
$x = 9$

28. The length of a rectangle is three times the width. The area is 48. Find the length and width.

W = 4
L = 12

$3x \cdot x = 48$
$3x^2 = 48$
$x^2 = 16$
$x = 4$

29. The sum of two consecutive numbers is 95. Find the numbers.

$x = 1^{st}$
$x - 1 = 2^{nd}$

47 and 48

30. Joe has twice as much money as Fred. Together, they have $1.05. How much does each have?

$x + 2x = 1.05$
$3x = 1.05$
$x = .35$

Fred = $.35 Joe = $.70

31. The length of a rectangle is twice the width. The perimeter is 120. Find the length and width.

$2(2x) + 2(x) = 120$
$6x = 120$
$x = 20$

L = 40 W = 20

32. The sum of three numbers is 180. The second number is twice the first, and the third number is three times the first. Find each number.

$x + 2x + 3x = 180$

30, 60, 90

33. The sum of two numbers is 16. The difference of the two numbers is 4. What are the two numbers?

$x + x + 4 = 16$

10 and 6

THE END

Mastery Test II **ALGEBRA**

Name ______________________ **Score** __________

1. **Add:**

$$\begin{array}{r} 2a - 3b \\ -5a + 2b \\ \underline{4a - b} \end{array}$$

a - 2b

2. **Subtract:**

$$\begin{array}{r} 7x - 5y \\ \underline{-2x + y} \end{array}$$

5x - 4y

3. **Multiply:**

$$\begin{array}{r} 12a^2 \\ \underline{-3a^3} \end{array}$$

$36a^5$

4. **Multiply:**

$$\begin{array}{r} 3a - 2b \\ \underline{a - b} \end{array}$$

$3a^2 - 5ab + 2b^2$

Multiply:

5. $$\begin{array}{r} 8x - 4y \\ \underline{3y} \end{array}$$

$24xy - 12y^2$

6. $$\begin{array}{r} 2x - 3x^2 \\ \underline{3x^2 - 5x^3} \end{array}$$

$6x^3 - 19x^4 + 15x^5$

7. $\frac{x^2}{4} \cdot \frac{8}{x} =$ 2x

8. $\frac{3(x-y)}{5} \cdot \frac{10}{x-y} =$ 6

Divide:

9. $5a^3 \overset{-6a^3}{\overline{)\,-30a^6}}$

10. $\frac{-15a^4}{-3a} = 5a^3$

11. $x+y \overset{x-4y}{\overline{)\,x^2-3xy-4y^2}}$

12. $\frac{x+y}{5} \div \frac{2(x+y)}{15} =$ $1\frac{1}{2}$

Perform the indicated operations:

13. $(-4x^2)^2 = 16x^4$

14. $\overset{10x^2y}{\sqrt{100x^4y^2}}$

Reduce to lowest terms:

15. $\frac{6x^4\ y^2}{9x^3\ y^3} = \frac{2x}{3y}$

16. $\frac{10xy + 20x^2y^2}{5xy} =$

2 + 4xy

Remove the parentheses and collect terms:

17. $3x - 4(x - y) - 3(x + 3y) - 3x$

3x - 4x + 4y - 3x - 9y - 3x

-7x - 5y

18. $4a - 2(a - 3b) + 4(a - 2b) - 3a$

3a - 2b

Write the products:

19. $(a + 4)\ (a - 4)$

$a^2 = 16$

20. $(x + 6)\ (x - 6)$

$x^2 = 36$

Find the factors:

21. $a^2 - 25$

$(a + 5)\ (a - 5)$

22. $x^2 - 5x + 4$

$(x - 4)\ (x - 1)$

Solve the equations:

23. $4a - 5 = 15$

$4a = 15 + 5$
$4a = 20$
$a = 5$

24. $3(x - 4) = x - 2$

$3x - 12 = x - 2$
$2x = 10$
$x = 5$

25. $\frac{x}{8} = \frac{3}{4}$

$4x = 24$
$x = 6$

26. $\begin{cases} a + b = 8 \\ a - b = 4 \end{cases}$

$a = 6$
$b = 2$

27. $\begin{cases} \frac{x}{4} + y = -2 \\ \frac{3x}{4} - y = 6 \end{cases}$

$x = -8$
$y = 0$

28. $x : 5 = 4 : 2$

$x = 10$

29. $x^2 - 5x + 4 = 0$

$x = 4$
$x = 1$

30. $x^2 - 7x = -6$

$x = 6$
$x = 1$

31. $x^3 = 27$

$x = 3$

Write the equations and solve:

32. The product of two consecutive numbers is 20. Find the numbers.

4 and 5

33. The length of a rectangle is 4 greater than the width, and the area is 60. Find the length and width of the rectangle.

L = 10
W = 6

THE END

Algebra Test #1
pgs. 23-36

*Answers are in capital letters for convenience in grading only.

Add:

1. $\begin{array}{r} 2A \\ 7A \\ 5A \\ 12A \\ \hline 26A \end{array}$

2. $\begin{array}{r} 3Ab \\ 4Ab \\ 32Ab \\ 9Ab \\ \hline 48AB \end{array}$

3. $\begin{array}{r} 16x^2y \\ 9x^2y \\ 14x^2y \\ 23x^2y \\ \hline 62X^2Y \end{array}$

4. $\begin{array}{r} -15a \\ +12a \\ -3a \\ +9a \\ \hline +3A \end{array}$

5. $\begin{array}{r} -15a \\ -12a \\ -3a \\ -9a \\ \hline -21A \end{array}$

6. $\begin{array}{r} 15abc \\ -6abc \\ -9abc \\ 3abc \\ \hline +3ABC \end{array}$

****

Subtract:

1. $\begin{array}{r} 15x \\ 7x \\ \hline +8X \end{array}$

2. $\begin{array}{r} 165xy \\ 39xy \\ \hline +126XY \end{array}$

3. $\begin{array}{r} 45ab \\ -19ab \\ \hline +64AB \end{array}$

4. $\begin{array}{r} -45ab \\ -19ab \\ \hline -26AB \end{array}$

5. $\begin{array}{r} -45ab \\ 19ab \\ \hline -64AB \end{array}$

6. $\begin{array}{r} 126x^2y \\ -120x^2y \\ \hline +246X^2Y \end{array}$

****

Multiply:

1. $\begin{array}{r} 15a \\ 3 \\ \hline 45A \end{array}$

2. $\begin{array}{r} 64ab \\ 3 \\ \hline 192AB \end{array}$

3. $\begin{array}{r} 19x^2 \\ 4 \\ \hline 76X^2 \end{array}$

4. $\begin{array}{r} 32xy \\ 5 \\ \hline 160XY \end{array}$

5. $\begin{array}{r} 18m^2n \\ 10 \\ \hline 180M^2N \end{array}$

6. $\begin{array}{r} 4.5a^2 \\ 2 \\ \hline 9A^2 \end{array}$

Test #1 (continued)

Divide:

1. $\frac{15x}{5} = \underline{3X}$
2. $\frac{63xy}{7} = \underline{9XY}$
3. $\frac{92a^2}{4} = \underline{23A^2}$
4. $\frac{25a^2b^2}{25} = \underline{A^2B^2}$
5. $\frac{225xy^2}{25} = \underline{9XY^2}$
6. $\frac{1.6x}{4} = \underline{.4X}$

* * *

Solve for the unknown:

1. $M + 13 = 32$

   $M = 19$

2. $x + 17 = 19$

   $X = 2$

3. $x + 3x = 18 + 6$

   $X = 6$

4. $8x - 6x = 3.8$

   $X = 1.9$

5. $15a - 3a = 60$

   $A = 5$

6. $10x = 115$

   $X = 11\frac{1}{2}$ -OR- 11.5

7. $15a = 45$

   $A = 3$

8. $\frac{x}{2} = 115$

   $X = 230$

9. $\frac{2x}{3} = 4$

   $X = 6$

10. $\frac{3y}{8} = 6$

    $Y = 16$

11. $3x - 5 = x + 7$

    $X = 6$

12. $15x - 5 = 10x + 15$

    $X = 4$

Test #1 (continued)

1. Together, Earl and Roy have $4.80. Earl has three times as much as Roy. How much does each?

ROY = x
EARL = 3x

4x = 4.80
x = 1.20

ROY = $1.20
EARL = $3.60

2. George and Harry inherited an estate of $1500. George is to receive three times as much as Harry. How much does each receive?

x = HARRY
3x = GEORGE

4x = 1500
x = 375

HARRY = $375
GEORGE = $1,125

3. Dorothy and Jane have $4.80. Dorothy has only one-fifth as much as Jane has. How much does each have?

x = JANE
1/5x = DOROTHY

5x + x = $4.80

JANE = $4.00
DOROTHY = .80

4. Mary is twice as old as her sister. The sum of their ages is 63 years. What is the age of each?

x = SISTER
2x = MARY

3x = 63
x = 21

SISTER = 21
MARY = 42

THE END

Algebra Test #2
pgs. 1-46

*Answers are in capital letters for convenience in grading only.

1. $A = lw$
$l = 30$
$w = 20$
$A =$ 600

2. $V = lwh$
$l = 15$
$w = 10$
$h = 5$
$V =$ 750

3. $A = \pi r^2$
$r = 7$
$A =$ 154

4. $V = \pi r^2 h$
$D = 14$
$h = 10$
$V =$ 1540

5. $I = PRT$
$P = \$1000$
$R = 6\%$
$T = 4$ mos.
$I =$ $20.00

6. $I = PRT$
$P = \$1200$
$R = 6\%$
$T = 18$ mos.
$I =$ $108

7. From the formula $A = Iw$, write the formula for finding L.

$$L = \frac{A}{w}$$

8. From the formula $I = PRT$, write a formula for finding R.

$$R = \frac{I}{PT}$$

9. From the formula $V = Lwh$, write a formula for finding h.

$$h = \frac{V}{Lw}$$

10. From the formula $C = \pi d$, write a formula for finding d.

$$d = \frac{C}{\pi}$$

Solve each of the following equations:

11. $5x = x + 20$ $x = 5$
$5x - x = 20$
$4x = 20$

12. $3x + 5 = 23$ $x = 6$
$3x = 18$

13. $4x - 5 = 3x + 1$
$4x - 3x = 1 + 5$
$x = 6$

14. $2(3x + 4) = 3x + 23$ $x = 5$
$6x + 8 = 3x + 23$
$6x - 3x = 23 - 8$
$3x = 15$

15. $3(2x - 4) = 2(2x + 2)$
$6x - 12 = 4x + 4$
$6x - 4x = 12 + 4$
$2x = 16$
$x = 8$

16. $\frac{2x}{4} = 16$
$2x = 16 \times 4$
$2x = 64$
$x = 32$

17. $\frac{x}{5} = 10$
$x = 50$

18. $\frac{3x}{3} = 7$
$3x = 21$
$x = 7$

19. $12x + 5x = 51$

$17x = 51$
$x = 3$

20. $14b - 6b = 48$

$8B = 48$
$B = 6$

21. Helen's father is three times as old as she. The sum of their ages is 56 years. How old is each?

$3x + x = 56$
$4x = 56$
$x = 14$

HELEN = 14
HELEN'S FATHER = 42

22. Henry is twice as old as his sister. The sum of their ages is 21 years. How old is each?

$2x + x = 21$
$3x = 21$
$x = 7$

SISTER = 7
HENRY = 14

23. The perimeter of a rectangle is 90. The length is twice as great as the width. Find the length and width.

$P = 2L + 2W$
$90 = 2(2x) + 2(x)$
$90 = 4x + 2x$
$90 = 6x$
$x = 15$

$W = 15$
$L = 30$

24. We have a lot whose perimeter is 360 ft. The length is 3.5 times the width. Find the length and width of the lot.

$P = 2L + 2W$

$2(3.5x) + 2(x) = 360$
$7x + 2x = 360$
$9x = 360$
$x = 40$

$L = 140$
$W = 40$

25. I am thinking of a number. Five times that number equals 240. What is the number?

$$5x = 240$$
$$x = 48$$

Solve these equations:

1. $\left(\frac{x}{2} + 3\right) = \left(\frac{x}{8} + 7\right)$

   $x = 10\frac{2}{3}$

2. $\left(\frac{x}{2} - 5\right) = \left(\frac{x}{5} - 11\right)$

   $x = -20$

3. $\left(x + 5\right) = \left(\frac{2x}{7} + 10\right)$

   $x = 7$

4. $\frac{x}{3} = \frac{2}{5}$

   $x = 1\frac{1}{5}$

5. $\frac{2x}{7} = \frac{6}{5}$

   $x = 4.2 = \frac{42}{10} = 4\frac{2}{5}$

6. $\frac{3x}{8} = \frac{6}{4}$

   $x = 4$

MT-27a
THE END

Algebra Test #3
pgs. 47-51

*Answers are in capital letters for convenience in grading only.

1. When multiplying expressions which contain exponents, you (add, subtract) the exponents.

2. When dividing expressions which contain exponents, you (add, subtract) the exponents.

Multiply:

3. $$\begin{array}{r} b^2de \\ \underline{bd^2} \\ +b^3d^3e \end{array}$$

4. $$\begin{array}{r} 2xy^3 \\ \underline{6yz^4} \\ +12xy^4z^4 \end{array}$$

5. $$\begin{array}{r} 14b^3 \\ \underline{4cd} \\ +56b^3cd \end{array}$$

6. $$\begin{array}{r} 19R^3N^3 \\ \underline{4MRN} \\ +76MR^4N^4 \end{array}$$

7. $$\begin{array}{r} 12d^3 \\ \underline{6ade} \\ +72ad^4e \end{array}$$

8. $$\begin{array}{r} 10bc^3d \\ \underline{5acd} \\ +50abc^4d^2 \end{array}$$

9. $$\begin{array}{r} 6ab^2 \\ \underline{9bc^2} \\ +54ab^3c^2 \end{array}$$

10. $$\begin{array}{r} 5x^2y^2 \\ \underline{4x^3y^3} \\ +20x^5y^5 \end{array}$$

Divide:

1. $\dfrac{d^5e^6f}{d^2ef} = d^3e^5$

2. $\dfrac{a^5b^2c}{a^3bc} = a^2b$

3. $\dfrac{cde^6f}{de} = ce^5f$

4. $\dfrac{8y^2}{16y} = \dfrac{y}{2}$

5. $\dfrac{45x^2y^2}{5xy} = 9xy$

6. $\dfrac{72x^2y}{8xy} = 9x$

Test #3 (continued)

Divide:

7. $\dfrac{14y^4z^3}{28y^3z^4} = \dfrac{y}{2z}$

8. $\dfrac{25c^2d^6}{75c^6d^2} = \dfrac{d^4}{3c^4}$

9. $\dfrac{18x^2}{2x^3} = \dfrac{9}{x}$

10. $\dfrac{16a^2b^3}{32a^3b^2} = \dfrac{b}{2a}$

MT-29a
THE END

*Answers are in capital letters for convenience in grading only.

Algebra Test #4
pgs. 51-63

Solve for the unknown:

1. $6x = 36$

   X = 6

2. $15a = 45$

   A = 3

3. $12x + 5x = 51$

   X = 3

4. $14x - 6x = 48$

   X = 6

5. $25m - 14m = 77$

   M = 7

6. $\frac{2n}{3} = 14$

   N = 21

7. $\frac{3z}{10} = 9$

   Z = 30

8. $\frac{2R}{3} = 48$

   R = 72

9. $3(x + 2) = 2(x + 5)$

   $3X + 6 = 2X + 10$
   X = 4

10. $5(3x + 1) = 5(2x + 3)$

    $15X + 5 = 10X + 15$
    $5X = 10$
    X = 2

11. Mrs. Lopez and her sister inherited $20,000. Mrs. Lopez received 4 times as much as her sister. How much did each receive?

    MRS. LOPEZ = 4X
    SISTER = X

    4X + X = 20,000
    5X = 20,000
    X = 4000

    SISTER = $4,000
    MRS. LOPEZ = $16,000

12. Five times a number is 35. What is the number?

    5X = 35
    X = 7

13. The area of a rectangle is 192. The width is one-third of the length. Find the length and width.

$A = LW$

$W = 1/3\ L$

OR $L = 3W$

$192 = 3W(W)$

$192 = 3W^2$

$8 = W$

WIDTH = 8

LENGTH = 24

14. Find the dimensions of the rectangular prism with a volume of 162, if the base width is one-half the base length and the height is three times the base width.

$W = X$

$L = 2X$

$H = 3X$

$162 = 6X^3$

$27 = X^3$

$3 = X$

WIDTH = 3

LENGTH = 6

HEIGHT = 9

15. $5x^3 - 17 = 3x^3 + 111$

$2x^3 = 128$

$x^3 = 64$

$x = 4$

16. $2x^3 + 5 = x^3 + 221$

$x^3 = 216$

$x = 6$

17. $4x^3 - 6 = 3x^3 + 119$

$x^3 = 125$

$x = 5$

18. $7x^2 - 15 = 5x^2 + 17$

$2x^2 = 32$

$x^2 = 16$

$x = 4$

Solve:

1. The area of a rectangular field is 1200 square rods. The length of the field is 40 rods. What is the width?

$A = LW$

1200 SQ. RODS = 40 W

$30 = W$

$W = \frac{A}{L}$

Test #4 (continued)

2. The area of a rectangle is 75. The length is three times the width. Find the two dimensions.

$L = 3W$

$W = x$

$A = LW$

$75 = (3x)x$

$25 = x^2$

$5 = x$

LENGTH = 15

WIDTH = 5

3. The width of a rectangle is one-third the length, and the area is 108. Find the two dimensions of the rectangle.

$W = x$

$L = 3x$

$3x^2 = 108$

$x^2 = 36$

$x = 6$

WIDTH = 6

LENGTH = 18

4. Find the three dimensions of the rectangular prism with a volume of 500, the length and height each being twice the width.

$W = x$

$L = 2x$

$H = 2x$

$V = LWH$

$500 = (2x)(x)(2x)$

$500 = 4x^2$

$125 = x^3$

$5 = x$

WIDTH = 5

LENGTH = 10

HEIGHT = 10

5. Find the three dimensions of the cube whose volume is 216.

$216 = x^3$

$6 = x$

THE END

Algebra Test #5
pgs. 1-63

*Answers are in capital letters for convenience in grading only.

Add:

1. 28xy
-14xy
-3xy
+ 11XY

2. -14ab
2ab
ab
-11AB

3. +12m
-3m
-6m
+3M

4. -14x
2x
10x
-2X

5. -6a
5a
-A

Subtract:

1. 15a
-3a
+18A

2. 26bc
13bc
+13BC

3. 4a
-6a
+10A

4. 16xy
8xy
+8XY

5. $\frac{1}{2}a$
$-\frac{1}{4}a$
+3/4 A

Multiply:

1. $$\begin{array}{r} -2.5a \\ \underline{\quad 2a} \\ -5A \end{array}$$

2. $$\begin{array}{r} -16b^{2} \\ \underline{\quad -5b} \\ +80B^{3} \end{array}$$

3. $$\begin{array}{r} -3xy^{3} \\ \underline{\quad 2x} \\ -6X^{2}Y^{3} \end{array}$$

4. $$\begin{array}{r} 1.5b \\ \underline{\quad -.7b} \\ -1.05B^{2} \end{array}$$

Divide:

1. $\frac{18b}{2} = 9b$

2. $\frac{15ac}{5bc} = \frac{3A}{B}$

3. $\frac{-32p}{8} = -4P$

4. $\frac{-60rs}{-5s} = 12R$

5. $\frac{-16r}{4} = -4R$

Solve:

1. $9x = 54$
   $x = 6$

2. $4x + 2x = 30$
   $x = 5$

3. $4x + 2 = 10$
   $x = 2$

4. $10x = 1,000$
   $x = 100$

5. $2x + 2\ (x+3) = x + 15$
   $2x + 2x + 6 = x + 15$
   $3x = 9$
   $x = 3$

6. $4 + 2x - 1 = 4\ (x+1)$
   $3 + 2x = 4x + 4$
   $-1 = 2x$
   $-\frac{1}{2} = x$

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7. If 2 is added to twice a number, the result is equal to the number increased by 12. Find the number.

$2 + 2x = x + 12$

$x = 10$

8. If five times a number is increased by four, then decreased by ten, the result will equal three times the number increased by four. What is the number?

$(5x + 4) - 10 = 3x + 4$

$2x = 10$

$x = 5$

9. If 14 is added to 3 times a number, the result is 53. What is the number?

$14 + 3x = 53$

$3x = 39$

$x = 13$

10. 12 less than twice a number is equal to the number increased by 6. What is the number?

$2x - 12 = x + 6$

$x = 18$

11. The length of a playground is 5 times the width. The perimeter is 300 rods. Find the length and the width of the playground.

$L = 5W$

$x = W$

$P = 2L + 2W$

$P = 2(5x) + 2x$

$300 = 10x + 2x$

$300 = 12x$

$25 = x$

WIDTH = 25

LENGTH = 125

12. The width of a rectangle is $\frac{1}{4}$ its length. The perimeter is 50 rods. What are the two dimensions?

$W = X$

$L = 4X$

$P = 2L + 2W$

$50 = 2(4X) + 2X$

$50 = 10X$

$5 = X$

WIDTH = 5

LENGTH = 20

13. The basement of a house is a rectangle with a perimeter of 144 ft. The length is $1\frac{1}{2}$ times the width. Find the width.

$W = X$

$L = 1.5X$

$144 = 2(1.5X) + 2(X)$

$144 = 3.0X + 2X$

$144 = 5X$

$28.8 = X$

$28\ 4/5 = X$

WIDTH = 28.8

OR

WIDTH = 28 4/5

LENGTH = 43.2

14. The perimeter of a rectangle is 80 yards. The length equals 3 times the width. What are the dimensions?

$P = 2L + 2W$

$80 = 2(3X) + 2X$

$80 = 8X$

$10 = X$

W = 10

L = 30

15. Jane is $2\frac{1}{2}$ times as old as her sister. The sum of their ages is 63 years. How old is Jane and how old is her sister?

$2.5X + X = 63$

$3.5X = 63$

$X = 18$

JANE = 45

SISTER = 18

THE END

Algebra Test #6
pgs. 1-71

*Answers are in capital letters for convenience in grading only.

Solve:

1. When 8 is subtracted from 4 times a number, the result is 36.

$4x - 8 = 36$

$4x = 44$

$x = 11$

2. If 9 times a number is decreased by 10, the result is equal to 4 times the number increased by 30.

$9x - 10 = 4x + 30$

$5x = 40$

$x = 8$

3. Three less than a number equals 42.

$x - 3 = 42$

$x = 45$

4. If 60 is decreased by 3 times a number, the result is 12 less than the number. Find the number.

$60 - 3x = x - 12$

$72 = 4x$

$18 = x$

5. The length of a rectangle is one less than twice the width. If the perimeter of the rectangle is 34", find the dimensions of the rectangle.

$L = 2x - 1$

$W = x$

$34 = 2\,(2x - 1) + 2x$

$34 = 4x - 2 + 2x$

$34 = 6x - 2$

$36 = 6x$

$6 = x$

WIDTH = 6

LENGTH = 11

Solve for X:

1. $6x = 78$

$x = 13$

2. $x - 3 = 42$

$x = 45$

3. $2x + 4 = 10$

$x = 3$

4. $5x + 13 = 43$

$x = 6$

5. $9x - 10 = 4x + 30$

$x = 8$

Add:

1. $$\begin{array}{r} +6b^2 \\ -5b^2 \\ +8b^2 \\ \hline +9B^2 \end{array}$$

2. $$\begin{array}{r} -10xy \\ +12xy \\ -8xy \\ \hline -6XY \end{array}$$

3. $$\begin{array}{r} +9mx^3 \\ -11mx^3 \\ +9mx^3 \\ -6mx^3 \\ \hline MX^3 \end{array}$$

4. $$\begin{array}{r} -3/4cd^4 \\ +3/8cd^4 \\ \hline -3/8CD^4 \end{array}$$

5. $$\begin{array}{r} -x^2y \\ -4x^2y \\ \hline -5X^2Y \end{array}$$

Subtract:

1. $$\begin{array}{r} +8x \\ -5x \\ \hline 13X \end{array}$$

2. $$\begin{array}{r} -2m \\ -6m \\ \hline 4M \end{array}$$

3. $$\begin{array}{r} 9x^2y \\ -12x^2y \\ \hline 21X^2Y \end{array}$$

4. $$\begin{array}{r} +5b^4 \\ -5b^4 \\ \hline 10B^4 \end{array}$$

5. $$\begin{array}{r} +23a \\ -29a \\ \hline 52A \end{array}$$

Multiply the following monomials:

1. $a^7 \cdot a^9 = A^{16}$

2. $b^6 \cdot b^5 = B^{11}$

3. $e \cdot e^5 \cdot e^7 = E^{13}$

4. $t^9 \cdot t^7 \cdot t^4 \cdot t^2 = T^{22}$

5. $a^m \cdot a^{m2} = A^{M3}$

6. $$\begin{array}{r} 2ab \\ \underline{-5b} \\ -10AB^2 \end{array}$$

7. $$\begin{array}{r} -3x^2y^2 \\ \underline{-2x^2y^2} \\ 6X^4Y^3 \end{array}$$

8. $$\begin{array}{r} +6rst \\ \underline{+3rst} \\ 18R^2S^2T^2 \end{array}$$

9. $$\begin{array}{r} -2x^2c \\ \underline{+3xc^2} \\ -6X^3C^3 \end{array}$$

10. $$\begin{array}{r} 2x^2c^3d^2 \\ \underline{6xcd} \\ 12X^3C^4D^3 \end{array}$$

Add the following polynomials:

1. $$\begin{array}{r} 7ab+4x^2 \\ \underline{2ab-2x^2} \\ 9AB+2X^2 \end{array}$$

2. $$\begin{array}{r} 5a+3b-2c \\ \underline{-2a+3b+2c} \\ 3A+6B \end{array}$$

3. $$\begin{array}{r} +2y^2-9 \\ -5y^2+4 \\ \underline{-6y^2-8} \\ -9Y^2-13 \end{array}$$

Subtract the following polynomials:

1. $$\begin{array}{r} x^2+y^2+z^2 \\ \underline{-3x^2+2y^2+z^2} \\ 4X^2-Y^2 \end{array}$$

2. $$\begin{array}{r} 5b^2+9 \\ \underline{+2b^2+9} \\ 3B^2 \end{array}$$

3. $$\begin{array}{r} -7y^2+3xy-2 \\ \underline{-y^2+xy+1} \\ -6Y^2+XY-3 \end{array}$$

Divide:

1. $\frac{15ac}{5bc} = \frac{3A}{B}$

2. $\frac{x^4y^3z^2}{xyz^2} = X^3Y^2$

3. $\frac{10a^2b^2}{5ab} = 2AB$

4. $\frac{8x^2y^2}{16x^2y} = \frac{Y}{2}$

5. $\frac{12xyz}{48x^2y^2z^2} = \frac{1}{4XYZ}$

Add the following binomials:

1. $2a+b$

$\underline{a-b}$

3A

2. $x+y$

$\underline{x-y}$

2x

3. $2x+2y$

$\underline{3x-2y}$

5x

4. $3x+2$

$\underline{x-2}$

4x

5. $4a-b$

$\underline{a-b}$

5A-2B

Solve for X:

1. $x+3(x-2) = 2x + 4$

$4x-6 = 2x+4$

$2X = 10$

$X = 5$

2. $\frac{3x}{4} = 12$

$x = 16$

3. $3x + (x-4) = 3x + 1$

$3x+x-4 = 3x+1$

$x = 5$

4. $4x - 2(x+3) = x - 4$

$2x-6 = x-4$

$x = 2$

5. $\frac{3x}{8} = 6$

$x = 16$

Solve:

1. How many square feet of carpet are needed to cover a living room 18 feet long and 12 feet wide?

   $A = LW$ $A = 216$ SQ. FT.

   $A = 18 \times 12$

2. The interest paid on $800 at 4% for 3 years.

   $I = PRT$ $I = \$96$

   $I = 800 \times .04 \times 3$

3. The interest paid on $1,200 at 6% for 2 years.

   $I = PRT$ $I = \$144$

   $I = 1{,}200 \times .06 \times 2$

4. The square of a number increased by 4 equals 40. Find the number.

   $x^2 + 4 = 40$ $x = 6$

   $x^2 = 36$

5. Find V when L = 25, w = 20, and h = 10.

   $V = 25 \times 20 \times 10$

   $V = 5{,}000$

6. In the formula $A = \pi r^2$, find A when r = 7.

   $A = \frac{22}{7} \times 49$

   $A = 154$

Solve: (continued)

7. The length of a rectangle is twice the width. The area is 450. Find the two dimensions.

$A = LW$ $\quad 450 = 2x^2$ $\quad$ WIDTH = 15

$A = (2x)$ $\quad 225 = x^2$ $\quad$ LENGTH = 30

$15 = x$

8. Find the three dimsnsions of the cube whose volume is 125.

$125 = x^3$ $\quad L = 5$

$5 = x$ $\quad W = 5$

$H = 5$

9. The width of a rectangle is only one-third the length, and the area is 4332; find the dimsnsions of the rectangle.

$W = x$ $\quad 4332 = 3x^2$ $\quad x =$ WIDTH = 38

$L = 3x$ $\quad 1444 = x^2$ $\quad 3x =$ LENGTH = 114

$38 = x$

Algebra Test #7
pgs. 1-85

Solve for X:

1. $\frac{x}{2} = 2$ x = 4

2. $13x = 39$ x = 3

3. $x + 25 = 45$ x = 20

4. $x - 21 = 6$ x = 27

5. $\frac{2x}{2} = 14$ x = 14

6. $9x - 11 = 70$ x = 9

7. $16x + 16 = 32$ x = 1

8. $7 + 5x = 42$ x = 7

9. $8x - 3x = 24 - 4$ x = 4

10. $3x - 4 = x + 8$ x = 6

11. $15x + 5 = 10x + 15$ x = 2

12. $5x - 7 = 2x + 11$ x = 6

13. $5x - (x + 6) = 10$ x = 4

Test #7

Solve for X: (continued)

14. $4(2x - 5) - 3(x + 10) = -15$ $x = 7$

15. $5(2x - 1) - 3(x + 2) = 31$ $x = 6$

Read and Solve:

1. The product of 6 and a number equals 78. Find the number. 13

2. Twice a number increased by 4 equals 10. Find the number. $x = 3$

3. The sum of five times a number and 13 is 43. Find the number. $x = 6$

4. If 5 times a number is decreased by 3 the result is the same as when twice the number is increased by 21. Find the number. $x = 8$

5. The length of a rectangle is represented by $3x + 5$ and the width by $6x - 20$. Find the length and the width if the perimeter is 78 feet.
length = 23   width = 16

Test #7 (Continued)

Multiply:

1. $$\begin{array}{r} 6x^3y^3 \\ 4x^2y \\ \hline 24x^5y^4 \end{array}$$

2. $$\begin{array}{r} -15x^{2y} \\ 6xy^3 \\ \hline -90x^3y^4 \end{array}$$

3. $$\begin{array}{r} -24x^2 \\ -4x^3y \\ \hline 96x^5y \end{array}$$

4. $$\begin{array}{r} -19xy^2 \\ -20x^2y^3z \\ \hline 380x^3y^5z \end{array}$$

5. $$\begin{array}{r} -23ab^2 \\ -4a^2bc^2 \\ \hline 92a^3b^3c^3 \end{array}$$

Divide:

1. $\frac{25x^2y^2z}{5xy^3z^4} = \frac{5x}{yz^3}$

2. $\frac{5x^4y^4c^4}{15x^6y^6c^6} = \frac{1}{3x^2y^2c^2}$

3. $\frac{-15x^2y^2}{5xy} = -3xy$

4. $\frac{-6x^2y^4c4}{-2xy^3c^5} = \frac{3xy}{c}$

5. $\frac{8x^3y^4c^6}{-16x^4y^8c^{10}} = \frac{1}{-2xy^4c^4}$

Add:

1. $$\begin{array}{r} 7a - 5b - 3c \\ -9a + b - 5c \\ -4a + 3b - 2c \\ 6a - 4b + c \\ \hline -5b - 5c \end{array} =$$

2. $$\begin{array}{r} 32a - 17b + 15c \\ -45a + 23b - 19c \\ 16a - 19b + 7c \\ -20a + 10b + 10c \\ 13a - 10b + 2c \\ -20a + 20b - 14c \\ \hline -24a + 7b + 6c \end{array} =$$

Test #7 (continued)

Subtract:

1. $$\begin{array}{r} -32b - 16c + 13d \\ \underline{-37b + 4c - 7d} \\ 5b - 20c + 20d \end{array}$$

2. $$\begin{array}{r} -26a + 17b - 33c \\ \underline{-9a + 6b - 16c} \\ -17a + 11b - 17c \end{array}$$

Solve for x and y:

1. $$\begin{array}{r} x + y = 35 \\ \underline{x - y = 5} \end{array}$$
$x = 20$
$y = 15$

2. $$\begin{array}{r} x + y = 25 \\ \underline{x - y = 15} \end{array}$$
$x = 20$
$y = 5$

3. $$\begin{array}{r} x - y = 2 \\ \underline{4x + y = 18} \end{array}$$
$x = 4$
$y = 2$

4. $$\begin{array}{r} 3x - 5y = 11 \\ \underline{x + 5y = 17} \end{array}$$
$x = 7$
$y = 2$

5. $$\begin{array}{r} x + 3y = 45 \\ \underline{x - 3y = -9} \end{array}$$
$x = 18$
$y = 9$

Solve for the unknowns:

1. $$\begin{array}{r} x + y = 5 \\ \underline{3x + 2y = 13} \end{array}$$
$x = 3$
$y = 2$

2. $$\begin{array}{r} 2m + 3n = 9 \\ \underline{m - 2n = 1} \end{array}$$
$n = 1$
$m = 3$

3. $$\begin{array}{r} a + 2b = 10 \\ \underline{a - b = 1} \end{array}$$
$b = 3$
$a = 4$

4. $$\begin{array}{r} 4a + 3b = 14 \\ \underline{2a + 3b = 10} \end{array}$$
$a = 2$
$b = 2$

5. $$\begin{array}{r} 2x - 3y = 1 \\ \underline{3x - 2y = 9} \end{array}$$
$y = 3$
$x = 5$

Solve:

1. The difference of two numbers is -3. Twice the first number increased by three times the second is 29. Find the numbers.

   $y = 7$

   $x = 4$

2. Four times a number increased by a second number equals 29. Three times the first number decreased by the second number equals 13. What are the numbers? $x = 6$ $y = 5$

3. The width of a rectangle is one-fourth the length, and the area is 256. Find the dimensions. Width = 8 Length = 32

4. The length of a rectangle is four times the width. The area is 900. Find the dimensions. Width = 15 Length = 60

5. The difference of two numbers is 8. The sum of the two numbers is 24. Find the numbers. $x = 16$

   $y = 8$

Algebra Test #8
pgs. 91-95

Add:

1. $\frac{a}{2} + y$

$\frac{a}{2} - y$

$\frac{2a}{2} = a$

2. $\frac{2x}{3} - y$

$\frac{x}{3} - y$

$\frac{\cancel{3}x}{\cancel{3}} -24 = x-24$

3. $\frac{5x}{8} - \frac{2y}{3}$

$\frac{3x}{8} - \frac{y}{3}$

$\frac{\cancel{8}x}{\cancel{8}} - \frac{\cancel{3}y}{\cancel{3}} = x-y$

4. $\frac{m}{3} - N$

$\frac{2m}{3} + N$

$\frac{\cancel{3}m}{\cancel{3}} = m$

5. $\frac{3a}{4} + b$

$\frac{a}{4} - b$

$\frac{\cancel{4}a}{\cancel{4}} = a$

Subtract:

1. $\frac{x}{3} + y$

$\frac{x}{3} - y$

$2y$

2. $\frac{3x}{8} - y$

$\frac{2x}{8} + y$

$\frac{x}{8} - 2y$

3. $\frac{5x}{9} + \frac{7y}{8}$

$\frac{x}{9} - \frac{y}{8}$

$\frac{4x}{9} + \frac{\cancel{8}y}{\cancel{8}} = \frac{4x}{9} + y$

4. $\frac{3a}{4} + b$

$\frac{a}{4} - b$

$\frac{2a}{4} + 26 =$

$\frac{1a}{2} + 26 =$

$\frac{a}{2} + 26$

5. $\frac{2a}{5} + \frac{2b}{3}$

$\frac{2a}{5} + \frac{b}{3}$

$\frac{b}{3}$

Solve:

1. Five-eights of a certain number increased by a second number equals 12. Three-eights of the same number decreased by the second number equals -4. Find the numbers. $x = 8$

   $y = 7$

2. One-half a certain number increased by a second number equals 5. One-half the same number decreased by the second number equals 1. Find the two numbers.

   $x = 6$

   $y = 2$

3. One half of a number decreased by 4 equals one-ninth of the number incresed by 3. What is the number? $\underline{x = 18}$

4. Three-fifths of a number added to a second number equals 11. Two-fifths of the first number decreased by the second number equals -1. Find the numbers.

   $x = 10$

   $y = 5$

5. Two-thirds of a number added to a second number equals zero. Two-thirds of the first number decreased by the second number equals 8. Find the numbers.

   $x = 6$

   $y = 4$

Solve for the unknowns:

1. $$\frac{1}{x} + \frac{1}{y} = 3/4$$
   $$\underline{\frac{1}{x} - \frac{1}{y} = 1/4}$$

   $x = 2$

   $y = 4$

2. $$\frac{1}{x} - \frac{1}{y} = 2$$
   $$\underline{\frac{1}{x} + \frac{1}{y} = 4}$$

   $x = 1/3$

   $y = 1$

Algebra Test #9

pgs. 1-107

Solve:

1. If 4 times a number is increased by 30, the result is the same as when 95 is decreased by the number; find the number. x = 13

2. The perimeter of a rectangle is 70 feet. The width and length are in ration of 2 to 3. Find the dimensions. W = 14 L = 21

3. The area of a rectangle is 200 square feet. The length and width are in the ratio of 2 to 1. Find the dimensions. W = 10 L = 20

4. If Jim earned $8.25 in 3 hours, how much would he earn in 40 hours?

   x = $110

5. A baseball player makes 24 hits in 98 times at bat. Express this ratio as a fraction, and then change it to a decimal correct to three places. .245

6. If it takes 16 gallons of gas to drive a car 288 miles, how many gallons will it tae to drive 558 miles at the same rate? 31 gallons

7. $\frac{AB}{EF} = \frac{BC}{FG}$

   AB = 2
   BC = 6
   EF = 1
   Find FG

   FG = 3

8. x : 14 = 6 : 21 x = 4

MT-50a

Review:

Add:

1.
$$\begin{array}{r} -15x \\ +\ 6x \\ -20x \\ +26x \\ +14x \\ \hline 11x \end{array}$$

2.
$$\begin{array}{l} 1/3a \\ \underline{1/3a} \\ \frac{2}{3a} \end{array}$$

3.
$$\begin{array}{l} \frac{m}{3} - N \\ \frac{2m}{3} + N \\ \hline m \end{array}$$

Subtract:

1.
$$\begin{array}{l} -14a^2b^3c^4 \\ -9a^2b^3c^4 \\ \hline -5a^2b^3c^4 \end{array}$$

2.
$$\begin{array}{l} +16x -12y + 9z \\ + 7x + 5y - 4z \\ \hline 9x - 17y + 13z \end{array}$$

3.
$$\begin{array}{ll} \frac{-4b}{5} & \frac{-4c}{3} \\ \frac{3b}{5} & \frac{c}{3} \\ \hline \frac{-7b}{5} & \frac{5c}{3} \end{array}$$

Multiply:

1.
$$\begin{array}{l} -9y^3z^2 \\ -3y^2z^5 \\ \hline 27y^5z^7 \end{array}$$

2.
$$\begin{array}{l} +8a^3z^2 \\ -2a^2z \\ \hline -16a^5z^6 \end{array}$$

Divide:

1. $\frac{-12x^4}{15x^2} = \frac{-4x^2}{5}$

2. $\frac{-25x^2y^3}{-50x^3y^2} = \frac{1y}{2x} = \frac{y}{2x}$

Solve for the unknown:

1. $.15m = 30$

$m = 200$

2. $\frac{3x}{4} = 3$

$x = 4$

3. $a - 4 = 7$

$a = 11$

4. $x + 2 = 5$

$x = 3$

5. $2x + 3 = x + 5$

$x = 2$

6. $4x - 2(x + 3) = x - 4$

$x = 2$

7. $5y + 7 - 3y - 5 = y + 4$

$y = 2$

8. $4x^3 - 6 = 3x^3 + 119$

$x = 5$

9. $5x^2 + 3 = 2x^2 + 30$

$x = 3$

10. $a + b = 15$
$\underline{a - b = 5}$

$a = 10$

$b = 5$

11. $4r + 6s = 58$
$\underline{4r - 6s = -2}$

$r = 7$

$s = 5$

12. $x - 3y = 3$
$\underline{2x + y = 13}$

$y = 1$

$x = 6$

Add and solve:

13. $\frac{3x}{4} + y = 6$
$\underline{\frac{x}{4} - y = -2}$

$x = 4$

$y = 3$

14. $\frac{x}{} + 3 = \frac{x}{8} + 7$

$x = 10\ 2/3$

15. $\frac{x}{2} - 5 = \frac{x}{5} + 1$

$x = 20$

16. $\frac{x}{3} = \frac{2}{5}$

$x = 1\ 1/5$ or 1.2

17. $\frac{2x}{7} = \frac{6}{5}$

$x = 4\ 1/5$ or 4.2

18. $\frac{3}{a} + \frac{2}{b} = 4$
$\underline{\frac{3}{a} - \frac{2}{b} = 2}$

$a = 1$

$b = 2$

19. $x : 16 : 4 : 8$

$x = 8$

20. $7 : 14 = 4 : x$

$x = 8$

Algebra Test #10

Pgs. 1-119

1. Five eighths of a certain number increased by a second number equals 12. Three-eighths of the same number decreased by the second number equals -4. Find the numbers. x = 8 y = 7

2. A man works for $2.80 per hour. What are his earnings for 80 hours of work? $224

3. The length of a rectangle is 30 feet and the width is 20 feet. A second rectangle with the same proportions has a width of 8 feet. What is the length of the second rectangle. x = 120

4. A man is 6' tall and casts a shadow 3' long and a tree nearby casts a shadow of 15'. How tall is the tree? x = 30'

5. Combine the following: 5d - 2c + 6d + 3c - 5d. c + 6d

6. A family spends 23.5% of their income on food. Their income is $135.00 per week. How much is their food bill per week? $31.73

7. Find two consecutive integers whose sum is 85. 1st = 42 2nd = 43

8. $x^2y^3 \div xy^2 = xy$

9. $14x^5y^3 \div 7x^3y^2 = 2x^2y$

10. The area of a rectangle is 720 square feet, and the width is 1/5 the length. Find the dimensions. W = 12 L = 60

11. $x + 4y = 12$
$\underline{2x - y = 6}$

$y = 2$
$x = 4$

12. Add: $2a + 3b$
$\underline{3a + 4b}$
$5a + 7b$

13. Find the distance traveled by a car going an average speed of 43.6 miles per hour for 3.4 hours. 148.24 mi.

14. Find the dimensions of a rectangle if the area is 144 square inches and the length and width are in the ratio of 4 to 1. W = 6 L = 24

15. The taxes on a piece of property valued at $6,000.00 are $150.00. At the same rate, what would be the taxes on property valued at $10,000.00? x = 250

16. $x^2 + 3y^2 = 12$
$x = 3y$
$\underline{y = 1}$
$\underline{x = 3}$

17. $x^2 + 2y^2 = 24$
$x = 2y$
$\underline{y = 2}$
$\underline{x = 4}$

18. $2x^2 + 3y^2 = 53$
$y = \frac{x}{5}$

$\underline{x = 5}$
$\underline{y = 1}$

19. $2x^2 + 3y^2 = 126$
$x = \frac{y}{2}$

$\underline{x = 3}$
$\underline{y = 6}$

20. The sum of two squares is 125. The first number is one-half the second. Find the numbers.

Solve:

1. $6a\,\overset{2a^4}{\overline{\big)\,12a^5}}$

2. $-3xy\,\overset{9xy^2}{\overline{\big)\,-27x^2y^3}}$

3. $-6B^2c\,\overset{-8BC^3}{\overline{\big)\,+48B^3c^4}}$

4. $\frac{4a}{x^2 + y^2} \quad \frac{2(x^2 + y^2)}{5} = \frac{8a}{5}$

5. $\frac{x-y}{2x} \quad \frac{4x^2}{x-y} = 2x$

6. $\frac{5x^2}{12a} \cdot \frac{6a^3}{10x^4} \cdot \frac{-8x}{3a} = \frac{-2a}{3x}$

7. $\frac{-4a^2}{x} \cdot \frac{2x^3}{a} \cdot \frac{3a}{16x} = \frac{3a^2x}{-2}$

8. $\frac{x}{a} \cdot \frac{a^2}{x^3} \cdot \frac{a}{x} = \frac{a^2}{x^3}$

9. $\frac{a^2}{x^2} \cdot \frac{x^4}{a^3} \cdot \frac{3x}{a} = \frac{3x^3}{a^2}$

10. $\frac{-5x}{a} \cdot \frac{-a^2}{10x^2} \cdot \frac{-2a}{x} = \frac{-a^2}{x^2}$

11. $\frac{a^2}{x} \div \frac{a}{x^2} = ax$

12. $\frac{2\,(a-b)}{x} \div \frac{a-b}{x^2} = 2x$

13. $\frac{a + b}{2} \div \frac{a + b}{4} = 2$

14. $\frac{5}{2x} \div \frac{15}{4x^2} = \frac{2x}{3}$

15. $\frac{3x}{4} \div \frac{x^2}{2} = \frac{3}{2x}$

16. $\frac{10x^2y}{3} \div 5xy = \frac{2x}{3}$

17. $\frac{-12a^3}{5} \div 4 = \frac{-3a^3}{5}$

Multiply:

18.
$$\begin{array}{r} -a + b + 2c \\ +\ 3b \\ \hline -3ab+3b^2+6bc \end{array}$$

19.
$$\begin{array}{r} a + b \\ -a \\ \hline -a^2-ab \end{array}$$

20.
$$\begin{array}{r} a - b \\ a \\ \hline a^2-ab \end{array}$$

21.
$$\begin{array}{r} 5a + 4b + 2c \\ -\ 3b \\ \hline -15ab-12b^2-6bc \end{array}$$

22.
$$\begin{array}{r} 5x - 2y + 3z \\ -2xy \\ \hline -10x^2y+4xy^2-6xyz \end{array}$$

23.
$$\begin{array}{r} a + b \\ a + b \\ \hline a^2+2ab+b^2 \end{array}$$

24.
$$\begin{array}{r} x + y \\ x - y \\ \hline x^2-y^2 \end{array}$$

25.
$$\begin{array}{r} 2x - y \\ x - y \\ \hline 2x^2-3xy+y^2 \end{array}$$

26.
$$\begin{array}{r} 3x + 2y \\ 2x - 3y \\ \hline 6x^2-5xy-6y^2 \end{array}$$

27.
$$\begin{array}{r} 2x + 4y \\ 3x - 2y \\ \hline 6x^2+8xy-8y^2 \end{array}$$

28.
$$\begin{array}{r} a + b \\ c + d \\ \hline ac+bc+ad+bd \end{array}$$

29.
$$\begin{array}{r} x - y + z \\ x + y - z \\ \hline x^2+2yz-y^2-z^2 \end{array}$$

30.
$$\begin{array}{r} 6 + 2a - 3a^2 \\ 2 - 3a + a^2 \\ \hline 12-14a-ba^2+11a^3-3a^4 \end{array}$$

Algebra Test #11

Pgs. 1-123

1. Find l when a = lw.

   $l = \frac{a}{w}$

2. -3.5a
   x2
   - 7a

3. Solve for x: $x-3 = 18$

   $x = 21$

4. $ab \times ab = a^2b^2$

5. Find three consecutive even integers whose sum is 54.

   1st = 17
   2nd = 18
   3rd = 19

6. If 22 is added to 2 times a certain number, the result is 88.
   What is the number? $x = 33$

7. Find the square root of 2401.

   49
   2401
   16
   801
   89 -801

8. Add:
   -6a - 2b
   -2a + b
   -8a - b

9. $\begin{array}{r} x+2 \\ x+1\overline{\big)\,x^2+3x+2} \end{array}$

10. $\begin{array}{r} 2x-1 \\ x-4\overline{\big)\,2x^2-9x+4} \end{array}$

11. $\begin{array}{r} x+3y \\ 2x-y\overline{\big)\,2x^2+5xy-3y^2} \end{array}$

Multiply:

12. $\begin{array}{r} x^2+2xy+y^2 \\ \underline{x+y} \\ x^3+3x^2y+3xy^2+y^3 \end{array}$

13. $\begin{array}{r} a+b+c \\ \underline{a-b-c} \\ a^2-2bc-b^2-c^2 \end{array}$

14. The length of a rectangular lot is two times the width. The area of the lot is 5,000 square feet. Find the dimensions.

x

$2x$

$5000=(x)(2x)$

$5000=2x^2$

$2500=x^2$

$50=x$

Width = 50 Length = 100

15. John bowled three games and had scores of 159, 168, and 173. What was his average score for the three games?

$$\begin{array}{r} 159 \\ 168 \\ 173 \\ 3\,\overline{\big)\,500} \end{array} = 166\ 2/3$$

16. The balance owed on a kitchen stove is $234.60 after a down payment. If it is paid for in 12 equal payments, how much is each payment?

$$\frac{234.60}{12} = 19.55$$

17. $\dfrac{22xyz}{11} = 2\,xyz$

18. Solve for a:

$9a = 63$

$a = 7$

19. Find the interest paid on $2,200.00 at 1 1/2% for 6 years.

2200 x.015 x 6 = $198

20. Sue is 3 times as old as her sister. The sum of their ages is 16. How old is Sue and how old is her sister?

x = sister
3x = Sue

$x+3x = 16$
$4x = 16$
$x = \underline{4}$

Sister is 4 yrs. old
Sue is 12 yrs. old

21. The perimeter of a rectangle is 60 yards. The length is 4 times the width. What are the dimensions?

P= 60 yrds.
l= 4xw=4x
w=x

60 yards = 2(4x) + 2(x)
6 = x

Width = 6 yards

Length = 24 yards

22. Solve for x: $4 + 2x - 3 = 6 (x + 1)$

$\underline{x = 1\ 1/4 \text{ or } \frac{-5}{4}}$

$1 + 2x = 6x + 6$
$- 5 = 4x$
$\frac{-5}{4}$ or $1\ 1/4 = x$

23. Write the exponent for x.x.x.x.x = x5

Multiply:

24. x^2y^2z
$\underline{x^2yz}$
$x^4y^3z^2$

25. $4x - y$
$\underline{3x + 2y}$
$12x^2+5xy-2y^2$

Subtract: $4a + 3b$
$\underline{3a - 2b}$
$a + 5b$

27. Find the square root of 16,384.

$\underline{128}$

28. What does $A^2 \times A^5$ =? $\underline{A^7}$

29. $3x + y = 8$ Solve for x and y:

$\underline{4x + 2y = 26}$ $-2(3x+y=8)= 6x-2y=16$

$\underline{4x+2y=26}$

$-2x=10$

$\underline{x=-5}$ $\underline{y = 23}$

30. $4x + y = 15$ Solve for x and y:

$\underline{x - 2y = -12}$ $2(4x+y=15) = 8x+2y=30$

$\underline{x-2y= 12}$

$9x=18$ $\underline{x = 2}$ $\underline{y = 7}$

Add:

31. $25xy$

$-14xy$

$70xy$

$\underline{-25xy}$

$56xy$

32. $7a - 5b - 9c$

$-6a + 9b - 7c$

$4a - 8b + 6c$

$\underline{-3a + b - c}$

$2a- 3b -11c$

33. $\frac{-18c^2d - 24c^2d + 12cd^3}{-6cd} =$

$\underline{3c + 4c-2d^2}$ or $\underline{7c-2d^2}$

34. $\frac{-35ab^3}{-7a^2b} =$

35. Reduce: $\frac{15ab^2c^3}{10a^3b^2c} = \frac{3c^2}{2a^2}$

36. $\frac{x}{y} \div \frac{x}{2} = \frac{2}{y}$

37. $\frac{2x}{3} \div \frac{x^2}{6} = \frac{4}{x}$

38. $\frac{3x}{8} \cdot \frac{4}{x^2} = \frac{3}{2x}$

39. $\frac{5x}{3} \cdot \frac{9}{10x^3} = \frac{3}{2x^2}$

Remove common factors:

40. $-9x^2 + 6xy - 18y^2 =$

$3(-3x^2+2xy-6y^2)$

41. $16a^2 - 24a^2b + 28a^3b^3 =$

$4a^2(4-6b + 7ab^3)$

Multiply:

42.
$$\begin{array}{r} 2a + 3a^2 + 4a^3 \\ \underline{a - a^2} \\ 2a^2+a^3+a^4-4a^5 \end{array}$$

43.
$$\begin{array}{r} -3a + 2a^2 - a^3 \\ \underline{-2a - 3a^2} \\ 6a^2+5a^3-4a^4+3a^5 \end{array}$$

44.
$$\begin{array}{r} 3x-2 \\ x+1 \overline{)3x^2 + x - 2} \end{array}$$

45.
$$\begin{array}{r} 2+3x \\ 3-2x \overline{)6 + 5x - 6x^2} \end{array}$$

46. $(-9ab)^2 = \underline{81a^2b^2}$

47. $(7x^2y^2z^2)^2 = \underline{49x^4y^4z^4}$

48. $\overset{4xy^2}{\sqrt[2]{16x^2y^4}}$

49. $\overset{x^2y}{\sqrt[2]{x^4y^2}}$

50. $\overset{5x^2yz^3}{\sqrt[2]{25x^4y^2z^6}}$

Algebra Test #12

pgs. 124-131

Perform the indicated operation:

1. $(x + 5)^2 = \quad x^2+10x+25$

2. $(3x + 4)^2 = \quad 9x^2+24x+16$

3. $(2x + 5)^2 = \quad 4x^2+20x+25$

4. $(x - 3)^2 = \quad x^2-6x+9$

5. $(2x - 3)^2 = \quad 4x^2-12x+9$

6. $(3x - 5)^2 = \quad 9x^2-30x+25$

7. $(2x + 1)(x + 3) = \quad 2x^2+7x+3$

8. $(3x - 2)(x + 1) = \quad 3x^2+x-2$

Solve the equations:

9. $x^2 + 6x + 9 = 0$
$(x+3)(x+3) = 0 \quad x+3 = 0 \quad \underline{x = 3}$

10. $x^2 - 25 = 0$
$(x+5)(x-5) = 0 \quad \underline{x = \pm 5}$

11. $x^2 - 8x = 12$
$(x-6)(x-2) = 0 \quad \dfrac{x = 6}{x = 2}$ or

12. $x^2 + 18x + 81 = 0$

$(x + 9)(x+9) = 0$ $\underline{x = -9}$

13. $x^2 - 100 = 0$

$(x+10)(x-10) = 0$ $\underline{x = +10}$

14. $2x^2 + 11x + 5 = 0$

$(x+5)(2x+1) = 0$ $\underline{x = -1/2}$ or $x = 5$

15. $x^2 - 4x = 12$

$(x-6)(x+2) = 0$ $\underline{x = 6}$ or $x = 2$

16. $x^2 - 81 = 0$

$(x+9)(x-9) = 0$ $\underline{x = \pm 9}$

17. $\frac{2(a-b)}{x} \div \frac{a-b}{x^2} = \underline{2x}$

18. $\frac{2x + 4y}{3} \div \frac{3x + 6y}{9} = \underline{\quad 2 \quad}$

19. $\overset{5x}{\sqrt[2]{25x^2}}$

20. $(2xy)^3 = 8x^3y^3$

Mastery Test I

WORKING WITH NUMBERS, Octagon Book

Name ______________________ Score ________

1. Add.

$$\begin{array}{r} 123.076 \\ 14.9 \\ 46.08 \\ 32.6 \\ 208.19 \\ \hline \end{array}$$

424.846

2. Add.

$$\begin{array}{c} \frac{5}{8} \\ \frac{3}{4} \\ \frac{1}{2} \\ \frac{5}{16} \\ \frac{1}{4} \\ \hline \end{array}$$

=2 7/16

3. Subtract.

$$\begin{array}{r} 125.974 \\ 85.995 \\ \hline \end{array}$$

39.979

4. Subtract.

$$\begin{array}{r} 9\frac{1}{2} \\ 3\frac{2}{3} \\ \hline \end{array}$$

=5 5/6

5. Multiply.

$$\begin{array}{r} 7.065 \\ .482 \\ \hline \end{array}$$

3.405330

6. Multiply.

$7\frac{1}{2} \times 1\frac{1}{5} =$ 9

7. Divide.

$4\frac{1}{2} \div \frac{3}{10} =$ 15

8. Divide.

$$\begin{array}{r} 3000 \\ .17\overline{)510} \end{array}$$

9. $12\frac{1}{2}\%$ of 400 = 50

10. $\frac{1}{2}\%$ of 200 = 1

11. \$25.00 less 10% = \$22.50

12.
$A = lw$
$l = 30$
$w = 20$

$A =$ 600

13.
$V = lwh$
$l = 15$
$w = 10$
$h = 5$

$V =$ 750

14.
$A = \pi r^2$
$r = 7$

$A =$ 154

15.
$V = \pi r^2 h$
$r = 7$
$h = 10$

$V =$ 1540

16.
Marked price = \$62.50
Discount = 20%

Net cost = \$50

17.
$I = prt$
$p = \$1000$
$r = 6\%$
$t = 4$ mos.

$I =$ \$20

18.
$I = prt$
$p = \$1200$
$r = 6\%$
$t = 18$ mos.

$I =$ \$108

19.
\$15 is what percent of \$45?

Answer: 33 1/3%

20.
75 is what percent of 25?

Answer: 300%

21. We pay taxes on a house assessed at $26,000. The tax rate is $2.50 per hundred. What is the amount of our tax bill?

Answer: $650

22. Our house is insured against fire in the amount of $15,000. The rate is $.32 per hundred. What is the annual premium?

Answer: $48

23. Mr. Porter bought a $32,000 house. He made a $5000 down payment and agreed to pay the balance in $180 monthly payments which included 7% interest. How much of his first payment will be applied to the balance due?

Answer: Cannot do

24. How much of Mr. Porter's second payment will be applied to the balance due?

Answer: Cannot do

25. What is the interest on $600 at 6¾% for 9 months?

Answer: $30.38

26. Taxes are often discounted if paid at an early date. Raul received a 3% discount on his tax bill of $120. How much tax did he have to pay?

Answer: $116.40

27. The Gem Store had a total business of $150,000 last year. Cost of the merchandise was $80,000. Overhead amounted to $50,000. The profit was how much?

Answer: $20,000

28. Mrs. Alexander used 6000 gallons of gasoline during the year. The total tax per gallon was 9 cents. How much tax did she pay?

Answer: $540

29. Tom took out a 20-year endowment policy in the amount of $5000. The rate was $28 per thousand. What was the annual premium?

Answer: $140

30. Last month our gas bill was $8. This month it is 12½% less. How much is this month's bill?

Answer: $7

Mastery Test II   *WORKING WITH NUMBERS, Octagon Book*

Name .... **Score** ....

1. Add.

15.906
4.3
8.04
12.
.954

41.200

2. Add.

$6\frac{5}{6}$
$4\frac{2}{3}$
$8\frac{1}{2}$
$4\frac{1}{5}$
$3\frac{1}{10}$

27 3/10

3. Subtract:

83.15
49.069

34.081

4. Subtract.

$24\frac{1}{3}$
$18\frac{5}{6}$ =5 1/2

5. Multiply.

26.047
2.9

75.5363

6. Multiply.

$4\frac{2}{3} \times 4\frac{1}{2} =$ 21

7. Divide.

$3\frac{2}{3} \div 7\frac{1}{3} =$ 1/2

8. Divide.

$$1.8\overline{)\,7\overset{\cdot}{2}0}$$
400

9. From the formula $A = lw$, write the formula for finding l.

$l = \frac{A}{W}$

10. From the formula $I = prt$, write a formula for finding r.

$r = \frac{I}{pt}$

11. From the formula $V = lwh$, write a formula for finding h.

$h = \frac{V}{lw}$

12. From the formula $C = \pi d$, write a formula for finding d.

$d = \frac{C}{\pi}$

Solve each of the following equations.

13. $5x = x + 20$

x = 5

14. $3x + 5 = 23$

x = 6

15. $10 : x = 5 : 20$

x = 40

16. $\frac{x}{6} = \frac{2}{3}$ x = 4

17. $\frac{x}{4} = \frac{5}{2}$ x = 10

18. $\frac{5}{2x} = \frac{1}{4}$ 10 = x
x

19. $4x - 5 = 3x + 1$

x = 6

20. $2(3x + 4) = 3x + 23$

x = 5

21. Bob paid $40 for his bicycle and Barney $50 for his. What is the ratio of the cost of Bob's bicycle to the cost of Barney's?

Answer: 4: 5

22. Henry is twice as old as his sister. The sum of their ages is 21 years. How old is each?

Henry: 14 *Sister:* 7

23. Helen's father is three times as old as she. The sum of their ages is 56 years. How old is each?

Helen: 14 *Father:* 42

24. The area of a rectangular field is 1200 square rods. The length of the field is 40 rods. What is the width?

Answer: 30 rods

25. The area of another rectangular field is 1500 square rods. The width is 30 rods. What is the length?

Answer: 50 rods

26. The interest on $500 for 2 years was $40. What was the interest rate?

Answer: 4%

27. If 25 is added to a certain number, the sum is 75. What is the number?

Answer: 50

28. The perimeter of a triangle is 72. The sides are in the ratio of 1 to 2 to 3. Find each side.

1st side: 12 *2nd side:* 24 *3rd side:* 36

29. Emmett earned $14 for working 6 hours. At the same rate, how much would he earn in 9 hours? Solve by proportion.

Answer: $21

30. From the mileage table, find the number of miles between New York and Tampa.

Answer: 1377

MILEAGE BETWEEN	CHICAGO, ILL.	DETROIT, MICH.	LOS ANGELES, CALIF.	NEW YORK, N. Y.
Atlanta, Ga.	744	726	2401	965
Chicago, Ill.		277	2243	859
Dallas, Texas	900	843	1509	1655
New Orleans, La.	972	1142	2035	1436
Tampa, Fla.	1426	1414	3012	1377

31. A tree casts a shadow of 40 feet. At the same time a pole casts a shadow of 20 feet. The pole is 18 feet high. How tall is the tree?

Answer: 36'

MASTERY TEST — PART I

FOR THE FIRST THREE PARTS

Working with Numbers

REFRESHER BOOK

Name .... **Date** ....

Instructions

This test is designed to give you the opportunity to mark your progress at the midpoint of your study of *Working with Numbers, Refresher Book.* The teacher, of course, will be interested in seeing just how well you have done. But you, yourself, will be most interested in seeing whether you have made the right amount of progress and in seeing how well you have retained all the skills studied to this point.

There is no time limit for this test. Use all the time necessary to complete it. If you come to a problem which you cannot work, go on to the others. Then, when you have finished them, come back to any you have skipped. Do your very best and *prove to yourself* that you are making the kind of progress you desire.

The test is divided into four parts: Whole Numbers, Fractions, Decimals, and Word Problems. Each part is divided into the four operations: Addition, Subtraction, Multiplication, and Division. When you have finished the test and it has been scored, enter your scores in the chart below. This will enable you to determine exactly where there are any weaknesses and where you need to acquire more skill.

		PROBLEMS	PERFECT SCORE	MY SCORE
Whole Numbers	Addition	1 - 7	7	
	Subtraction	8 - 13	6	
	Multiplication	14 - 19	6	
	Division	20 - 25	6	
Fractions	Addition	26 - 32	7	
	Subtraction	33 - 38	6	
	Multiplication	39 - 44	6	
	Division	45 - 50	6	
Decimals	Addition	51 - 57	7	
	Subtraction	58 - 63	6	
	Multiplication	64 - 69	6	
	Division	70 - 75	6	
Word Problems		76 - 100	25	
TOTAL SCORE			100	

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MT-68a

Whole Numbers

Add.

1.
9
7
5
4
6
2
8
41

2.
12
55
74
37
49
86
313

3.
464
647
528
786
953
275
3,653

4.
285346
465987
751,333

5.
380240
76095
456,335

6.
5237
4785
6876
8569
25,467

7.
4105
3086
2470
5909
15,570

Subtract.

8.
17
9
8

9.
99
53
46

10.
1289
435
854

11.
745
246
499

12.
620
443
177

13.
84239
36544
47,695

Whole Numbers

Multiply.

14.
$$\begin{array}{r} 9 \\ \underline{7} \\ 63 \end{array}$$

15.
$$\begin{array}{r} 14 \\ \underline{9} \\ 126 \end{array}$$

16.
$$\begin{array}{r} 34 \\ \underline{23} \\ 102 \\ \underline{68} \\ 782 \end{array}$$

17.
$$\begin{array}{r} 75 \\ \underline{37} \\ 525 \\ \underline{225} \\ 2{,}775 \end{array}$$

18.
$$\begin{array}{r} 762 \\ \underline{56} \\ 4572 \\ \underline{3810} \\ 42{,}672 \end{array}$$

19.
$$\begin{array}{r} 206 \\ \underline{250} \\ 10300 \\ \underline{412} \\ 51{,}500 \end{array}$$

Divide.

20.
$$\begin{array}{r} 9 \\ 7\overline{)63} \end{array}$$

21.
$$\begin{array}{r} 41 \\ 7\overline{)287} \end{array}$$

22.
$$\begin{array}{r} 32 \\ 12\overline{)384} \end{array}$$

23.
$$\begin{array}{r} 223 \\ 9\overline{)2007} \end{array}$$

24.
$$\begin{array}{r} 304 \\ 16\overline{)4864} \end{array}$$

25.
$$\begin{array}{r} 2060 \\ 172\overline{)354320} \end{array}$$

Fractions

Add. (Simplify Answers)

26.

$$\begin{array}{r}\frac{1}{8}\\ \frac{3}{8}\\ \frac{5}{8}\\ \hline\end{array}$$

$\frac{9}{8} = 1\ \frac{1}{8}$

27.

$$\begin{array}{rl}\frac{1}{2} & \frac{2}{4}\\ \frac{1}{4} = & \frac{1}{4}\\ \hline & \frac{3}{4}\end{array}$$

28.

$$\begin{array}{rl}\frac{2}{3} = & \frac{8}{12}\\ \frac{1}{4} & \frac{3}{12}\\ \hline & \frac{11}{12}\end{array}$$

29.

$$\begin{array}{r}15\frac{3}{4}\\ 12\frac{1}{4}\\ \hline 28\end{array}$$

30.

$$\begin{array}{r}19\frac{7}{8}\\ 12\\ \hline 31\ \frac{7}{8}\end{array}$$

31.

$$\begin{array}{r}27\frac{1}{4}\\ 13\frac{3}{8}\\ \hline 40\ \frac{5}{8}\end{array}$$

32.

$$\begin{array}{r}30\frac{1}{2}\\ 16\frac{1}{3}\\ \hline 46\ \frac{5}{6}\end{array}$$

Subtract.

33.

$$\begin{array}{r}\frac{3}{4}\\ \frac{1}{4}\\ \hline \frac{1}{2}\end{array}$$

34.

$$\begin{array}{r}\frac{3}{4}\\ \frac{3}{8}\\ \hline \frac{3}{8}\end{array}$$

35.

$$\begin{array}{r}\frac{1}{3}\\ \frac{1}{4}\\ \hline \frac{1}{12}\end{array}$$

36.

$$\begin{array}{r}21\frac{3}{4}\\ 9\\ \hline 12\ \frac{3}{4}\end{array}$$

37.

$$\begin{array}{r}19\\ 7\frac{2}{3}\\ \hline 11\ \frac{1}{3}\end{array}$$

38.

$$\begin{array}{r}30\frac{1}{2}\\ 14\frac{1}{4}\\ \hline 16\ \frac{1}{4}\end{array}$$

Multiply.

39. $5 \times \frac{3}{10} = 1\ \frac{1}{2}$

40. $\frac{1}{2} \times \frac{4}{5} = \frac{2}{5}$

41. $\frac{1}{2} \times 25 = 12\ \frac{1}{2}$

42. $9 \times 2\frac{1}{3} = 21$

43. $8\frac{1}{3} \times \frac{3}{5} = \frac{25}{3} \times \frac{3}{5} = 5$

44. $3\frac{1}{3} \times 2\frac{2}{5} = \frac{10}{3} \times \frac{12}{5} = 8$

Fractions and Decimals

Fractions

Divide.

45. $\frac{3}{8} \div 3 = \frac{1}{8}$

46. $3 \div \frac{3}{8} =$ 8

47. $\frac{1}{2} \div \frac{1}{6} =$ 3

48. $4\frac{2}{3} \div \frac{7}{8} = \frac{14}{3} \text{ x } \frac{8}{7} = 5\ \frac{1}{3}$

49. $5\frac{1}{3} \div \frac{1}{2} = \frac{16}{3} \text{ x } 2 = 10\ \frac{2}{3}$

50. $10\frac{1}{2} \div 1\frac{3}{4} = \frac{21}{2} \text{ x } \frac{4}{7} = 6$

Decimals

Add.

51.

$$\begin{array}{r} .4 \\ .5 \\ \hline .9 \end{array}$$

52.

$$\begin{array}{r} .2 \\ .6 \\ .3 \\ .8 \\ \hline 1.9 \end{array}$$

53.

$$\begin{array}{r} 108.75 \\ 235.87 \\ \hline 344.62 \end{array}$$

54.

$$\begin{array}{r} 214.08 \\ 17.9 \\ \hline 231.98 \end{array}$$

55.

$$\begin{array}{r} 1.046 \\ .87 \\ .9 \\ 4. \\ \hline 6.816 \end{array}$$

56.

$$\begin{array}{r} \$217.43 \\ 7.15 \\ 29.00 \\ 4.35 \\ \hline \$257.93 \end{array}$$

57.

$$\begin{array}{r} 263.0005 \\ 17.6 \\ \hline 280.6005 \end{array}$$

Subtract.

58.

$$\begin{array}{r} .9 \\ .4 \\ \hline .5 \end{array}$$

59.

$$\begin{array}{r} 128.9 \\ 19.8 \\ \hline 109.1 \end{array}$$

60.

$$\begin{array}{r} 216.5 \\ 19. \\ \hline 197.5 \end{array}$$

61.

$$\begin{array}{r} 137. \\ 15.8 \\ \hline 121.2 \end{array}$$

62.

$$\begin{array}{r} 25.004 \\ 16. \\ \hline 9.004 \end{array}$$

63.

$$\begin{array}{r} 372. \\ 135.006 \\ \hline 236.994 \end{array}$$

Decimals

Multiply.

64.

.4
.9
.36

65.

1.5
.9
1.35

66.

.025
.3
.0075

67.

.15
.06
.0090

68.

.004
.03
.00012

69.

3.5
2.4
140
70
8.40

Divide.

70.

.5
.5).25

71.

.15
15) 2.25

72.

.03
1.3).039

73.

40
1.2) 48

74.

500
.15) 75

75.

18.283
9.9) 181

Word Problems

Do Your Work Here

76. Sam's father has a truck that weighs 4725 pounds. The truck will carry a load of 7500 pounds. What is the total weight of truck and full load? . . . . 12,225 lbs.

77. The population of New York is approximately 7,867,760. Chicago has a population of approximately 3,366,957. What would the combined population be? 11,234,717

78. The Cascade Tunnel in Washington is 41,152 feet long. The Moffat Tunnel in Colorado is 32,798 feet long. How much longer is the Cascade Tunnel? . . . . 8,354 ft.

79. Esther gave the storekeeper a twenty-dollar bill for a purchase of $14.51. How much change should she have received? $5.49

80. Mrs. George made a payment of $37.50 on a bill of $73.94. How much did she still owe? . . . $36.44

81. Bob Holmstrom sold his farm of 87 acres for $250 an acre. How much did he receive for the farm? $21,750

82. A jet airliner flew from New York to London in just 7 hours. The average speed was 499.4 miles an hour. How many miles long was the trip? 3495.8 miles

83. Sam can plant 261 cabbage plants in 1 row of his garden. How many rows in his garden must be used to plant 1044 plants? . . . . . . 4 rows

84. Last summer we took an auto trip of 3,427 miles. After we had driven 1,578 miles, how many miles were left? . . . . 1,849 miles

85. For the first four months of the year, rain fell as follows: $2\frac{1}{2}$ in., $3\frac{1}{4}$ in., $1\frac{1}{8}$ in., and $1\frac{1}{2}$ in. What was the total rainfall for these four months? . . . . $8\frac{3}{8}$ inches

86. Ruth used $1\frac{3}{4}$ cups of sugar in making a cake and $\frac{3}{4}$ cup in making the icing. How much sugar did she use in all? . . . . $2\frac{1}{2}$ cups

87. Last year Milwaukee had a total fall of snow amounting to 92 inches. The normal fall is $52\frac{1}{4}$ inches. How much above normal was the snowfall last year? . . . . $39\frac{3}{4}$ inches

Word Problems

Do Your Work Here

88. The storekeeper sold 9¼ yards from a piece of material containing 53 yards. How much was left in the original piece? . . . . $43\frac{3}{4}$ yds.

89. Hay sells for $56 per ton. How much will Ted Steinson have to pay for five and one-half tons? $308

90. Last week we drove to Detroit. At an average speed of 42 miles per hour, the trip took 4½ hours. How far was it? . . . . 189 mi.

91. Mr. Thomas shoveled 12½ yards of sand in 2½ hours. At that rate how many yards did he shovel in one hour? . . . . 5 yds.

92. How much material was required for each of two dresses of the same size if it took 5½ yards for both? . . . . $2\frac{3}{4}$ yds.

93. Rain fell last month as follows: 2.4 in., .19 in., 3.7 in., and .9 in. What was the total rainfall? . . . . 7.19 inches

94. From Chicago to Memphis is 527.4 miles. From Memphis to New Orleans is 393.8 miles farther. How many miles is it from Chicago to New Orleans? . . . . 921.2 miles

95. The heaviest rainfall in one day on record for one city is 11.17 inches. The next heaviest is 8.32 inches. How much greater is the record fall? . . . . 2.85 in.

96. Mr. Richard Lavilla took a trip by train from Montreal to Vancouver, a distance of 2937.67 miles. Mr. John Henry Larkin went by train from Chicago to Denver, a distance of 1014.9 miles. Lavilla went how much farther than Larkin? . 1922.77 mi.

97. Since water weighs 8.355 lb. to the gallon, how much do 9.4 gallons weigh? . . . . 78.5370 lbs.

98. Milk weighs 8.605 lb. to the gallon. How much do 12.2 gallons weigh? . . . . 104.9810 lbs.

99. Mr. Lee drove his car 297.6 miles while using 16 gallons of gas. How many miles does he get to the gallon? . . . . 18.6 m.p.g.

100. If a train travels 217.5 miles in 3 hours, what average speed per hour does it make? . 72.5 mph

MASTERY TEST — PART II

FOR THE ENTIRE BOOK

Working with Numbers

REFRESHER BOOK

Name .... Date ....

Instructions

This test is designed to give you the opportunity to measure the amount of progress you have made in the study of *Working with Numbers, Refresher Book.* It is to be used after you have finished studying the course so you can tell exactly how much you have achieved. The teacher, of course, will want to know how well you have studied. But you will be most interested in proving to *yourself* how much you have learned, and in finding out whether you have acquired all of the skills presented.

There is no time limit for this test. Use all the time necessary to complete it. If you come to a problem which you cannot work, go on to the others. Then, when you have finished them, go back to any you have skipped. Do your very best and *prove to yourself* just how well you have studied.

The test is divided into six parts: Whole Numbers; Fractions; Decimals; Percent; Measurements and Formulas; and Ratio, Proportion, and Simple Equations. When you have finished the test and it has been scored, enter your score in the chart below. This will enable you to determine exactly where there are any weaknesses and where you need to acquire more skill.

		PROBLEMS	PERFECT SCORE	MY SCORE
Whole Numbers	Addition	1 - 4	4	
	Subtraction	5 - 8	4	
	Multiplication	9 - 12	4	
	Division	13 - 16	4	
Fractions	Addition	17 - 20	4	
	Subtraction	21 -24	4	
	Multiplication	25 - 28	4	
	Division	29 - 32	4	
Decimals	Addition	33 - 36	4	
	Subtraction	37 - 40	4	
	Multiplication	41 - 44	4	
	Division	45 - 48	4	
Percent		49 - 68	20	
Measurements and Formulas		69 - 84	16	
Ratio, Proportion, and Simple Equations		85 - 100	16	
TOTAL SCORE			100	

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Whole Numbers

Add.

1.
17
29
78
85
34
243

2.
2176
3429
5,605

3.
2943
34
176
2053
5,206

4.
27965
30609
22079
30705
50078
161,436

Subtract.

5.
546
227
319

6.
1943
849
1,094

7.
2050
1564
486

8.
907086
548299
358787

Multiply.

9.
247
24
988
494
5,928

10.
306
24
1224
612
7,344

11.
305
240
12200
610
73,200

12.
2516
123
7548
5032
2516
309,468

Whole Numbers and Fractions

Whole Numbers

Divide.

13.
$$\begin{array}{r} 15 \\ 15\overline{)225} \end{array}$$

14.
$$\begin{array}{r} 21 \\ 24\overline{)504} \end{array}$$

15.
$$\begin{array}{r} 203 \\ 18\overline{)3654} \end{array}$$

16.
$$\begin{array}{r} 613 \\ 32\overline{)19616} \end{array}$$

Fractions (Reduce Answers)

Add.

17.
$$\begin{array}{r} \frac{5}{8} \\ \frac{1}{8} \\ \frac{3}{8} \\ \underline{\frac{7}{8}} \end{array} = 2$$

18.
$$\begin{array}{r} \frac{1}{2} \\ \frac{1}{4} \\ \frac{1}{8} \\ \underline{\frac{3}{4}} \end{array} = 1\frac{5}{8}$$

19.
$$\begin{array}{r} 2\frac{1}{8} \\ 4\frac{3}{8} \\ 7\frac{5}{8} \\ \underline{6\frac{7}{8}} \end{array} = 21$$

20.
$$\begin{array}{r} 2\frac{1}{2} \\ 3\frac{1}{4} \\ 15\frac{1}{8} \\ 12\frac{5}{16} \\ \underline{3\frac{3}{4}} \end{array} = 36\frac{15}{16}$$

Subtract.

21.
$$\begin{array}{r} \frac{5}{8} \\ \underline{\frac{3}{8}} \end{array} = \frac{1}{4}$$

22.
$$\begin{array}{r} \frac{3}{4} \\ \underline{\frac{2}{3}} \end{array} = \frac{1}{12}$$

23.
$$\begin{array}{r} 12\frac{9}{16} \\ \underline{4\frac{3}{8}} \end{array} = 8\frac{3}{16}$$

24.
$$\begin{array}{r} 9\frac{1}{2} \\ \underline{2\frac{2}{3}} \end{array} = 6\frac{5}{6}$$

Multiply.

25. $\frac{1}{3} \times 12 = 4$

26. $14 \times \frac{3}{7} = 6$

27. $\frac{3}{4} \times 4\frac{2}{3} = 3\frac{1}{2}$

28. $5\frac{1}{4} \times 2\frac{2}{3} = 14$

Divide.

29. $18 \div \frac{1}{3} = 54$

30. $\frac{2}{3} \div 4 = \frac{1}{6}$

31. $\frac{3}{5} \div 4\frac{1}{2} = \frac{2}{15}$

32. $10\frac{1}{2} \div 5\frac{1}{4} = 2$

Decimals

Add.

33.
.8
.7
.3
.2
2.

34.
.9
.15
.008
.54
.3
1.898

35.
.15
.05
.025
.2008
.4
.8258

36.
252.495
28.17
9.6
15.24
106.80
412.305

Subtract.

37.
6.276
1.042
5.234

38.
94.78
15.
79.78

39.
8.14
6.1
2.04

40.
1.9
.674
1.226

Multiply.

41.
.7
.5
.35

42.
.84
5
4.20

43.
.862
2
1.724

44.
.0004
.3
.00012

Divide.

45.
2.2
3) 6.6

46.
.6
16) 9.6

47.
.8103
.8) .64824

48.
5
1.8) 9

Percent

Change to Percent.

49. $\frac{1}{2} =$ 50%

50. $.75 =$ 75%

51. $.375 =$ 37 $\frac{1}{2}$%

Change to Decimals.

52. $\frac{1}{4} =$ .25

53. $60\% =$ .60

54. $87\frac{1}{2}\% =$ .875

Change to Fractions.

55. $.75 =$ $\frac{3}{4}$

56. $40\% =$ $\frac{2}{5}$

57. $62\frac{1}{2}\% =$ $\frac{5}{8}$

Solve.

58. 14% of 300 = 42

59. 75% of 440 = 330

60. Taxes are often discounted if paid at an early date. Father received a 3% discount on his tax bill of $120. How much tax did he have to pay?

Answer: $116.40

61. One of the fastest growing cities in the United States had a population of 587,718 in 1960. The 1970 census listed the population as 111.3% of the 1960 count. How large was the city in 1970?

Answer: 654,130+

62. A merchant was allowed ½% discount for prompt payment of a bill amounting to $400. What was the amount of his discount?

Answer: $2.00

63. Last year, a salesman sold merchandise amounting to $160,500. His commission of 6% amounted to how much?

Answer: $96.30

64. Last month our gas and light bill was $8.00. This month shows a decrease of 12½%. How much is this month's bill?

Answer: $7.00

Do Your Work Here

Percent; Measurements and Formulas

Percent

65. The distance from New York to San Francisco is 5200 miles by way of the Panama Canal. This is 40% of the distance by way of the Strait of Magellan. How far is it by way of the Strait of Magellan?

Answer: 13,000 mi.

66. Mr. Evans, the grocer, borrowed $1250 at 6% for 6 months. How much interest would he owe on the loan?

Answer: $37.50

67. Dan's bank pays 4½% interest. How much interest will he have on his savings account of $360 at the end of a year?

Answer: $16.20

68. To help build some houses, Mr. Raymond borrowed $12,000 from the bank and agreed to pay it back in two and one-half years, with interest at 5%. How much interest will he pay?

Answer: $1500

Measurements and Formulas

69.

$P = 2l + 2w$

$l = 400'$

$w = 300'$

$P =$ 1400'

70.

$A = lw$

$l = 60'$

$w = 50'$

$A =$ 3000 sq. ft.

71.

$V = lwh$

$l = 30$

$w = 20$

$h = 10$

$V =$ 6000 cu. ft.

72.

$C = \pi d$

$\pi = \frac{22}{7}$

$d = 14'$

$C =$ 44 ft.

73.

$A = \pi r^2$

$\pi = \frac{22}{7}$

$r = 7$

$A =$ 154

74.

$V = \pi r^2 h$

$\pi = \frac{22}{7}$

$r = 7$

$h = 10$

$V =$ 1540

75.

$I = prt$

$p = \$500$

$r = 4\%$

$t = 24$ mo.

$I =$ $40

Do Your Work Here

Measurements and Formulas; Ratio, Proportion, and Simple Equations

Measurements and Formulas

76. From the formula $A = lw$, write the formula for finding w.

$w = \frac{A}{l}$

77. What is the perimeter of the rectangle whose length is 60 ft. and whose width is one-half the length?

Answer: 180 ft.

78. How many feet of wire will be required to enclose a triangular-shaped park whose sides are 150′, 140′, and 200′?

Answer: 490'

79. A gymnasium floor measures 90 ft. by 150 ft. How many square feet are there in this floor area?

Answer: 13,500 sq. ft.

80. How much did Mother pay at $4 per square yard for a rug which measured 9′ by 12′?

Answer: $48

81. A cord of wood is a pile 8′ long, 4′ wide, and 4′ high. How many cubic ft. are in a cord?

Answer: 128 cu. ft.

82. How many cubic yards of dirt will be required to fill a hole 27′ by 30′ by 10′?

Answer: 300 cu. yds.

83. A coal car has inside dimensions of 40 ft. by 6 ft. and is filled level to a 5-ft. depth. Allowing 40 cu. ft. to the ton, how many tons of coal are there in this car?

Answer: 30 tons

84. A water standpipe is 60 feet high and has a diameter of 14 feet. What is the volume of the standpipe (in cubic feet)?

Answer: 9240 cu. ft.

Ratio, Proportion, and Simple Equations

85. $(120 \div 3) - (7 \times 5)$

$= 5$

86. $5x + 3 = 18$

$x = 3$

87. $5x - 1 = 4x + 3$

$x = 4$

88. $5(x + 4) = 4(x + 6)$

$x = 4$

Do Your Work Here

Ratio, Proportion, and Simple Equations (Continued)

89. Together Pat and Jerry have $5.25. Pat has $.25 more than Jerry. How much does each have?

Pat = $2.75

Jerry = $2.50

Answer:

90. The sum of Father's age and my age is 65 years. Father is 25 years older than I am. How old is each?

45 - father

Answer: 20

91. The perimeter of a rectangular field is 60 rods. The length is twice the width. Find the length and width.

width = 10 rods

Answer: length = 20 rods

92. Joe has $3.00, and John has $2.00. What is the ratio of Joe's money to that of John?

Answer: 1.5 : 1

93. The perimeter of a triangle is 81 feet. The three sides are in the ratio of 2 to 3 to 4. Find the length of each side.

Answer: 18 dr.,27ft.36 ft.

94. Frank, George, and Tom together have $45. The ratio of their amounts is 4 : 5 : 6. How much does each one have?

Answer: $12, $15, $18

95. $30 : x = 2 : 3$

$x =$ 45

96. $\frac{x}{6} = \frac{5}{3}$

$x =$ 10

97. If a photograph 3" by 4" were enlarged so that the 3" side would be 6", how large would the other side be?

Answer: 8"

98. The area of a square varies as the length of a side. Square A has a side twice as great as that of square B. The area of square A is how many times the area of square B?

Answer: 4

99. The sum of three numbers is 175. The second number is twice the first, and the third is twice the second. What are the numbers?

Answer: 25, 50, 100

100. A telephone pole casts a shadow of 30 feet at the same time a fence post casts a shadow of 3 feet. The fence post is 6 feet tall. How tall is the telephone pole?

Answer: 60 ft.

FRACTIONAL NUMBERS - ANSWER KEY

Practice 1

1. 1/3
2. 2/4 = 1/2
3. 3/4
4. 1/2
5. 4/8 = 1/2
6. 13/16

Practice 2

1. 3 1/2
2. 2 1/3
3. 1 3/10
4. 2 1/8
5. 4 1/4
6. 5 1/2
7. 6 2/5
8. 5 7/8
9. 1 1/5
10. 1 4/5
11. 11 1/2
12. 4 1/8
13. 6 1/4
14. 4 1/3
15. 1 5/7
16. 3 3/4
17. 1 5/6
18. 4 1/2
19. 1 3/5
20. 5 1/4

Practice 3

1. 7/4
2. 41/8
3. 57/8
4. 10/3
5. 5/2
6. 11/2
7. 25/3
8. 37/8
9. 49/8
10. 14/3
11. 37/4
12. 8/5
13. 35/4
14. 31/8
15. 29/5
16. 15/8
17. 28/3
18. 20/3
19. 23/5
20. 13/5

Practice 4

1. 9/12
2. 18/27
3. 10/16
4. 6/10
5. 10/12
6. 3/6
7. 21/24
8. 3/27
9. 6/21
10. 6/15

Practice 5

1. 1/3
2. 7/16
3. 3/5
4. 2/3 = 1/4
5. 1/4
6. 1/3
7. 1/2
8. 2/3
9. 1/4
10. 1/7

Practice 6

1. 9
2. 21
3. 24
4. 40
5. 8
6. 12
7. 12 or 24
8. 40 or 80
9. 8
10. 12 or 72
    (other possibilities)

Practice 7

1. 5/6
2. 5/6
3. 3/4
4. 1/3
5. 2/5
6. 1/2
7. 3/5
8. 1/5
9. 4/5
10. 9/10
11. 1/75, 14/15, 1/104
12. 6/7, 17/31

FRACTIONS - ADDITION

ANSWER KEY

Practice 1:

1. 1/2
2. 3/4
3. 4/5
4. 8/9
5. 1
6. 1 1/7
7. 1 2/5
8. 1 1/8
9. 9/11
10. 3/7

Practice 2:

1. 1 1/12
2. 11/12
3. 7/8
4. 7/10
5. 15/16
6. 8/9
7. 1/6
8. 17/20
9. 1 1/18
10. 1 1/24
11. 7/10
12. 13/24
13. 9/14
14. 17/18
15. 1 1/15
16. 1 1/12
17. 1 5/12
18. 1 7/12
19. 1 7/24
20. 7/10
21. 1 23/30
22. 11/16
23. 1 1/24
24. 1 7/12
25. 1 7/12

Practice 3:

1. 14 4/15
2. 15 13/24
3. 14 3/10
4. 10 1/6
5. 6 7/12
6. 16 1/10
7. 5 1/9
8. 12 1/12
9. 11 1/6
10. 9 1/12
11. 9 1/10
12. 13 1/12
13. 12 7/12
14. 14 13/15
15. 20 7/12

Fractions - Subtraction - Answer Key

Practice 1

1. 3/8
2. 1/30
3. 1/12
4. 1/15
5. 1/8
6. 1/12
7. 6/8 = 3/4
8. 6/20 = 3/10
9. 2/12 = 1/6
10. 2/6 = 1/3
11. 5 1/4
12. 1/12
13. 1/30
14. 4/27
15. 2/9

Practice 2

1. 1 3/10
2. 8 2/12 = 8 1/6
3. 6 5/24
4. 6 1/15
5. 3 7/20
6. 2 1/6
7. 2 5/18
8. 1/12
9. 1 11/30
10. 1 4/8 = 1 1/2
11. 3 2/5
12. 4 1/8

Practice 3

1. 3 8/12 = 3 2/3
2. 2/3
3. 6 2/4 = 6 1/2
4. 7 4/7
5. 4 2/8 = 4 1/4
6. 1 16/24 = 1 2/3
7. 6 6/10 = 6 3/5
8. 1 3/5
9. 12 6/9 = 12 2/3
10. 1 2/6 = 1 1/3
11. 1 6/8 = 1 3/4
12. 12 2/4 = 12 1/2
13. 6 5/7
14. 3/4
15. 90 2/3

Practice 4

1. 1 7/12
2. 2 7/10
3. 2 9/10
4. 3 7/8
5. 3 9/10
6. 3 19/24
7. 4 7/10
8. 2 11/24
9. 3 9/10
10. 3 11/12
11. 4 5/6
12. 2 8/15
13. 4 1/8
14. 2 3/4
15. 3 17/30

Practice 5

1. 6 2/3
2. 4 1/4
3. 13 1/3
4. 3 7/8
5. 1 1/2
6. 2 1/6
7. 3 1/3
8. 3 3/5
9. 4 5/7
10. 1 8/9

FRACTIONS - MULTIPLICATION - ANSWER KEY

PRACTICE 1

1. 3/4
2. 35/8 = 4 3/8
3. 16/5 = 3 1/5
4. 21/8 = 2 5/8
5. 9/2 = 4 1/2
6. 15/4 = 3 3/4
7. 15/8 = 1 7/8
8. 24/5 = 4 4/5
9. 7/4 = 1 3/4
10. 16/3 = 5 1/3

PRACTICE 2

1. 3/8
2. 1/2
3. 3/5
4. 2/7
5. 15/29
6. 1/2
7. 49/128
8. 7/22
9. 10/27
10. 3/5
11. 16/51
12. 2/5
13. 3/5
14. 1/34
15. 2 1/2
16. 1/32
17. 9/25
18. 2/5
19. 1/4
20. 9/20
21. 16/27
22. 1/2
23. 2/13
24. 1/12
25. 2/3

PRACTICE 3

1. 63/8 = 7 7/8
2. 21/2 = 10 1/2
3. 11/4 = 2 3/4
4. 21
5. 15
6. 55/8 = 6 7/8
7. 49/3 = 16 1/3
8. 72
9. 47/3 = 15 2/3
10. 9/11
11. 65/2 = 32 1/2
12. 672/25 = 26 22/25
13. 49/5 = 9 4/5
14. 36
15. 15
16. 25/8 = 3 1/8
17. 80/3 = 26 2/3
18. 4
19. 3
20. 91/30 = 3 1/30

MULTIPLICATION OF FRACTIONS II
ANSWER KEY

Page F33	Page F34
1. 1/34	1. 7 7/8
2. 2½	2. 4 2/3
3. 6/25	3. 95
4. 11/384	4. 72
5. 3/8	5. 15
6. 7/22	6. 15 2/3
7. 49/128	7. 9/11
8. 2/7	8. 36
9. 8	9. 32.5
10. 3/5	10. 13 3/5
11. 12 3/5	11. 34 2/5
12. 32/165	12. 9 4/5

Fractions--Division

Answer Key

Practice 1

1. 15/2 = 7 1/2
2. 16
3. 18
4. 12
5. 14
6. 14
7. 20
8. 16
9. 15
10. 4

Practice 2

1. 1/14
2. 2/27
3. 1/3
4. 1/12
5. 1/16
6. 1/9
7. 1/15
8. 1/36
9. 1/18
10. 1/32

Practice 3

1. 5/6
2. 4/5
3. 10/9 = 1 1/9
4. 2/3
5. 3/4
6. 9/14
7. 3/4
8. 3/2 = 1 1/2
9. 8/9
10. 8/25
11. 6/5 = 1 1/5
12. 1/2
13. 20/9 = 2 2/9
14. 14/3 = 4 2/3
15. 5/3 = 1 2/3

Practice 4

1. 15/88
2. 7/4 = 1 3/4
3. 4/3 = 1 1/3
4. 7/4 = 1 3/4
5. 14/3 = 4 2/3
6. 34/13 = 2 8/13
7. 4/5
8. 23/10 = 2 3/10
9. 1/2
10. 3/16
11. 9/20
12. 2
13. 11/15
14. 13/2 = 6 1/2
15. 4/11
16. 6/5 = 1 1/5
17. 17/11 = 1 6/11
18. 11/6 = 1 5/6
19. 4/9
20. 10

Introduction to Decimals I

Answer Key

Exercise 1

1. three tenths
2. seventeen thousandths
3. forty-eight hundredths
4. six hundred one thousandths
5. five and eight hundred thirty-nine thousandths
6. six and eight thousandths

Exercise 2

1. ------
2. .13
3. .105
4. .0002
5. 17/10,000 = .0017
6. 1/10 = .1
7. 53/100 = .53
8. .001454
9. 40/100 = .40
10. 95/1000 = .095

Exercise 3

1. 36.724
2. 1.8069
3. .3777
4. 35.238
5. 16.958
6. 2.5267
7. 23.69
8. 340.912
9. 51.007

Exercise 4

1. 2.1
2. 3.5
3. 8.649
4. 54.37
5. 3.15
6. 171.425
7. 4.428
8. 12.05
9. 15.129
10. 202.669

Exercise 5

27'
2.8
1.35
1.28
1.20
1.1
1.028
.37
.128

THE END

Introduction to Decimal II

Answer Key

Exercise 1

1. 7/10
2. 50/100 = 1/2
3. 20/100 = 1/5
4. 75/100 = 3/4
5. 56/100 = 14/25
6. 85/100 = 17/20
7. 376/1000 = 47/125
8. 450/1000 = 9/20
9. 135/1000 = 27/200
10. 40/100 = 2/5

NOTE: When a decimal doesn't come out even, the division answer will often repeat numbers. The bar is used to indicate the part that repeats. (See Exercise p.5 answers 3, 7, 9, 10, 11, 15)

Exercise - Page 5

1. .75
2. .125
3. .28 4/7 or $.\overline{285714}$
4. .6
5. .83 1/3 or $.8\overline{3}$
6. .85
7. .083 1/3, .08 1/3, or $.08\overline{3}$
8. .875
9. .22 2/9 or $.\overline{22}$
10. .26 2/3 or $.\overline{26}$
11. .09 1/11 or $.\overline{09}$
12. .05
13. 4.25
14. 1.7
15. 1.33 1/3 or $1.\overline{33}$

Exercise - Page 6

2. 16.73 1/3 or 16 11/15
3. .1375 or 11/80
4. .55 or 11/20
5. 3.85 or 3 17/20
6. 2.75 or 2 3/4
7. 19.3
8. 1.34 or 1 17/50
9. 1.66 2/3 or 1 2/3
10. 27.95 or 27 19/20

THE END

Multiplying Decimal Numbers

Answer Key

Section One

1.	.48	2.	.68	3.	.96	4.	.96	5.	.88
6.	.134	7.	.252	8.	.156	9.	.312	10.	.184
11.	1.28	12.	.28	13.	15.9	14.	10.5	15.	16.8
16.	11.88	17.	11.61	18.	9.36	19.	5.035	20.	257.25

Section Two

1.	40.448	2.	.1488	3.	110.24	4.	.3995
5.	27.304	6.	3670.8	7.	14.175	8.	.10044
9.	6.055	10.	.7152				

Section Three

1.	.09	2.	.06	3.	.09	4.	.16	5.	.08
6.	.064	7.	.086	8.	.069	9.	.099	10.	.044
11.	.0096	12.	.0018	13.	.0135	14.	.0273	15.	.0125
16.	.0805	17.	.0828	18.	.0775	19.	.0832	20.	.02544

Section Four

1.	$93.96	2.	$4.47	3.	$69.93	4.	$105.30
5.	$559.92	6.	$40.32	7.	$1,494.25	8.	$3,391.50
9.	$32.01	10.	$1,557.92				

Multiplying Decimal Numbers

Answer Key (continued)

Section Five

1. (3)
$$\begin{array}{r} \$\ \ .39 \\ \text{X}\ \ \ \ 4 \\ \hline \$1.56 \end{array}$$

2. (2)
$$\begin{array}{r} \$2.49 \\ \text{X}\ \ \ \ 3 \\ \hline \$7.47 \end{array} \qquad \begin{array}{r} \$10.00 \\ -\ 7.47 \\ \hline \$\ 2.53 \end{array}$$

3. (2)
$$\begin{array}{r} \$3.20 \\ \text{X}\ \ 5.00 \\ \hline \$16.00\ 00 \end{array}$$

4. (5)
$$\begin{array}{r} 4 \\ 5. \\ 6 \\ +\ \ 4 \\ \hline 19 \end{array} \qquad \begin{array}{r} \$\ 1.65 \\ \text{X}\ \ \ \ 19 \\ 14\ 85 \\ 16\ 50 \\ \hline \$31.35 \end{array}$$

5. (2)
$$\begin{array}{r} \$1.92 \\ \text{X}\ \ 1.\ 5 \\ 960 \\ 1920 \\ \hline \$2.880 \end{array}$$

6. (1)
$$\begin{array}{r} \$12.92 \\ \text{X}\ \ \ \ 7 \\ \hline \$90.44 \end{array}$$

7. (2)
$$\begin{array}{r} \$\ .55 \\ \text{X}\ \ \ 20 \\ \hline \$11.00 \end{array} \qquad \begin{array}{r} \$11.00 \\ -\ \ 9.50 \\ \hline \$\ 1.50 \end{array}$$

8. (3)
$$\begin{array}{r} \$.45 \\ -\ .20 \\ \hline \$.25 \end{array} \qquad \begin{array}{r} \$\ .25 \\ \text{X}\ \ \ 10 \\ \hline \$2.50 \end{array}$$

9. (2)
$$\begin{array}{r} 1\ 6.5 \\ \text{X}\ \ 1\ 2.5 \\ 8\ 2\ 5 \\ 33\ 0\ 0 \\ 165\ 0\ 0 \\ \hline 206.2\ 5 \end{array}$$

10. (1)
$$\begin{array}{r} \$137.42 \\ +\ \ 1.24 \\ \hline \$138.66 \end{array} \qquad \begin{array}{r} \$138.66 \\ \text{X}\ \ \ \ \ 5\ 63 \\ 415\ 98 \\ 8319\ 60 \\ 69330\ 00 \\ \hline \$78,065.58 \end{array}$$

D-4a

THE END

Dividing Decimal Numbers

Answer Key

Section 1

1. .4
2. .2
3. .11
4. .21
5. .31
6. 1.3
7. 4.2
8. 1.1
9. 2.1
10. 3.1
11. .91
12. .81
13. .61
14. .91
15. .62

Section 2

1. .061
2. .073
3. .071
4. .041
5. .082
6. .007
7. .006
8. .004
9. .007
10. .003

Section 3

1. .5
2. .8
3. .2
4. .6
5. .5

Section 4

1. .35
2. .076
3. 6.5
4. .085
5. .201

Section 5

1. 1.157
2. 8.217
3. .0829
4. .333
5. 3.667

Section 6

1. 4
2. 7
3. .9
4. 8
5. 4
6. .4
7. 6
8. .8
9. 2.06
10. .72
11. 6
12. 35.6

Dividing Decimal Numbers

Answer Key (continued)

Section 7

1. 2 2. 5

3. 632 4. 1.33

5. 3000 6. 5

7. 8 8. 5

9. 125 10. 205

11. 15 12. 35

Section 8

1. $.05 2. $1.10

3. $2.50 4. $16.85

5. $120.00 6. $12.50

7. $13.11 8. $32.24

9. $2.50 10. $3,042.22

Section 9

1. (1) $\begin{array}{r} 51.2 \\ 4\overline{)204.8} \end{array}$

2. (2) $\begin{array}{r} \$.20 \\ 5\overline{)\$1.00} \end{array}$

3. (3) $\begin{array}{r} 17 \\ \$.30\overline{)\$5.10} \end{array}$

4. (3) $\begin{array}{r} 26 \\ .03\overline{)\$.78} \end{array}$

5. (4) $\begin{array}{r} 20 \\ .349\overline{)6.980} \end{array}$

D-6a

THE END

DIVIDING OF DECIMAL HANDOUT

MEASURING YOUR PROGRESS

Test A

Add or Subtract:

1. 2.1
2. 12.37
3. 7.014
4. 64.37
5. 1985.4
6. .4
7. 2.5
8. .281
9. 2.025
10. 81.80

Multiply and Divide

11. .111
12. 1.518
13. 6.315
14. 37.544
15. 126.4653
16. .61
17. .0071
18. 89
19. .05
20. 4.37
21. 150
22. 600

Test B

1. 3
2. 2
3. 3
4. 1
5. 2
6. 2
7. 3
8. 2
9. 1
10. 3

THE END

ANSWER SHEET FOR DECIMALS, FRACTIONS, PERCENTS

Lesson 1

1. 1/20
2. 13/50
3. 1/25
4. 24/25
5. 2/25
6. 9/20
7. 1/8
8. 3/8
9. 5/8
10. 7/8
11. 3/200
12. 1/40

Lesson 2

1. .125
2. .375
3. .95
4. .083 1/3 or .0833
5. .75
6. .45

Lesson 3

1. 6%
2. 25%
3. 50%
4. 75%
5. 12.5%
6. 62.5%
7. 87.5%
8. 17.5%
9. 250%
10. 350%
11. 100%
12. 150%

Lesson 4

1. .75
2. .06
3. .055
4. .50
5. .018
6. .032
7. .99
8. .06 1/4 or .0625
9. .70
10. .90
11. 1.00
12. 1.50

Lesson 5

1. 1/200 .005
2. 1/400 .0025
3. 3/400 .0075
4. 5/600 .0083 1/3
5. 3/800 .00625
6. 2/300 .0066 2/3
7. 5/800 .00625
8. 3/1000 .003

Lesson 6

1. 1/8 .125
2. 1/3 .33 1/3
3. 3/200 .015
4. 7/8 .875
5. 5/8 .625

Lesson 7

1. 438
2. 125
3. 130
4. 144
5. 625
6. 168
7. 142
8. 546
9. 15
10. 50
11. 560
12. 462
13. 426
14. 24
15. 42
16. 16.50

Lesson 8

1. 4/5 80%
2. 4/5 80%
3. 25%
4. 75%
5. 75%
6. 12½%
7. 12½% or 12.5%
8. 16 2/3%
9. 87½%
10. 75% 25%
11. Janet (Note: Elsie had 75%, Janet 80%
12. 40%

Lesson 9

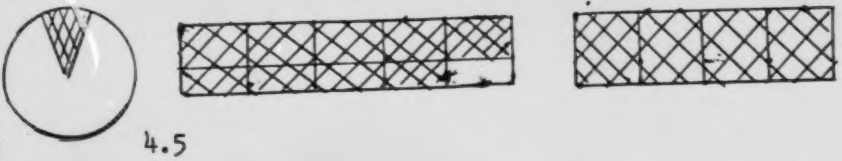
1. .25 25%
   .37½ 37.5% .375
   .005 .00½
   2.00

2. 50% 5% 125% 70% 75%
   62½% 33 1/3% 15%

3. 4/25 .16 7/8 .87½ 9/20 .45

4. 75% 37½% 60%

No. 5



5. 16.20 4.5
   $30. 6
   13.50 15
   25 60

7. 48%
   33 1/3%
   25%
   150%

THE END

INTRODUCTION TO MEASUREMENTS
(FOR USE WITH HEXAGON pp. 94-100)

Units of measures and weights

The letters in parentheses are short ways of writing the unit of measures, weights, and time.

Time
60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
4 weeks (about) = 1 month (mo.)
12 months = 1 year (yr.)
365 days = 1 year (yr.)
366 days = 1 leap year
(leap year occurs every four years by adding one extra day to February)

LENGTH

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
16½ feet = 1 rod (rd.)
5½ yd. = 1 rod
5,280 feet = 1 mile (mi.)
1,760 yards = 1 mile
320 rods = 1 mile

LIQUID MEASUREMENTS

4 gills = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

DRY MEASUREMENTS

2 pints = 1 quart
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

WEIGHT

16 ounces (oz.) = 1 pound (lb.)
100 lb. - 1 hundredweight (cwt.)
2000 lb. - 1 Ton (T.)
20 cwt. = 1 Ton
2240 lb. = 1 long Ton

Do you know what time it is now? Do you know how long it takes to get to work in the morning? How many hours do you sleep?

When you answer these questions, you are measuring time.

10 min. 25 sec. +15 min. 40 sec. 65 sec.	Step 1: Add the seconds: 25 + 40 = 65
60 sec. = 1 minute	Step 2: Think how many seconds are in a minute.
65 sec=1 min. 5 secs. 1 min. 65 -60 5 secs.	Step 3: Change 65 seconds into 1 minute 5 secs.
1 min. 10 min. 25 sec. +15 min. 40 sec. 26 min. 65 sec. -60 sec. = 1 min. 5 sec.	Step 4: Add the 1 minute to the other minutes.

Answer is:
1 min.
10 min. 25 sec.
+ 15 min. 40 sec.
25 min. 65 sec. = 26 min. 5 sec.

In adding measures we have dealt with one measure at a time. Now we are going to deal with two or more of the measures at the same time, such as gallons and quarts or minutes and seconds.

Problems such as these are common every day life.

Example: It takes 1 hour and 40 minutes to go from Washington to Maplewood and 2 hours and 45 minutes to go from Maplewood to Mount Vernon. How long does it take to go from Washington to Mount Vernon?

The process:	Washington to Maplewood	= 1 hr. 40 min.
	Maplewood to Mt. Vernon	= 2 hr. 45 min.
adding	Washington to Mt. Vernon	= 3 hr. 85 min.= 4 hr. 25 min.

Since there are 60 minutes in one hour, 85 minutes are equal to 1 hour and 25 minutes. This added to the 3 hours gives us 4 hours and 25 minutes.

3 hours 85 minutes

60 min. = 1 hr.

85 min. - 60 min. = 1 hour and 25 minutes

+ 3 hours
1 + 25 min.
4 hours 25 minutes

Measuring Length:

Which is longer 4 feet or 1 yard? Again you add and then change to larger units.

+ 1 yard 2 feet
3 yard 2 feet
4 feet

Step 1: Add the feet:
2 + 2 = 4

3 feet = 1 yard

Step. 2: Think how many feet are in a yard

1 yd.
3) 4 ft.
3
1 ft.

Step 3: Change 4 feet into 1 yard and 1 foot

+ 1 yard 2 feet
3 yard 2 feet
5 yard 4 feet
-3 = 1 yard
1 foot

Step 4: Add the 1 yard to the other yards

Answer:
+ 1 yard 2 feet
3 yard 2 feet
4 yard 4 feet = 5 yard 1 foot

Add the following in the same manner:

1. 5 gal. 2 qt.
2 gal. 3 qt.
7 gal. 5 qt.

8 gal. 1 qt.

2. 4 pk. 4 qt.
2 pk. 5 qt.
6 pk. 9 qt.

7 pk. 1 qt.

3. 4 hr. 30 min.
5 hr. 50 min.
9 hrs.80 min.

10 hrs.20 min.

4. 5 yd. 1 ft.
3 yd. 2 ft.
8 yd. 3 ft.

9 yd.

5. 22 hrs. 30 mins.
10 hrs. 40 mins.
32 hrs. 70 mins.

33 hrs. 10 mins.

6. 1 yd. 2 ft. 4 in.
2 yd. 1 ft. 9 in.
3 yd. 3 ft. 13 in.

3 yd. 4 ft. 1 in.

4 yd. 1 ft. 1 in.

7. 3 hr. 3 min. 18 sec.
5 hr.56 min. 42 sec.
8 hrs. 59 min. 60 sec.

9 hrs.

8. 2 lb. 6 oz.
3 lb.12 oz.
4 lb.14 oz.
9 lb.32 oz.

11 lb.

Word Problems

1. Yesterday you bought 1 yard 3 inches of wool. Today you bought 2 yards 6 inches of cotton and 2 yards of velvet. How much material have you bought altogether?

5 yd. 9 in.

2. You buy 2 pieces of lumber to make some shelves. The first piece is 1 foot 5 inches long. The second piece is 1 foot 7 inches long. How much lumber have you bought?

3 ft.

3. In the morning, you spend 15 minutes washing and 10 minutes dressing. It takes you 20 minutes to eat breakfast. You walk 15 minutes to work. How much time do you spend before work?

1 hr.

Subtraction of Measures

Measures are subtracted one unit at a time. You can borrow from a larger unit to subtract a smaller unit.

$$\begin{array}{r} 3\text{ hr. }10\text{ min.}\\ -\underline{1\text{ hr. }49\text{ min.}}\end{array}$$

Step 1: Borrow 1 hour and change it to minutes.
1 hour = 60 minutes

$$\begin{array}{r} +60\\ \underline{10}\end{array}$$

Step 2: Add the 60 minutes to 10 minutes you have already.

$$\begin{array}{rr} 2 & 70\\ -\cancel{3}\text{ hr.} & \cancel{10}\text{ min.}\\ \underline{1\text{ hr.}} & \underline{49\text{ min.}}\\ & 21\text{ min.}\end{array}$$

Step 3: Subtract the minutes.

$$\begin{array}{rr} 2 & 70\\ -\cancel{3}\text{ hr.} & \cancel{10}\text{ min.}\\ \underline{1\text{ hr.}} & \underline{49\text{ min.}}\\ 1\text{ hr.} & 21\text{ min.}\end{array}$$

Step 4: Now subtract the hours.

The answer is 1 hour and 21 minutes.

Example:

Bill is 22 years and 10 months old. Mary is 20 years and 11 months old. How much younger is Mary?

The process:

Since 11 months cannot be subtracted from 10 months, we must change 22 yr. 10 mo. to 21 years 22 mo. by renaming 1 yr.

$$\begin{array}{lclcr} \text{Bill's age} & = & 22\text{ yr. }10\text{ mo.} & = & 21\text{ yr. }22\text{ mo.}\\ \text{Mary's age} & = & 20\text{ yr. }11\text{ mo.} & = & \underline{20\text{ yr. }11\text{ mo.}}\\ & & \text{Difference} & & 1\text{ yr. }11\text{ mo.}\end{array}$$

Subtract:

1. $\begin{array}{r} 24\text{ lb. }8\text{ oz.}\\ \underline{10\text{ lb.}12\text{ oz.}}\\ 13\text{ lb.}12\text{ oz.}\end{array}$

2. $\begin{array}{r} 8\text{ hrs. }20\text{ min.}\\ \underline{4\text{ hrs. }40\text{ min.}}\\ 3\text{ hrs. }40\text{ min.}\end{array}$

3. $\begin{array}{r} 12\text{ bu. }2\text{ pk. }6\text{ qt.}\\ \underline{8\text{ bu. }3\text{ pk. }5\text{ qt.}}\\ 3\text{ bu. }3\text{ pk. }1\text{ qt.}\end{array}$

4. 6 yd. 2 ft. 9 in.
2 yd. 1 ft. 4 in.
4 yd. 1 ft. 5 in.

5. 7 qt.
2 qt. 1 pt.
4 qt. 1 pt.

6. 9 ft. 6 in.
2 ft. 8 in.
6 ft. 10 in.

7. 4 gal. 1 qt.
3 gal. 2 qt.
3 qt.

8. 10 min. 15 sec.
5 min. 40 sec.
4 min. 35 sec.

1. You have 3 hours to do some shopping. If you spend 1 hr. and 20 min. in one store, how much time do you have left to shop?

1 hr. 40 min.

2. You have 2 lbs 20 oz. of ground chuck. A recipe calls for 1 lb. 8 oz. of ground chuck. How much will you have left over?

10 oz.

3. Jim Jackson runs a mile in 7 min. 45 sec. Bob Smith runs a mile in 8 min. 20 sec. How much faster can Jim Jackson run the mile?

35 sec.

Multiplication of Measures

Units of time, length, weight and liquid measures are multiplied separately.

Example:

$$\begin{array}{rrr} 2\text{ hr.} & 40\text{ min.} & 22\text{ sec.}\\ & & \times 3\\ \hline 6\text{ hr.} & 120\text{ min.} & 66\text{ sec.}\end{array}$$

Step 1: Multiply each unit of time separately.
3 x 22 = 66; 3 x 40 = 120;
3 x 2 = 6

$$\begin{array}{rrrl} 2\text{ hr.} & 40\text{ min.} & 22\text{ sec.} & \\ & & \times 3 & \\ \hline & & 66\text{ sec.} & = 1\text{ min. } 6\text{ sec.}\end{array}$$

Step 2: Is to regroup the measures.
66 sec. = 1 min. 6 sec.

$$\begin{array}{rrrl} 2\text{ hr.} & 40\text{ min.} & 22\text{ sec.} & \\ & & \times 3 & \\ \hline 6\text{ hr.} & 120\text{ min.} & 66\text{ sec.} & = 1\text{ min. } 6\text{ sec.}\\ & \underline{+1} & & \\ & 121\text{ min.} & & \end{array}$$

Add the 1 min. to the 120 min.

$$\begin{array}{rrrr} & 2\text{ hr.} & 40\text{ min.} & 22\text{ sec.}\\ & & & \times 3\\ \hline & 6\text{ hr.} & 121\text{ min.} & 6\text{ sec.}\\ + & 2\text{ hr.} & -120 & \\ \hline & 8\text{ hr.} & 1\text{ min.} & 6\text{ sec.}\end{array}$$

Regroup again; 121 min. =
2 hr. 1 min.
Think how many minutes are in one hour, then find how many hours are in 121 min.

$$\begin{array}{rl} 2 & \text{hrs. 1 min.}\\ 60\overline{)121} & \\ \underline{120} & \\ 1 & \end{array}$$

Now work the problem.

$$\begin{array}{rrrl} 2\text{ hr.} & 40\text{ min.} & 22\text{ sec.} & \\ & & \times 3 & \\ \hline 6\text{ hr.} & 120\text{ min.} & 66\text{ sec.} & =\\ 8\text{ hrs.} & 1\text{ min.} & 6\text{ secs.} & \end{array}$$

Solve these problems by Multiplying:

1. 4 lb. 2 oz.
x 4
16 lb. 8 oz.

2. 8 lb. 6 oz.
x 5
40 lb. 30 oz.
41 lb. 14 oz.

3. 7 ft. 3 in.
x 4
28 ft. 12 in.
29 ft.

4. 4 mi. 80 rod
x 4
16 mi. 320 rod
17 mi.

5. 4 yd. 18 in.
x 6
24 yd. 108 in.
27 yd.

6. 6 qt. 1 pt.
x 4
24 qt. 4 pt.
26 qt.

7. 8 gal. 1 pt.
x 12
96 gal. 12 pt.
97 gal. 4 pt.

8. 6 T. 500 lb.
x 8
48 T. 4000 lb.
50 T.

9. 3 yd. 2 ft.
x 4
12 yd. 8 ft.
14 yd. 2 ft.

1. It takes 30 minutes to do one load of wash in an automatic washing machine. How long will it take to do 4 loads?

120 min. = 2 hrs.

2. You have 3 pieces of wire. Each piece of wire is 2 yd. 1 ft. 4 in. long. What is the total length of the wire you have.

7 yd. 1 ft.

Dividing Measures

Units of time, length, weight, and liquid measure are divided separately. Look at the example below.

$$\begin{array}{rll} & 4 & \\ \cline{2-3} 3 & \big)\,13 \text{ hr.} & 15 \text{ min.} \\ & \underline{12} & \\ & 1 \text{ hr.} & \end{array}$$

Step 1: Divide 3 into 13 hr. Then change the 1 hour you have left over into minutes. Think: How many minutes in 1 hour

$$\begin{array}{rll} & 4 \text{ hr.} & \\ \cline{2-3} 3 & \big)\,13 \text{ hr.} & 15 \text{ min.} \\ & \underline{-12} & \underline{+60} \\ & 1 \text{ hr.} = & 75 \text{ min.} \end{array}$$

Step 2: Then add to the 15 minutes Now its like a whole new division problem.

$$\begin{array}{rll} & 4 \text{ hr.} & 2 \\ \cline{2-3} 3 & \big)\,13 \text{ hr.} & 15 \text{ min.} \\ & \underline{12} & \underline{60} \\ & 1 & \underline{75} \\ & & \underline{6} \\ & & 15 \end{array}$$

Step 3: Divide 3 into 7, subtract 6 from 7, bring down the 5 along with the one.

$$\begin{array}{rll} & 4 \text{ hr.} & 25 \text{ min.} \\ \cline{2-3} 3 & \big)\,13 \text{ hr.} & 15 \text{ min.} \\ & \underline{12} & \underline{60} \\ & 1 & 75 \\ & & \underline{6} \\ & & 15 \\ & & \underline{15} \end{array}$$

Then divide 3 into 15 to get your answer.

Example 2

Gladys lives 8 miles from Martha. She drives this distance in 12 minutes 24 seconds. How long does it take her to go a mile:

Process:

$$\begin{array}{rll} & 1 \text{ min.} & 33 \text{ sec.} \\ \cline{2-3} 8 & \big)\,12 \text{ min.} & 24 \text{ sec.} \\ & \underline{8} & \underline{240} \\ & 4 & 264 \\ & & \underline{264} \\ & & 24 \end{array}$$

8 into 12 = 1 remainder 4
4 min. = 240 sec.
240 + 24 = 264
8 into 264 = 33

Solve these problems:

1. $$\begin{array}{r} \text{2 min. 13 sec.} \\ 5\overline{)\text{11 min. 5 sec.}} \end{array}$$

2. $$\begin{array}{r} \text{1 gal. 1 qt.} \\ 6\overline{)\text{7 gal. 2 qt.}} \end{array}$$

3. $$\begin{array}{r} \text{1 T. 600 lb.} \\ 4\overline{)\text{5 T. 400 lb.}} \end{array}$$

4. $$\begin{array}{r} \text{1 da. 5 hrs.} \\ 9\overline{)\text{10 da. 21 hr.}} \end{array}$$

5. $$\begin{array}{r} \text{1 bu. 3 pk.} \\ 3\overline{)\text{5 bu. 1 pk.}} \end{array}$$

6. $$\begin{array}{r} \text{1 yd. 1 ft.} \\ 7\overline{)\text{9 yd. 1 ft.}} \end{array}$$

7. A new born baby needs to be fed 6 times a day. How many hours are there between feedings?

$$\begin{array}{r} 4 \\ 6\overline{)24} \end{array}$$

4 hrs.

8. You have 4 lb. 2 oz. of meat. You want to use this meat for 3 meals. How much meat will you serve at each meal?

$$\begin{array}{r} \text{1 lb. 6 oz.} \\ 3\overline{)\text{4 lb. 2 oz.}} \end{array}$$

1 lb. 6 oz.

THE METRIC SYSTEM

The metric system is based on the use of decimals. The principal unit of length is called the meter.

Area is measured in square meters.

Volume is measured in cubic meters.

Capacity is measured in liters.

Weight is expressed in grams.

Smaller units are expressed by adding prefixes:

Deci - 1/10 (one-tenth)
Centi - 1/100 (one-hundredth)
Milli - 1/1000 (one-thousandth)

Larger quantities are expressed by adding prefixes meaning:

Deca - 10 (ten)
Hecto - 100 (hundred)
Kilo - 1000 (thousand)

Instruction: The metric system of measure is based upon units of ten. The basic unit of length in the metric system is the meter (m). Notice that the symbol (m) for meter is written without a period, as are all metric symbols.

Units of length larger and smaller than a meter are formed by adding prefixes to the basic unit.

The table below show the prefixes and their meanings

Prefixes		Meaning		Example
kilo	-	1,000 times basic unit	-	1 kilometer (km) = 1,000 meter
hecto	-	100 times basic unit	-	1 hectometer (hm) = 100 meter
deka	-	10 times basic unit	-	1 dekameter (dkam) = 10 meter
deci	-	.1 times basic unit	-	1 decimeter (dm) = .1 meter
centi	-	.01 times basic unit	-	1 centimeter (cm) = .01 meter
milli	-	.001 times basic unit	-	1 millimeter (mm) = .001 meter

During gradual conversion from the English system of measurement to the metric system, you may find it necessary to convert measures from one system to the other.

The table on the next page lists the conversion factors for most common measures in each system.

To use the table, find the unit you know in the left-hand column and multiply it by the given number in the middle column. The answer will be the approximate number of units shown in the right hand column.

(When you know)	(Multiply by)	(To find)
Length and Distance		
inches (in.)	2.5	centimeters (cm)
inches (in.)	25	millimeters (mm)
feet (ft.)	30	centimeters
yards (yd.)	0.9	meters (m)
miles (mi.)	1.6	kilometers (km)
millimeters (mm)	0.04	inches
centimeters (cm)	0.4	inches
meters (m)	3.3	feet
meters	1.1	yards
kilometers (km)	0.6	miles
Area		
square inches (sq. in.)	6.5	square centimeters
square feet (sq. ft.)	0.09	square meters
square yards (sq. yd.)	0.8	square meters
square miles (sq. mi.)	2.6	square kilometers
acres	0.4	hectares
square centimeters (cm^2)	0.16	square inches
square meters (m^2)	1.2	square yards
square kilometers (km^2)	0.4	square miles
hectares (ha)	2.5	acres
Capacity (Liquid)		
teaspoon (tsp.)	5	milliliters (ml)
tablespoon (tbsp.)	15	milliliters
fluid ounces (fl. oz.)	30	milliliters
cups (c.)	0.24	liters (1)
pints (pt.)	0.47	liters
quarts (qt.)	0.95	liters
gallons (gal.)	3.8	liters
milliliters (ml)	0.03	fluid oz.
liters (1)	2.1	pints
liters (1)	1.06	quarts
liters (1)	0.26	gallons
Mass (Weight)		
ounces (oz.)	28	grams (g)
pounds (lb.)	0.45	kilograms (kg)
tons (T)	0.9	metric tons
grams (g)	0.035	ounces
kilograms (kg)	2.2	pounds
metric ton (t)	1.1	tons
Temperature		
degrees Fahrenheit (°F)	5/9 (after subtracting 32)	degrees Celsuis (C)
degrees Celsuis (°C)	9/5 (then add 32)	degrees Fahrenheit

A. WORD PROBLEMS

1. From Chicago to New York is 908.2 miles. From Chicago to Pittsburgh, en route, is 468.4 miles. How far is Pittsburgh from New York in kilometers? 703.68 km.

2. Our car has a gas tank which measures 42 ins. by 11 ins. by 15 ins. There are 231 cu. ins. equaling 1 gallon. How many liters will it hold? 114.0 liters

3. Allowing 40 cu. ft. to the ton, how many metric tons of coal are there in a bin measuring 60 ft. by 30 ft. by 25 ft.? 1012.5 metric tons

4. A train traveling a certain speed must have 0.7 kilometer in which to stop. How far is the stopping distance in meters? 700 meters

5. Normal body temperature is 98.6° F. What is normal body temperature in Celsuis degrees? 37° C

6. A desk is 1.5 meters long. What is the length in centimeters? 150 cm.

7. What is the volume of a box which measures 1 meter by 2 meters by 0.5 meters?

1 cu. meter

8. A litter contains 1,000 milliliters. How many milliliters would be in three-fourths liter 750 milliliters

9. Dick caught one fish weighing 2 3/4 lb. and another fish weighing 6 2/3 lbs. How much did the two fish weigh in kilograms together? (Round off to nearest hundredths) 4.2375 kilograms

M-13a

10. Carpeting is to completely cover a floor which measures 18 ft. by 39 ft. How many square yards are needed? How many square meters is this 78 sq. yd. 62.4 sq. meters

11. Jan jogged 2 kilometers this morning. How many meters is that? 2000 meters

12. Find the cost of 4 meters of dress material if it cost $2.20 per meter? $8.80

B. Convert to metric:

Add: 1. + 5 gal. 5 qt.
2 gal. 3 qt.
7 gal. 8 qt.
9 gal. = 34.2 liters

2. + 2 yds. 5 ft.
6 yds. 4 ft.
8 yds. 9 ft. = 11 yds. = 9.9 meters

Multiply: 3. 20 mi.
x4
80 mi. = 128 km.

4. 22 lbs.
x 14
88
22
308 lbs. = 138.60 kg.

Subtract: 5. − 88 mi.
33 mi.
55 mi. = 88 kilometers

6. − 75 mi.
25 mi.
50 mi = 80 kilometers

Answer Key

Exercise 1:

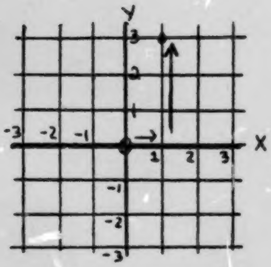
a. (3, -2)

b. (-2, 3)

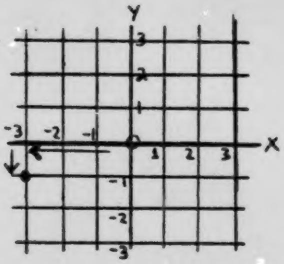
c. (1,3)

Exercise 2:

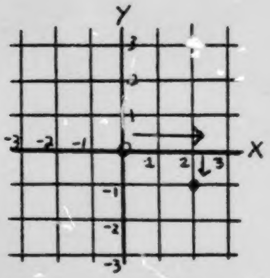
a. (1, 3)


Y
X
3
2
1
-3 -2 -1
1 2 3
-1
-2
-3

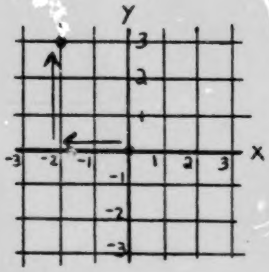
b. (-3, -1)


Y
X
3
2
1
-3 -2 -1
1 2 3
-1
-2
-3

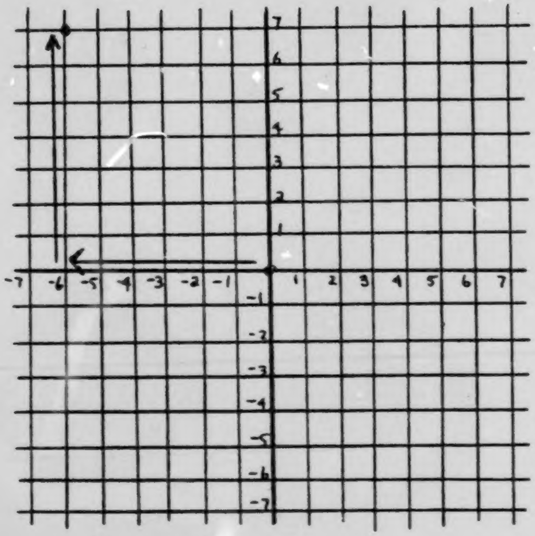
c. (2,-1)


Y
X
3
2
1
-3 -2 -1
1 2 3
-1
-2
-3

d. (-2, 3)


Y
X
3
2
1
-3 -2 -1
1 2 3
-1
-2
-3

e. (-6, 7)

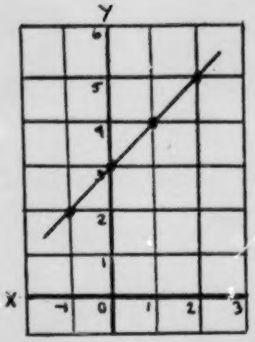

7
6
5
4
3
2
1
-7 -6 -5 -4 -3 -2 -1
1 2 3 4 5 6 7
-1
-2
-3
-4
-5
-6
-7

Answers:

Exercise 3

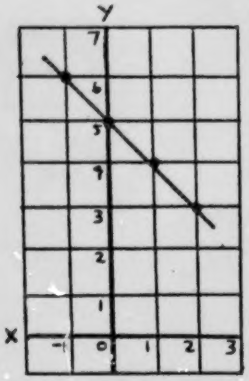
1. $x - y = -3$

x	y
0	3
1	4
2	5
-1	2



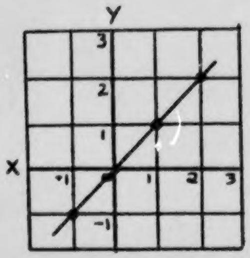
2. $x + y = 5$

x	y
0	5
1	4
2	3
-1	6



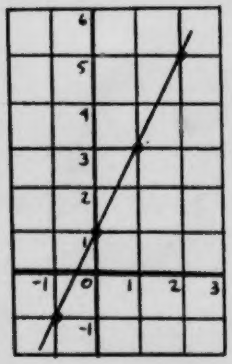
3. $x - y = 0$

x	y
0	0
1	1
2	2
-1	-1



4. $2x + 1 = y$

x	y
-1	-1
0	1
1	3
2	5



Exercise 4:

1. C
2. A
3. D
4. B

Page 1

1. 1 in. = 20 ft.
2. 2
3. 200 ft.
4. is answered
5. 76 ft.
6. 200 rd.
7. 1000 ft.
8. 2640 ft.

Page 2

1. 620 Rds.
2. 1080 ft.
3. 1832 ft.
4. $54
5. 54
   118
   9
   124
   84
   200
   45.8
   200
   80
   334
   67.6
   112

Page 3

1. is answered
2. 8 1/3 yds.
3. 190 ft.
4. 1120 rd. or 3.5 mi.
5. 7/10 mile
6. 460 ft.
7. 900 yds. or 2700 ft.

Page 4

1. $1.74
2. $ .90
3. 350 ft.
4. $6500
5. $302.50
6. 97 ft.
7. 1464 ft.
8. $1760.

Page 5

1. a. 1 in. = 4 ft.
   b. 12
   c. 8
   d. 12
   e. 8
   f. 96
   g. equal
2. 432 sq. in.
3. 280 sq. ft.
4. 22750 sq. ft.
5. 2808 sq. in.
6. 504 sq. in.
7. 152 sq. ft.

Page 6

1. a. 144
   b. 3
   c. 144
   d. 1440
   e. 9
   f. 3
   g. 9
   h. 36
   i. 30 1/4
   j. 4
2. is answered
3. $5.40
4. 100 sq. ft.
5. $225.
6. $70.
7. $108

Page 7

1. a. 1
   b. 1
   c. 4
   d. 4
   e. 8
   f. 8
   g. 32
2. 32 YES
3. Volume
   1000
   6000
   15232
   15
   2500
   1440
   1920
   6336
   281.6

Page 8

1. is answered
2. 8 cards
3. 12,600 cu. ft.
4. 6650 cu. ft.
5. yes
6. 2808 cu. ft.
7. 2100 cu. ft.
8. 480 cu. ft.

Page 9

1. is answered
2. 40 cu. yds.
3. 20 cords
4. $70.
5. $2880.
6. 880 cu. ft.
7. 24 gals.
8. 320 loads
9. 720 bu.
10. 60,000 lbs.

FORMULA WORKSHEET

(For use with Heptagon pp. 76-79)

Find the area and circumference of the circles.

1. $A=\pi r^2$

$\pi = 22/7$

$r = 21$

$A = 1386$

2. $A=\pi r^2$

$\pi = 22/7$

$r = 28$

$A = 2464$

3. $A=\pi r^2$

$\pi = 22/7$

$r = 35$

$A = 3850$

4. $A=\pi r^2$

$\pi = 22/7$

$r = 63$

$A = 12474$

5. $A=\pi r^2$

$\pi = 22/7$

$r = 56$

$A = 9856$

6. $A=\pi r^2$

$\pi = 22/7$

$r = 3.5$

$A = 38.5$

7. $C = \pi d$

$\pi = 22/7$

$d = 14$

$C = 44$

8. $C = \pi d$

$\pi = 22/7$

$d = 7$

$C = 22$

9. $C = \pi d$

$\pi = 22/7$

$d = 21$

$C = 66$

10. $C = \pi d$

$\pi = 22/7$

$d = 35$

$C = 110$

11. $C = \pi d$

$\pi = 22/7$

$d = 175$

$C = 550$

12. $C = \pi d$

$\pi = 22/7$

$d = 140$

$C = 440$

Find the perimeter and the area of the rectangles.

L	W	P	A		L	W	P	A
15	12	54	180		17	10	54	170
18	24	84	432		16	9	50	144
29	27	112	783		25	20	90	500
60	40	200	2400		23	40	126	920

Find the volume of the rectangular prisms.

L	W	h	V
8	9	10	720
10	10	10	1000
6.5	8	10	520
6	8	5	240

Find the volume of the cylinders. Fill in the blanks.

r	d	h	V
7	14	10	1540
3.5	7	10	385
10	20	14	4400
6	12	21	2376

Find the circumference and area of the circles. Fill in the blanks.

r	d	C	a
1.75	3.5	11	9.625
3.5	7	22	38.5
10.5	21	66	346.5
7	14	44	145
14	28	88	616

Find the area of the triangles.

a	b	A
8	16	64
20	15	150
14	5	35

Fill in the perimeter of the rectangles

L	W	P
15	12	54
27	32	118
2.5	2	9
29	33	124
18	24	84
60	40	200
12.4	10.5	45.8
54	46	200
22	18	80
92	75	334
18.6	15.2	69.6
29	27	112

Find the volume of the rectangular prisms.

L	W	h	V
8	9	10	720
10	10	10	1000
6.5	8	10	520
6	8	5	240
8	12	10	960
20	15	6.5	1950

Find circumference & area of circles.

r	d	C	A
1.75	3.5	11	9.625
3.5	7	22	38.5
10.5	21	66	346.5

Find the area of the rectangle.

L	W	A
17	10	170
16	9	144
8.5	6	51
9.5	12	11.4
25	20	500
23	40	920
12	6.5	78
18	6 1/2	117
15	6	90

Find the volume of the cylinders. Fill in blanks.

r	d	h	V
7	14	10	1540
3.5	7	10	385
10	20	14	4400
6	12	21	2375

Find the area of the triangles.

a	b	A
8	16	64
20	15	150
14	5	35

Volume of rectangles.

1. $V = L W h$

$L = 16'$

$W = 20'$

$h = 6'$

$V =$

1920

2. $V = L W h$

$L = 12'$

$W = 15'$

$h = 8'$

$V =$

1440

3. $V = L W h$

$L = 45'$

$W = 30'$

$h = 6'$

$V =$

8100

4. $V = L W h$

$L = 20'$

$W = 10'$

$h = 5'$

$V =$

1000

5. $V = L W h$

$L = 12'$

$W = 12'$

$h = 6'$

$V =$

864

$V = L W h$

$L = 22'$

$W = 10'$

$h = 5'$

$V =$

1100

Volume of cylinders.

1. $V = \pi r^2 h$

$\pi = 22/7$

$r = 7'$

$h = 5'$

$V =$

770

2. $V = \pi r^2 h$

$\pi = 22/7$

$r = 14'$

$h = 10'$

$V =$

6160

3. $V = \pi r^2 h$

$\pi = 22/7$

$r = 5'$

$h = 7'$

$V =$

550

4. $V = \pi r^2 h$

$\pi = 22/7$

$r = 6'$

$h = 28'$

$V =$

3168

5. $V = \pi r^2 h$

$\pi = 22/7$

$r = 7'$

$h = 20'$

$V =$

3080

6. $V = \pi r^2 h$

$\pi = 22/7$

$r = 21'$

$h = 60'$

$V =$

83160

ABE #104

Find the perimeter and area of the rectangles.

1. $P = 2L + 2W$
$L = 15'$
$W = 12'$
$P =$
54

2. $P = 2L + 2W$
$L - 27'$
$W = 32'$
$P =$
118

3. $P = 2L + 2W$
$L = 2.5'$
$W = 2'$
$P =$
9

4. $P = 2L + 2W$
$L = 29'$
$W = 27'$
$P =$
112

5. $P = 2L + 2W$
$L = 12'$
$W = 8'$
$P =$
40

6. $P = 2L + 2W$
$L = 30'$
$W = 18'$
$P =$
96

7. $A = L\ W$
$L = 15'$
$W = 9'$
$A =$
135

8. $A = L\ W$
$L = 15'$
$W = 10'$
$A =$
150

9. $A = L\ W$
$L = 20'$
$W = 15'$
$A =$
300

10. $A = L\ W$
$L = 12'$
$W = 9'$
$A =$
108

11. $A = L\ W$
$L = 17'$
$W = 20'$
$A =$
340

12. $A = L\ W$
$L = 12'$
$W = 6.5'$
$A =$
78

Find the perimeter and area of the triangles.

1. P = S+S+S

$s^1 = 6'$

$s^2 = 6'$

$s^3 = 6'$

P =

18

2. P = S+S+S

$s^1 = 8'$

$s^2 = 8'$

$s^3 = 6'$

P =

22

3. P = S+S+S

$s^1 = 3.5'$

$s^2 = 3.5'$

$s^3 = 3.5'$

P =

10.5

4. P = S+S+S

$s^1 = 15'$

$s^2 = 15'$

$s^3 = 12'$

P =

42

5. P = S+S+S

$s^1 = 6'5''$

$s^2 = 6'5''$

$s^3 = 3'5''$

P =

16'3"

P = S+S+S

$s^1 = 24'$

$s^2 = 24'$

$s^3 = 20'$

P =

68

7. $A = \frac{ab}{2}$

a = 18'

b = 10'

A =

90

8. $A = \frac{ab}{2}$

a = 14'

b = 22'

A =

154

9. $A = \frac{ab}{2}$

a = 30'

b = 15'

A =

225

10. $A = \frac{ab}{2}$

a = 14'

b = 24'

A =

168

11. $A = \frac{ab}{2}$

a = 15'

b = 8'

A =

60

12. $A = \frac{ab}{2}$

a = 40'

b = 20'

A =

400

ADDING AND SUBTRACTING
SIGNED NUMBERS
(For use with algebra pp. 22-24)

A newspaper reported that the temperature at 5 P.M. was +2°(or 2°above zero). The temperature at 6 P.M. was reported at -2° (or 2°below zero). Notice that these two temperatures were on opposite sides of the zero reading on the thermometer.

In math, numbers that are opposites are called positive(+)or negative (-). If you gain $10 it could be written +$10. If you lose $10, if could be written -$10.

A positive number will have a plus (+) sign before it or nothing before it. For example: +2 or 2, +5 or 5, +72 or 72, +103 or 103 are all positive numbers.

A negative number will always have a minus (-) sign before it. For example: -2, -5, -72, -103 are all negative numbers.

Positive and negative numbers are called signed numbers.

1. If +$8 is a profit, what does -$8 mean? loss
2. If +40°means north of the equator, what does -40° mean? south
3. If +$75 is a deposit in the bank, what is -$75? withdrawal
4. If +90 means 90 miles west, what does -90 mean? east

Express the following as a signed number:

1. 75°above zero +75
2. a loss of $75 -$15.
3. a rise of 5 feet +5 ft.
4. 5° below zero -5°
5. a gain of $50 +$50

Rules - for adding signed numbers:

A. If the signs of the numbers are all the same just add the numbers.

+2 +2 +4	2 2 4	-2 -2 -4	+2 +2 +2 +6	2 2 2 6	-2 -2 -2 -6

B. If the signs of the numbers are not alike subtract the larger number from the smaller, then use the sign of the larger number in the answer.

The Sign of the Larger Number	+5 -3 +2	-5 +3 -2	The Sign of the Largest Sum of Numbers	+5 +1 -5 +1	-5 -1 +5 -1

C. Two signed numbers that have the same number but opposite signs add up to zero.

+5 -5 0	+3 -3 0	-6 +6 0	-10 +10 0	+14 -14 0

ADD:

1. +15 + 9 +24	2. +8 +17 +25	3. 5 2 7	4. 5 8 13	5. +8 1/4 +9 3/4 + 18
6. -17 - 8 -25	7. -28 -38 -66	8. -8 -7 -15	9. -5.4 -2.6 -8.0	10. -12 1/2 -19 3/4 -31 5/4 -32 1/4
11. +70 -20 +50	12. -55 +20 -35	13. -8.5 +6.1 -2.4	14. -6.9 +9.4 +2.5	15. +23 5/6 - 9 1/6 14 1/6 = 14 2/3
16. -25 42 +17	17. 18 -32 -14	18. 8.3 -8.3 0	19. 7.1 -9.4 -2.3	20. -33 1/3 19 2/3 -13 2/3
21. +27 - 9 -12 + 6	22. -45 +12 +13 -20	23. +20 -12 - 8 0	24. -1.5 +3.7 -8.3 -6.1	25. +8 1/2 -4 1/4 +7 3/4 + 12

26. 9	27. -21	28. -24	29. - .7	30. + 8 1/6
7	-13	15	+3.1	-13 1/6
-3	17	19	-9.6	+ 3 1/3
-5	10	-12	+ .5	- 9 5/6
+8	-7	-2	-6.7	-11 1/2

RULE: For subracting signed numbers

A. Change the sign of the second number to the opposite sign and then finish the problem as you would in adding.

	Change Sign		Change Sign
-2	-2	-5	-5
-2	+2	-2	+2
	0		-3

SUBTRACT:

1. +30	2. +45	3. -19	4. -19	5. -7
+12	-20	17	- 7	-7
+18	+65	-36	-12	0

6. 50	7. 18	8. 15	9. 8.7	10. 8 5/6
30	29	15	9.5	3 1/6
+20	-11	0	- .8	+5 2/3

11. +35	12. +27	13. +19	14. +8.3	15. +7 3/4
-15	- 8	- 9	-6.2	-2 1/4
+50	+35	+28	+14.5	+10

16. -39	17. -45	18. -6	19. -6.9	20. -6 1/3
+15	17	6	+3.7	+3 1/3
-54	-62	-12	-10.6	-9 2/3

21. -26	22. -15	23. -8	24. -3.6	25. -8 7/8
-18	-31	-8	-5.2	-3 1/8
- 8	+16	0	+1.6	-5 3.4

MULTIPLYING AND DIVIDING
SIGNED NUMBERS
(For use with Algebra pp. 37-39)

Rules for Multiplication:

A. When multiplying two numbers that have positive signs, just multiply and keep the same sign.

B. When multiplying two numbers that have unlike signs, just multiply and make the answer negative (-).

C. When multiplying two numbers that have negative signs, the answer will always be positive.

D. When multiplying more than two numbers, just take two at a time and follow rules A, B, or C to determine the answer sign.

Multiply:

1. $\begin{array}{r} 9 \\ 8 \\ \hline 72 \end{array}$ 2. $\begin{array}{r} +23 \\ +5 \\ \hline +115 \end{array}$ 3. $\begin{array}{r} 17 \\ 9 \\ \hline +153 \end{array}$ 4. $\begin{array}{r} +12 \\ +7 \\ \hline +84 \end{array}$ 5. $\begin{array}{r} +9 \\ 0 \\ \hline 0 \end{array}$

6. $\begin{array}{r} -11 \\ -7 \\ \hline +77 \end{array}$ 7. $\begin{array}{r} -15 \\ -8 \\ \hline +120 \end{array}$ 8. $\begin{array}{r} -24 \\ -4 \\ \hline +96 \end{array}$ 9. $\begin{array}{r} -75 \\ -3 \\ \hline +225 \end{array}$ 10. $\begin{array}{r} -19 \\ -6 \\ \hline +114 \end{array}$

11. $\begin{array}{r} -17 \\ +3 \\ \hline -51 \end{array}$ 12. $\begin{array}{r} +36 \\ -2 \\ \hline -72 \end{array}$ 13. $\begin{array}{r} -24 \\ 8 \\ \hline -192 \end{array}$ 14. $\begin{array}{r} 15 \\ -9 \\ \hline -135 \end{array}$ 15. $\begin{array}{r} -25 \\ +6 \\ \hline -150 \end{array}$

16. (8) (1/4)= +2
17. (-3/5) (-20) = +12
18. (1/2) (-2/3) = 1/3
19. (-4 1/2) (2/3) = 3
20. (-15) (-3 1/5) = +84
21. (4) (3) (2) = +24
22. (-1) (-7) (-8) = -56
23. (-3) (-5) (+4) = +60
24. (-7) (+2) (-5) = +70
25. (+8) (-9) (0) = 0
26. $(+4)^2$ = 16
27. $(-3)^2$ = +9

28. $(+5)^3 = +125$

29. $(-2)^3 = -8$

30. $(-5)^3 = -125$

31. $(-1)^2 = +1$

32. $(+\ 1/2)^2 = +1/4$

33. $(-\ 1/3)^2 = +\ 1/9$

34. $(+\ 2/3)^3 = +\ 8/27$

35. $(-\ 3/5)^3 = -27/125$

36. $(-\ 1/4)^3 = -\ 1/64$

37. $(-\ 1/5)^2 = +\ 1/25$

Rules for Division:

A. If dividing a positive number by a positive number, the answer will be positive.

B. If dividing a negative number by a negative number, the answer will be positive.

C. If dividing a number by a number with a different sign, the answer will be negative.

Divide:

1. $18 \div 6 = 3$

2. $-36 \div -3 = 12$

3. $52 \div -4 = -13$

4. $84 \div -12 = -7$

5. $-30 \div -6 = +5$

6. $100 \div -25 = -4$

7. $55 \div -11 = -5$

8. $-84 \div -12 = +7$

9. $-144 \div 9 = -16$

10. $4 \div -8 = -.5$

11. $-6 \div -9 = +.66\ 2/3$

12. $-15 \div -12 = +1.25$ or $1\ 1/4$

13. $+8.4 \div -4 = -2.1$

14. $-9.6 \div -.3 = +32$

15. $-3.6 \div +12 = -.3$

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16. $\frac{75}{15} = 5$

17. 17. $\frac{-100}{-20} = 5$

18. $\frac{+144}{-8} = -18$

19. $\frac{-108}{+9} = -12$

20. $\frac{-65}{+5} = -13$

21. $\frac{18}{-4} = -4.5$

22. $\frac{-16}{+5} = -3.2$

23. $\frac{-34}{+4} = -8.5$

24. $\frac{+100}{-8} = -12.5$

25. $\frac{-36}{-8} = 4.5$

26. $\frac{-4.8}{-4} = 1.2$

27. $\frac{+9.6}{-3} = 3.2$

28. $\frac{-1.8}{+9} = -.2$

29. $\frac{-2.4}{-.6} = 4$

30. $\frac{-7.2}{-1.2} = 6$

ALGEBRA HANDOUT
EQUATIONS
(Octagon, p. 91-111)
(For use with Algebra)

The word equation means a statement of equality. Thus, 2 x 2 = 4 is an equation. While the word equation may be new to us, its use is not at all new. The only important difference between a formula and an equation is that the formula is a rule which applies to any situation for which it is intended, and an equation applies to a particular situation. Thus, A = lw is the formula for finding the area of any rectangle. But when we use that formula to find the area of a rectangle whose length is 15 and whose width is 10 and substitute those values in the formula, the result is the equation: A = 15 X 10.

1. John's father is three times as old as John. The sum of their age is 76 years. How old is each?

THE PROCESS

Let a = John's age
Then, 3a = Father's age.
Then, [illegible] + 3a = 76
4a = 76
a = 76 ÷ 4
a = 19 (John's age)
3a = 57 (Father's age)
Proof: 19 + 57 = 76

2. Mary is twice as old as her sister. The sum of their ages is 63 years. What is the age of each?

Mary: 42 yrs. Mary's sister: 21 yrs.

3. Together, Earl and Roy have $4.80. Earl has three times as much as Roy. How much does each have?

Earl: $3.60 Roy: $1.20

4. Frank and Fred together have $7.50. Fred has half as much as Frank. How much does each have? (By changing the problem around, Frank has twice as much as Fred. In this way you can avoid fractions.)

Frank: $5.00 Fred: $2.50

5. Dorothy and Jane have $8.40. Dorothy has only one fifth as much as Jane has. How much does each have?

Dorothy: $1.40 Jane: $7.00

6. George and Harry inherited an estate of $1500. George is to receive three times as much as Harry. How much does each receive?

George: $ 1125 Harry: $375

7. Mr. Brown has two fields for pasture. The area of one field is 3 times that of the second field. Their combined areas equal 60 acres. How many acres are there in each field?

1st field: 45 acres 2nd field: 15 acres

Number Right ______________

SOLVING EQUATIONS

We have now learned how to add the terms of the equation and finally to divide for the value of the unknown quantity (letter). Not all equations have addition, or nothing but addition. You will come into contact with equations where all four operations -- addition, subtraction, multiplication, and division--are necessary.

Solve the following for the value of the letter used in the equation. You should be able to do all these mentally.

1. $5a + 4a = 45$ $a = 5$
2. $3x + 7x = 50$ $x = 5$
3. $23a + 27a = 100$ $a = 2$
4. $19a - 7a = 84$ $a = 7$
5. $16c + 12c = 84$ $c = 3$
6. $35b - 18b = 68$ $b = 4$
7. $21b + 9b = 120$ $b = 4$
8. $17z + 12z = 87$ $z = 3$
9. $12x + 5x = 51$ $x = 3$
10. $3b + 4b = 42$ $b = 6$
11. $15c + 10c = 100$ $c = 4$
12. $8a - 5a = 60$ $a = 20$
13. $14b - 6b = 48$ $b = 6$
14. $5x - 2x = 120$ $x = 40$
15. $7x - 4x = 30$ $x = 10$
16. $14m - 6m = 80$ $m = 10$
17. $23x - 7x = 80$ $x = 5$
18. $14a + 19a = 165$ $a = 5$
19. $42c - 19c = 92$ $c = 4$
20. $25n - 14n = 77$ $m = 7$
21. $4m + 7m = 66$ $m = 6$
22. $15a - 3a = 60$ $a = 5$
23. $6b + 5b = 5.5$ $b = .5$
24. $14b - 9b = 6.5$ $b = 1.3$
25. $6x + 7x = 52$ $x = 4$
26. $5x - x = 20$ $x = 5$
27. $3x + 4x = 49$ $x = 7$
28. $2x + 3x = 20$ $x = 4$
29. $8x - 6x = 3.8$ $x = 1.9$
30. $3x - x = .46$ $x = .23$
31. $7.5a + 2.5a = 60$ $a = 6$
32. $3.8a + 2.2a = 60$ $a = 10$
33. $1.4b + 3.6b = 35$ $b = 7$
34. $9.6x - 3.6x = 36$ $x = 6$
35. $12.4z - 4.4z = 64$ $z = 8$
36. $16.9d - 13.9d = 42$ $d = 14$

Now, go back and prove each answer by substituting the found value for the letter in each equation. Remember that an equation means equality.

Thus, in problem 1:

$$5a + 4a = 45$$
$$5(5) + 4(5) = 45$$
$$25 + 20 = 45$$
$$45 = 45$$

Number Right ____________

SIGNS OF OPERATION

When working equations we need to know more about how to proceed when there are several terms (or parts) and when there are several operations to be performed. Some equations require addition, subtraction, multiplication, and division, all in the one equation.

1. Frank had \$25. He spent \$6.50 for a pair of shoes, \$2.50 for a shirt, and \$1 for socks. How much did he have left?

THE PROCESS

Amount left = \$25 - (\$6.50 + \$2.50 + \$1)
Amount left = \$25 - \$10 = \$15

2. A merchant bought 15 dozen ties at \$30 per dozen and sold them at \$50 per dozen. How much was his profit?

THE PROCESS

Profit = (\$50 X 15) - (\$30 X 15)
Profit = \$750 - \$450 = ?

Solve the following equations. (Always perform operations within parenthesis first.)

3. (36 ÷ 6) - (2 X 2) = 2

4. (4 X 2) ÷ (4 ÷ 2) = 4

5. (15 - 3) ÷ (2 X 3) = 2

6. (27 + 3) ÷ (6 + 4) = 3

7. (16 X 3) ÷ (2 X 4) = 6

8. (32 ÷ 8) X (20 ÷ 5) = 16

9. 25 + (8 X 3) = 49

10. (27 ÷ 3) X (4 + 2) = 54

11. (80 X 2) + (40 ÷ 2) = 180

12. (120 ÷ 3) - (3 X 2) = 34

13. 12a + 5b + 2(a + 2b) =
12a + 5b + 2a + 4b = 14a + 9b
Note that 2 (a + 2b) means 2 times (a + 2b)

14. 5x + 3y + 4 (x + 2y) = 9x + 11y

15. 3x + 5y + 2(3x + y) = 9x + 7y

16. 5a + 3b + 3(a + b) = 8a + bb

17. 6x + 9y + 2(x -y) =
6x + 9y + 2x - 2y = 8x + 7y

18. 5a + 7b + 3(2a - b) = 11a + 4b

19. 12a + 4b + 4(a - b) = 16a

20. 6x + 3y + 2(x + 2y) = 8a + 75

21. 3(x + y) + 2(x + y) = 5x + 5y

22. 4(x + 2y) + 3(x + 5y) = 7x + 23y

Number Right ____________

COLLECTING TERMS IN EQUATIONS

The form of an equation may be changed by addition, subtraction, multiplication, or division, as long as the same operation is performed to both sides (or members) of the equation. The form of the equation $x = y$ may be changed in any of the following ways.

Addition	Subtraction	Multiplication	Division
$x = y$	$x = y$	$x = y$	$x = y$
$x + 2 = y + 2$	$x - 2 = y - 2$	$2x = 2y$	$\frac{x}{2} - \frac{y}{2}$

To solve an equation in the form $x + 4 = 22 - x$, we must first collect similar terms so all known quantities are on one side of the equation and all unknown quantities are on the other. In this equation we can do this by subtracting 4 from both sides of the equation and adding x. Thus, the equation becomes $x + 4 - 4 + x = 22 - x - 4 + x$, or $x + x = 22 - 4$.

The terms of an equation may also be collected by transposition, or moving a term from one side of an equation to the other by changing the sign of the quantity. Transposition is a short way of collecting terms by addition and subtraction. Thus, $y - 8 = 35$ becomes $y = 35 + 8$.

1. Solve $4x - 5 = 3x + 1$.

The Process

The Long Way

$4x - 5 = 3x + 1$
$4x - 5 - 3x = 3x + 1 - 3x$
$4x - 5 - 3x + 5 = 3x + 1 - 3x + 5$
$x = 6$

The Process

The Short Way

$4x - 5 = 3x + 1$
$4x = 3x + 1 + 5$
$4x - 3x = 1 + 5$
$x = 6$

Using either of these methods, solve the following equations.

2. $5x + 3 = 18$
   $x = 3$
3. $3x + 5 = x + 3$
   $x = 1$
4. $8x - 5 = 5x + 10$
   $x = 5$
5. $5y - 16 = y$
   $y = 4$
6. $5(a + 4) = 4(a + 6)$
   $a = 4$
7. $3(x + 2) = x + 8$
   $x = 1$
8. $5x - 6 = 2(x + 6)$
   $x = 2$
9. $3x - 2 = 2(x + 2)$
   $x = 6$
10. $4(x - 3) = 2(x + 1)$
    $x = 7$
11. $5x - 2 = 3x + 2$
    $x = 2$
12. $7x - 4 = 3x$
    $x = 1$
13. $2x - 9 = x + 1$
    $x = 10$
14. $5(3x + 1) = 5(2x + 3)$
    $x = 2$
15. $4x + 5 = 41$
    $x = 9$
16. $2x + 8 = x + 4$
    $x = -4$

Number Right ___________

USING SIMPLE EQUATIONS

1. Jack and Bill together have \$250. Jack has \$10 more than 3 times as much as Bill. How much does each have?

The Process

Let x = Bill's money
Then $3x + \$10$ = Jack's money
$x + 3x + \$10 = \$250$
$4x + \$10 = \$250$
$4x = \$250 - \$10$
$4x = \$240$
$x = \$60$ (Bill's money)
$3x + \$10 = \$190$ (Jack's money)

2. Two fields together have an area of 90 acres. One field is twice as large as the other field. How many acres are there in each field?

1st field: 30 acres 2nd field: 60 acres

3. The perimeter of a certain field is 160 rods. The length of the field is 3 times the width. What are the dimensions of the field?

Length: 60 rds. Width: 20 rds.

4. The perimeter of another field is 300 rods. The width is one fourth as much as the length. What are the dimensions of the field? (Let x = width. In this way, avoid fractions.)

Length: 120 rds. Width: 30 rds.

5. Jerry has three times as much money as does Pat. Together they have \$12.80. How much does each have?

Jerry: \$9.60 Pat: \$3.20

6. The perimeter of a field is 60 rods. The length of the field is twice as great as the width. What are the dimensions of the field?

Length: 20 rds. Width: 10 rds.

7. We have a lot whose perimeter is 360 ft. The length is 3.5 times as great as the width. Find the length and width of the lot.

Length: 140 ft. Width: 40 ft.

8. I am thinking of a number. Five times that number equals 240. What is the number?

Answer: 48

Number Right ____________

PROBLEMS WITH EQUATIONS

1. Jane has five times as much money as does Helen. Together they have $42. How much does each have?

Jane: $35 Helen: $7

2. Ruth has $15 more than Julia. Together they have $55. How much does each have?

Ruth: $35 Julia: $20

---

3. One field contains 20 acres more than a second field. Together they contain 80 acres. How many acres does each field contain?

1st field: 30 acres 2nd field: 50 acres

4. One lot contains 500 square feet less than a second lot. Together they have a total area of 15,500 square feet. How many square feet are there in each lot?

1st lot: 8000 sq. ft. 2nd lot: 7500 sq. ft.

---

5. Joe's mother is 3 times as old as Joe. The sum of their ages is 84 years. How old is each?

Mother: 63 Joe: 21

6. Frank's father lacks 2 years of being three times as old as Frank. The sum of their ages is 78 years. How old is each?

Father: 58 yrs. Frank: 20 yrs.

---

7. Three numbers, added, total 180. The second number is twice the first, and the third is three times the first. What is each number? (Let x = the first number)

1st no.: 30 2nd no.: 60 3rd no.: 90

8. Three numbers, added total 140. The second number is twice the first, and the third number is twice the second. What are the three numbers?

1st no.: 20 2nd no.: 40 3rd no.: 80

---

9. If 24 is added to a certain number, the result is 52. What is the number?

Answer: 28

10. Terry has $25 more than twice as much as Tom has. Together they have $145. How much does each have?

Terry: $105 Tom: $40

Number Right ____________

CHECKING UP

1. From the formula base area x height = V, write the formula for finding h.

$$\frac{V}{\text{base area}} = h$$

2. Find h when V is 240 and base area is 24.

$$\frac{240}{24} = h \qquad 10 = h$$

---

3. A television set priced at $300 was purchased for a down payment of $100 and a payment of $20 for each of the following eleven months. What was the rate of interest paid?

Answer: 10%

4. Fred is 3 years older than twice Frank's age. The sum of their ages is 45 years. How old is each?

Fred: 31 Frank: 14

---

5. $x + 2 = 8$

$x = 6$

6. $3x + 2 = x + 10$

$x = 4$

7. $5x + 4 = 4x + 8$

$x = 4$

8. $4x = x + 9$

$x = 3$

9. $3(2x - 4) = 2(2x + 2)$

$x = 8$

10. $3(2x - 2) = 4x + 4$

$x = 5$

11. $3(x - 2) = x + 4$

$x = 5$

12. $5(x - 4) = 3x$

$x = 10$

13. $2(x + 5) = x + 15$

$x = 5$

Number Light__________

THE MEANING OF RATIO

There are two methods of comparing the sizes of two similar quantities--by subtraction and by division. If we subtract, we get their difference. If we divide, we get their ratio. Ratio is defined as a comparison of two similar quantities by division.

For example, Erick Johnston is 20 years old, and his younger brother Nathan is 10. Using subtraction, we say that Erick is 10 years older than Nathan. If we compare their ages by division then Erick is twice as old as Nathan. Thus the ratio of Erick's age to Nathan is 2 to 1. This may also be written as 2: 1, or simply as the fraction 2/1.

Notice, however, that if we compare Nathan's age to Erick's, then the ratio is 1 to 2, which may also be written as 1: 2 or 1/2.

The following problems are designed to help you understand the meaning of ratio. As you read, be sure that you understand which amount is being compared to another; that is, notice which amount is the numerator of your fraction.

1. A tree is 30 ft. tall, and a nearby telephone pole is 25 ft. tall. What is the ratio of the height of the tree to that of the pole?

The Process

Stated in the fractional form, the ratio is $\frac{30}{25}$, or 30 to 25. Reducing this to lowest terms, the ratio is 6: 5, or 6/5. Do you see that this ratio could also be stated as 1 1/5 to 1, or simply 1 1/5?

2. In problem 1, what is the ratio of the height of the pole to that of the tree?

The Process

This time we need to divide 25 by 30, and the ratio of the pole to the tree is 5 to 6. This may also be written as 5: 6 and as the fraction 5/6.

---

3. Fred Aarons is 6 ft. tall, and his wife is 64 in. tall. What is the ratio of Fred's height to that of his wife? (Remember to change to the same units--either feet or inches.)

Answer: 9:8

4. The ratio of Jim's age to his father's age is 2 :5. If Jim is 20 years old, how old is his father?

Answer: 50 yrs.

---

5. In problem 3, what is the ratio of the wife's height to Fred's?

Answer: 8:9

6. The diameter of a certain circle is 16 inches. The diameter of a smaller one is 4 inches. What is the ratio of the radius of the larger circle to that of the smaller circle?

Answer: 4:1

---

7. The total paid attendance at a recent football game was 6,000. However, more adults attended the game than did students. If the ratio of the number of adults to the number of students was 7: 5, how many adults attended the game?

Answer: 3,500

8. The ratio of the height of a certain elm tree to that of a nearby oak tree is 3: 2. If the elm tree is 60 ft. tall, how tall is the oak tree?

Answer: 40 ft.

Number Right

EQUATIONS INVOLVING RATIO

1. The perimeter of a rectangle is 300 in. The length and width are in the ratio of 2 to 1. Find the length and width.

The Process

Let x = width
2x = length
Then, 2x + 2x + x + x = 300 in.
6x = 300 in.
x = ? (the width)
2x = ? (the length)

2. The perimeter of a rectangle is 400 rods. The length and width are in the ratio of 3 to 2. Find the length and width.

Length: 120 rds. Width: 80 rds.

---

3. The length and width of a rectangle are in the ratio of 3 to 1. The perimeter is 80. Find the length and width.

Length: 30 Width: 10

4. The perimeter of a triangle is 48 in. The sides are in the ratio of 1 to 2 to 3. Find the length of each side.

8 in. 16 in. 24 in.

---

5. The perimeter of another triangle is 480 rods. The sides are in the ratio of 5 to 4 to 3. How long is each side?

120 rds. 160 rds. 200 rds.

6. Tom, Dick, and Harry together have $2.40, and the amount each one has is in the ratio of 1 to 2 to 3. How much does each have?

The Process

Let x = Tom's money
2x = Dick's money
3x = Harry's money
Then, 6x = $2.40
x = ? (Tom's money)
2x = ? (Dick's money)
3x = ? (Harry's money)

---

7. Mary, Jane, and Catherine together have $2.70. Their monies are in the ratio of 2 to 3 to 4. How much does each have? (Let 2x = Mary's money)

Mary: $.60 Jane: $.90 Catherine: $1.20

8. Three men contributed $84 to the Community Chest drive. Their contributions were in the ratio of 3 to 4 to 5. How much did each contribute?

1st man: 21 2nd man: 28 3rd man: 35

---

9. Three persons gave $48 to the Red Cross. What the three persons gave was in the ratio of 3 to 4 to 5. How much did each give?

1st: $12 2nd: $16 3rd: $20

10. The population of a certain town is 3600. The number of men, women, and children is in the ratio of 1 to 1.5 to 3.5. How many are in each of these three groups?

Men: 600 Women: 900 Children: 2100

Number Right __________

THE MEANING OF PROPORTION

When two ratios are equal, they are said to form a proportion. Thus, the ratios 1 to 2 and 8 to 16 form a proportion. Proportion is usually written in this form: 1 :2 = 8 : 16, and it is read, "1 is to 2 as 8 is to 16." Proportion is also written in fractional form like this: 1/2 = 8/16.

The first and last terms of a proportion are called the extremes, and the second and third terms are called the **means:** extreme: mean=mean : extreme or $\frac{\text{extreme}}{\text{mean}} = \frac{\text{mean}}{\text{extreme}}$

In the proportion 1 :2 = 8 : 16 notice that 1 X 16 = 2 X 8. That is, the product of the means equals the product of the extremes. This is true in all proportions. This rule may be used to solve a proportion in which three of the members are known and the fourth is unknown.

Use this rule to find the missing term in each of the following proportions.

1. 5 : 10 = 10 : x
The Process
5x = 100
x = 20
Proof: 5: 10 = 10 :20
100 = 100

2. 25 : 3 = 100 : x
25x = 300
x = 12

3. 5 : 12 = 10 : x
5x = 120
x = 24

4. 7 : 9 = 21 : x
7x = 189
x = 27

5. 4 : 15 = 8 : x
4x = 120
x = 30

6. 5: 12 = 10 : x
5x = 120
x = 24

7. 5 : x = 15 : 9
45 = 15x
3 = x

8. 20 : x = 5: 4
80 = 5x
16 = x

9. 100 : x = 25 : 3
300 = 25x
12 = x

10. 12: 4 = x : 7
84 = 4x
21 = x

11. 6: 9 = x : 3
18 = 9x
2 = x

12. 7 : 3 = x : 6
42 = 3x
14 = x

13. x : 4 = 5 : 10
10 x = 20
x = 2

14. x : 20 = 4 : 5
5x = 80
x = 10

15. x : 100 = 3 :25
300 = 25x
12 = x

16. x : 6 = 2 : 3
3x = 12
x = 4

17. x : 3.2 = 5 : 8
8x = 16
x = 2

18. 6 : 7 = x : 14
84 = 7x
12 = x

19. x : 3 = 10 : 15
15x = 30
x = 2

20. 7.5 : x = 2 : 4
30 = 2x
15 = x

Number Right ________

FRACTIONAL EQUATIONS

Because proportions are often written in the form of fractions, we should study the solving of fractional equations. The proportion 4 : 5 = 8 : x can be written $\frac{4}{5} = \frac{8}{x}$. Using the rule the product of the means equals the product of the extremes, we find that $4x = 40$, or $x = 10$. Thus, the numerator of one fraction is multiplied by the denominator of the other fraction. This procedure is called cross multiplication

Solve each of the following fractional equations to find the unknown value.

1. $\frac{x}{3} = \frac{10}{5}$

The Process

Cross-Multiply.

$5\ X\ x = 5x$

$3\ X\ 10 = 30$

$5x = 30$

$x = 6$

2. $\frac{x}{8} = \frac{5}{4}$

$4x = 40$

$x = 10$

3. $\frac{x}{16} = \frac{3}{8}$

$8x = 48$

$x = 6$

4. $\frac{x}{6} = \frac{5}{3}$

$3x = 30$

$x = 10$

---

5. $\frac{x}{9} = \frac{1}{3}$

$3x = 9$

$x = 3$

6. $\frac{x}{3} = \frac{2}{5}$

$5x = 6$

$x = 1\ 1/5$

7. $\frac{x}{12} = \frac{1}{6}$

$6x = 12$

$x = 2$

8. $\frac{x}{4} = \frac{3}{2}$

$2x = 12$

$x = 6$

---

9. $\frac{2x}{3} = \frac{10}{5}$

$10x = 30$

$x = 3$

10. $\frac{x}{5} = \frac{7}{5}$

$5x = 35$

$x = 7$

11. $\frac{8}{x} = \frac{4}{5}$

$40 = 4x$

$10 = x$

12. $\frac{5}{x} = \frac{7}{14}$

$70\ x\ 7x$

$10 = x$

---

13. $\frac{6}{x} = \frac{3}{5}$

$30 = 3x$

$10 = x$

14. $\frac{4}{x} = \frac{2}{3}$

$12 = 2x$

$6 = x$

15. $\frac{3}{x} = \frac{5}{2}$

$6 = 5x$

$1\ 1/5 = x$

16. $\frac{16}{x} = \frac{8}{3}$

$48 = 8x$

$6 = x$

---

17. $\frac{15}{9} = \frac{5}{x}$

$15\ x = 45$

$x = 3$

18. $\frac{5}{3} = \frac{25}{x}$

$5x = 75$

$x = 15$

19. $\frac{7}{6} = \frac{14}{x}$

$7x = 84$

$x = 12$

20. $\frac{6.5}{5} = \frac{13}{x}$

$6.5x = 65$

$x = 10$

Number Right ________

MORE FRACTIONAL EQUATIONS

Find the unknown value in each of the following fractional equations.

1. $\frac{3}{8} = \frac{x}{16}$

$48 = 84$
$6 = x$

2. $\frac{3}{5} = \frac{6}{x}$

$3x = 30$
$x = 10$

3. $\frac{2}{3} = \frac{4}{x}$

$2x = 12$
$x = 6$

4. $\frac{2}{5} = \frac{x}{3}$

$6 = 5x$
$1\ 1/5 = x$

---

5. $\frac{3}{4} = \frac{9}{x}$

$3x = 36$
$x = 12$

6. $\frac{2x}{9} = \frac{2}{3}$

$6x = 18$
$x = 3$

7. $\frac{2x}{3} = \frac{4}{5}$

$10x = 12$
$x - 1\ 1/5$
or
1.2

8. $\frac{5}{2x} = \frac{6}{12}$

$60 = 12x$
$5 = x$

---

9. $\frac{2}{3} = \frac{2x}{9}$

$18 = 6x$
$3 = x$

10. $\frac{3}{2} = \frac{9}{2x}$

$6x = 18$
$x = 3$

11. $\frac{x}{5} = \frac{2}{3}$

$3x = 10$
$x = 3\ 1/3$

12. $\frac{4}{2x} = \frac{1}{4}$

$16 = 2x$
$8 = x$

---

13. $\frac{5}{4} = \frac{2x}{8}$

$40 = 8x$
$5 = x$

14. $\frac{9}{4} = \frac{27}{3a}$

$27a = 108$
$a = 4$

15. $\frac{5}{3} = \frac{4a}{24}$

$120 = 12a$
$10 = a$

16. $\frac{7}{3a} = \frac{2}{12}$

$84 = b\ a$
$14 = a$

---

17. $\frac{12}{x} = 6$

The Process
Cross multiply.
$\frac{12}{x} = \frac{6}{1}$
$6x = 12$
$x = 2$

18. $\frac{9}{x} = 3$

$9 = 3x$
$3 = x$

19. $\frac{16}{x} = 4$

$16 = 4x$
$4 = x$

20. $\frac{21}{x} = 7$

$21 = 7x$
$3 = x$

Number Right ________

SOLVING PROBLEMS BY PROPORTION

If Bob earns $8 in 5 days, at the same rate how much will he earn in 4 days?

The proportion from which this problem may be solved can be written in a number of different ways.

$$\frac{\text{dollars}}{\text{time}} = \frac{\text{dollars}}{\text{time}} \text{ or } \frac{8}{5} = \frac{x}{4} \qquad \frac{\text{time}}{\text{dollars}} = \frac{\text{time}}{\text{dollars}} \text{ or } \frac{5}{8} = \frac{4}{x}$$

$$\frac{\text{dollars}}{\text{dollars}} = \frac{\text{time}}{\text{time}} \text{ or } \frac{8}{x} = \frac{5}{4} \qquad \frac{\text{dollars}}{\text{dollars}} = \frac{\text{time}}{\text{time}} \text{ or } \frac{x}{8} = \frac{4}{5}$$

It is important to remember that the order of comparison must be the same in both the first and the second ratios. If the first ratio compares dollars to time, then the second ratio must also compare dollars to time in the same order.

2. If Paul can walk 15 miles in 6 hours, how far can he walk, at the same rate, in 8 hours?

The Process

$15 : 6 = x : 8$
$6x = 120$
$x = 120$
He can walk 20 miles in 8 hours.

3. In the 1972 Olympic Games the Soviet Union's Valeri Borzov won first place in the 100-meter dash with a time of 10.1 seconds. If he ran at this same rate, how long would it take him to run 800 meters (about 1/2 mile)?

Answer: 80.8 sec.

---

4. Tom runs the 50-yard race in 6 seconds. At the same speed, how long will it take him to run 75 yards?

Answer: 9 sec.

5. Mamie saved $5 in 4 weeks. At the same rate how much will she save in 6 weeks?

Answer: $7.50

---

6. Joe spent $1.36 for gasoline in driving 68 miles. How much would he spend in driving 85 miles?

Answer: $1.70

7. The taxes on a piece of property valued at $4000 were $66. At this same rate, what would be the taxes on property valued at $5500?

Answer: $90.75

---

8. A train is traveling at the rate of 75 miles per hour. At the same speed, how far will it travel in 40 minutes? (Change the 1 hour to minutes).

Answer: 50

9. Frank Budd's best time in running 100 yards is 9.2 seconds. If he ran at this same rate, how many seconds would it take him to run 220 yards?

Answer: 20.24 sec.

Number Right__________

PRACTICE IN PROPORTION

Solve each of the following proportions to find the value of the unknown.

1. $\frac{9}{x} = \frac{3}{4}$

$3x = 36$
$x = 12$

2. $\frac{6}{5} = \frac{x}{10}$

$60 = 5x$
$12 = x$

3. $\frac{x}{12} = \frac{3}{2}$

$2x = 36$
$x = 18$

4. $\frac{17}{10} = \frac{8.5}{x}$

$17x = 85$
$x = 5$

---

5. $\frac{6}{11} = \frac{9}{x}$

$6x = 99$
$x = 16\ 1/2$

6. $\frac{9}{2} = \frac{36}{x}$

$9x = 72$
$x = 8$

7. $\frac{15}{40} = \frac{x}{8}$

$120 = 40x$
$3 = x$

8. $\frac{35}{14} = \frac{50}{x}$

$35x = 700$
$x = 20$

---

9. $\frac{20}{19} = \frac{30}{x}$

$20x = 570$
$x = 28.5$

10. $\frac{27}{x} = \frac{15}{10}$

$270 = 15x$
$18 = x$

11. $\frac{3}{9} = \frac{x}{16}$

$48 = 9x$
$5\ 1/3 = x$

12. $\frac{21}{75} = \frac{x}{25}$

$75x = 525$
$x = 7$

---

13. $5 : 1 = 30 : x$

$5x = 30$
$x = 6$

14. $x : 21 = 2 : 3$

$3x = 42$
$x = 14$

15. $12 : x = 16 : 4$

$48 = 16x$
$3 = x$

16. $7 : x = 28 : 8$

$56 = 28x$
$2 = x$

---

17. $12 : 6 = 8 : x$

$12x = 48$
$x = 4$

18. $8 : x = 32 : 12$

$86 = 32x$
$3 = x$

19. $4 : x = 16 : 12$

$48 = 16x$
$3 = x$

20. $35 : 14 = 50 : x$

$35x = 700$
$x = 20$

Number Right________

PROPORTION IN MEASUREMENT

Remember that a square is a rectangle with all sides equal. This means that the length and width are the same. Therefore, to find the area of a square we need to know only one side, since all sides are equal. Hence, to find the area of a square we multiply a side by itself; s X s, or s^2. Remember that s^2 means s squared or s multiplied by itself. Thus, 2^2 = 2 X 2, or 4; 3^2 = 3 X 3, or 9, etc.

1. Notice the figures at the right. Square A has a side of 1", and square B has a side of 2". The area of B is how many times the area of A?

The Process

Using proportion, we have:
$B : A = 2^2 : 1^2$
$B : A = 4 : 1$
The area of B is 4 times that of A.

2. One square has a side of 3" and a second square has a side of 6". The area of the second is how many times that of the first?

Answer: 4

3. One square has a side of 2 1/2 times the side of a second square. The area of the first is how many times that of the second? (Change 2 1/2 to 2.5.)

Answer: 6.25

A cube is a prism having equal length, width, and height. The volume of a cube is s X s X s, or s^3 (read, "s cubed").

4. One cube has a side of 1", and a second cube has a side of 2". The volume of the second cube is how many times that of the first cube?

The Process
$V : V = 1^3 : 2^3$
$V : V = 1 : 8$
The volume of the second cube is 8 times that of the first cube.

5. The side of one cube is 2", and the side of a second cube is 4". The volume of the second cube is how many times that of the first?

V:V 43 : 23
64 : 8

Answer: 8

---

6. The side of one cube is 2", and the side of a second cube is 3". The volume of the second cube is how many times that of the first? (Do not be afraid of fractions in the answer.)

Answer: 3.375 or 3 3/6

7. A picture 7" by 12" is enlarged so that the 7" side will be 14". What will be the other dimention of the enlarged picture

Answer: 24

---

8. A mixture of 2 parts sand and 3 parts gravel is used to make 25 cubic yards of concrete. How many cubic yards of each are needed?

The Process
2 parts + 3 = 5 parts
Then 2 : 5 = x : 25 (sand)
And 3 : 5 = x : 25 (gravel)

10 sand 15 gravel

9. In making solder, 2 parts of tin and 3 parts of lead are used. How much of each is used in making 65 pounds of solder?

Answer: tin = 26 lead = 37

Number Right ______________

VARIATION

The distance traveled is determined by the speed and the time. The distance varies as the speed or the time varies. Thus, if a car goes at the rate of 50 miles an hour, in 2 hours it will go 100 miles. Here the distance varies as the time varies because the rate of speed is the same. If the speed is changed to 60 miles an hour, the car will go 120 miles in 2 hours.

The circumference of a circle is equal to d. The value of is always the same (or constant). Therefore, the circumference of a circle is said to vary directly as the diameter varies.

1. From the table below we learn that the circumference of a circle whose diameter is 7 is 22. If we doubt the diameter, what is done to the circumference?

doubled (44)

2. If the diameter is tripled, what happens to the circumference?

tripled (66)

3. Fill in the table.

Circle	A	B	C	D	E	F
Diameter	7	14	21	28	35	42
Circumference	22	44	66	88	110	132

4. The area of a circle equals . In the areas of two circles, we have this proportion:

$$\frac{A}{A} = \underline{\quad}$$

Since the value of is the same (or constant), we have the proportion:

$$\frac{A}{A} = \frac{r^2}{r^2}$$

Or, the area of a circle varies as the square of the radius. If the radius is doubled, what happens to the area? (Double means times 2. The square of 2 is how much?)

4

5. If the radius is increased to 3 times, how much is the area increased?
Answer: 8 times

Do Your Work Here

6. Fill in the table below. Prove the first one you fill in by the formula method so as to be sure you are on the right track.

Circle	A	B	C	D
Radius	7	14	21	28
Area	154	616	1386	2464

7. Since the area of a circle varies as the square of the radius, the area of a second circle is how many times that of the first circle if the radius of the second is 3 times that of the first?

The Process

1st A : 2nd A = 1st r^2 : 2nd r^2
1st A : 2nd A = $1^2 : 3^2$
1st A : 2nd A = 1 : 9

The area of the second circle is 9 times that of the first circle. Now go back to the table and check your work to see if you proceeded correctly.

8. If we double the radius of a circle, how many times have we increased the area? 4 times

INVERSE VARIATION

We have seen how direct variation works. Thus, if a train travels at a fixed rate of speed, the distance the train travels is dependent upon the number of hours it travels. The distance varies directly as the time varies.

However, if the train doubles its speed, it requires only half as much time to go a fixed distance. The faster the speed, the less the time required to go the fixed distance. Hence, the time required to go the fixed distance varies inversely as the speed. Double the speed, and the time becomes one half; triple the speed, and the time becomes one third, etc.

1. Fill in the table below, showing the time required to go the fixed distance of 600 miles at the different rates of speed shown.

Distance	600	600	600	600	600
Speed	10	20	30	40	80
Time	60	30	20	15	7 1/2

2. Fill in the table below to show the rate of speed required to go the fixed distance at the times shown.

Distance	400	400	400	400	400
Time	40	20	10	5	2.5
Speed	10	20	40	80	160

3. You have probably read the expression "man-hours." The term is used extensively in industry. One man working one hour is said to be a man-hour.

A contractor is engaged on a job that has been estimated to require 6000 man-hours. The time required to finish the job will vary inversely as the number of men working. Fill in the table below to show the number of hours required to finish the job for the different numbers of men employed.

Man-hours	6000	6000	6000	6000	6000
Men	20	40	80	160	
Hours	300	150	75	37.5	18.75

4. When Mr. Allen built his home, he figured on spending $100 for fruit trees. The number of trees he can buy varies inversely as the price increases. Fill in the table below, showing the number of trees that can be bought at the prices shown, the total price being fixed at $100.

Total	$100	$100	$100	$100	$100
Price	$4	$5	$6.25	$10	$12.50
Number	25	20	16	10	8

SIMILAR FIGURES

Similar triangles are triangles of the same shape. Triangles are said to be similar when any two sides of one triangle are in proportion to the corresponding two sides of the other triangle. In the triangles at the right the sides $\frac{AB}{XY} = \frac{BC}{YZ}$. If two sides of one triangle are in proportion to the two corresponding sides of the other triangle, the third sides are likewise in proportion. In the two triangles at the right: $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$. AB = 1", BC = 1.5", AC = 2", and XY = 2", YZ = 3", XZ = 4". Substitute these values in the proportion above to prove that it is a true proportion and the triangles are similar.

You may also note that the corresponding angles of two similar triangles are equal. Thus, in the triangles at the right, A= X, B = Y, and C = Z.

Complete these proportions for the similar triangles at the right.

1. $\frac{AB}{XY} = \frac{BC}{y2}$ 2. $\frac{AB}{BC} = \frac{XY}{x2}$ 3. $\frac{AB}{XY} = \frac{AC}{XZ}$ 4. $\frac{AB}{AC} = \frac{XY}{xc}$ 5. $\frac{AC}{XZ} :: \frac{AB}{XY}$ 6. $\frac{BC}{x2} = \frac{AB}{XY}$

---

7. In the similar triangles ABC and XYZ, AB is 5", XY is 6", and BC is 10". What is the length of YZ? The process is at the right. Study it carefully.

The Process

$\frac{AB}{XY} = \frac{BC}{YZ}$ $\frac{AB}{BC} = \frac{XY}{YZ}$

Then, $\frac{5}{6} = \frac{10}{YZ}$

5YZ = 60 YZ = 12

Do Your Work Here

8. In the same triangles, if AC is 15", how much is XZ? (Use any pair of corresponding sides of problem 7 for the proportion comparing AC and XZ.) 18

9. Two similar triangles ABC and XYZ have AB = 2", BC = 3", and XY = 3". Find how much YZ is. (AB/XY = BC/YZ) . . . . 4.5 or 4 1/2

10. In these same triangles find AC if XZ is 6". (AB/XY =AC/XZ) 4

11. In similar triangles ABC and XYZ, AB is 4", XY is 3", and BC is 6". Find YZ. . . . . . ~~4.5 or 4 1~~/2

12. In these same triangles (problem 11), if AC is ", find XZ. (AB/XY = AC/XZ) 6.75

USING PROPORTION IN SIMILAR FIGURES

One Saturday afternoon Jack Robertson and Jim Blackwood noticed the shadows cast by a tall oak tree and a nearby fence post.

"Jim," said Jack, "we have always wondered how high that tree is. I believe we can find out by using similar triangles. If we measure the two shadows, that will give the basis of the triangles. Then if we measure the height of the post, we can make a proportion and find the height of the tree."

"But where are the two triangles?" Jim asked. "The shadow and the height make only two sides. Where is the third side?"

"The rays of the sun shining over the top of the tree and the top of the post make the third side," Jack said.

Jack and Jim measured the two shadows and the height of the post and made sketches like those shown.

1. Jack measured the tree's shadow to be 60'. The shadow of the post was 12', and the height of the post was 8'. How high was the tree? (The process is on the right.)

The Process

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\frac{12}{60} = \frac{8}{EF}$$

$$12\ EF = 480$$

$EF = 40$ The tree was 40 ft. ta

Do Your Work Here

2. A flag pole casts a 210' shadow at the same time a pole 20' tall casts a shadow of 30'. How tall is the pole? (It will be easier if you make a sketch.) . . . . 140'

3. A telephone pole casts a shadow 30 ft. long, while a fence post 4 ft. high casts a shadow 3 ft. long. How high is the pole? . . . . 40'

4. The Washington Monument casts a shadow 111 feet long at the same time a 50-foot telephone pole casts a shadow 10 feet long. How high is the Washington Monument? . . . . 555'

5. A smokestack casts a shadow of 40 feet while a fence post nearby casts a shadow of 2 feet. The fence post is 5 feet high. How tall is the smokestack? . . . . 100'

6. Frank is 6 feet tall. His shadow was 4 feet long when the shadow of a building was 100 feet. How tall is the building? . . . . 150'

In triangles ABC and DEF, above:

7. $\frac{EF}{DE} = \frac{BC}{AB}$ 8. $\frac{DF}{AC} = \frac{DE}{AB}$

9. $\frac{DE}{AB} = \frac{DF}{AC}$ 10. $\frac{AC}{DF} = \frac{AB}{DE}$

CHECKING UP

1. Mary has $2.50. Jane has $2. What is the ratio of Mary's money to Jane's?

Answer: 5:4

2. What is the ratio of Jane's money to Mary's?

Answer: 4:5

---

3. Trenton has a population of 40,000 persons. There are 6,000 children of school age in Trenton. What is the ratio of the school children to the total population?

Answer: 3:20

4. The combined ages of Mr. Green and his eldest son is 80 years. The father is three times as old as the son. How old is each?

Mr. Green: 60 yrs. Son: 20 yrs.

---

5. Mr. Franklin is one-third as old as his father. The sum of their ages is 100 years. How old is each?

Mr. Franklin: 25 yrs. His father: 75 yrs.

6. Ted, Bill, and James together have $8.40. Bill has twice as much as Ted. James has twice as much as Bill. How much does each have?

Ted: $1.20 Bill: $2.40 James: $4.80

---

7. Complete this proportion.
7 : 15 = 21 : x

Answer: 45

8. Find the missing quantity in this proportion.
x : 10 = 15 : 30

Answer: 5

---

9. In the similar triangles ABC and DEF, AB is 2", BC is 3", and DE is 4". Find the length of EF.

Answer: 6"

10. In the same similar triangles find AC if DF is 5".

Answer: 2.5

---

11. The side of a square is 1". The side of a second square is 2". The area of the second square is how many times that of the first?

Answer: 4 times

12. If 5 is added to 2 times a certain number, the result is 55. What is the number?

Answer: 25

---

13. The sum of three numbers is 180. The three numbers are in the ratio of 1 : 2 : 3. What are the three numbers?

Answer: 30, 60, 90

14. The ratio of peanuts to pecans to cashews in a certain nut assortment is 5 : 3 : 1. Per pound, peanuts cost $.35, pecans $.46, and cashews $.65. How much would 4.5 pounds of this mixture be worth?

Answer: $1.89

PROGRESS TEST

1. Joe has $5 and Tom has $4. What is the ratio of Joe's money to Tom's?

Answer: 5:4

2. What is the ratio of Tom's money to that of Joe (problem 1)?

Answer: 4:5

---

3. Jerry and Pat together have $15. Jerry's money is to Pat's money as 3 is to 2. How much does each have?

Jerry: $9.00 Pat: $6.00

4. June's father is three times as old as she. The sum of their ages is 56 years. How old is each?

June: 14 Father: 42

---

5. Bill, Frank, and George together have $24. Frank has twice as much as Bill. George has three times as much as Bill. How much does each have?

Bill: $4 Frank: $8 George: $12

6. The area of a rectangle is twice the area of a second rectangle. The sum of their areas is 600. What is the area of each?

1st rectangle: 200 2nd rectangle: 400

---

7. Jim earned $30 in 12 hours. By using proportion, show how much he would earn in 15 hours.

Answer: $37.50

8. Find the value of x in these proportions.

$\frac{10}{20} = \frac{x}{16}$ $x =$ 8 $\frac{x}{15} = \frac{10}{3}$ $x =$ 50

---

9. The area of a square varies as its side squared. The side of one square is twice the size of another square. The area of the first square is how many times that of the second?

Answer: 4 times

10. The Empire State Building casts a shadow 250 feet long, while a nearby 100-foot flag pole casts a shadow 20 feet long. How tall is the Empire State Building?

Answer: 1250 ft.

GEOMETRY I

Section A

LINES:

When we think of lines, we think of: Horizontal lines, vertical lines, parallel lines, and intersecting lines,

1. Horizontal lines are lines that extend from left to right as the horizon does. Examples of horizontal lines are lines on writing paper and all level lines which we find in man-made structures.

2. Vertical lines are lines that go straight up and down with respect to the earth. Examples of vertical lines are telephone poles and trees.

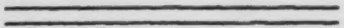
3. Parallel lines are lines that have the same direction and are always the same distance apart. An example of parallel lines are railroad tracks. They are equal distance apart and will never meet no matter how far they extend. They may be vertical, horizontal, or oblique (slanting). The symbol // means " is parallel to."

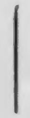
4. Intersecting lines are lines that cross each other. They meet only at one point. An example of intersecting lines would be the lines of roads intersect at corners. Also vertical and horizontals of graph paper intersect.

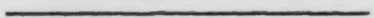
SECTION A
EXERCISES

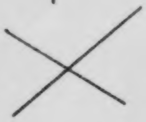
Complete the following exercises. If you miss any questions, discuss those questions and any others you feel unsure about with your instructor. If you do not miss any questions, and if you feel comfortable with the material you have covered, go on to Section B.

1. Label each of the following lines by writing the name under it.

(A) 
Parallel

(B) 
Vertical

(C) 
Horizontal

(D) 
Intersecting

2. Parallel lines may be horizontal, vertical, or oblique (slanting).

3. Lines on writing paper are examples of horizontal or parallel lines.

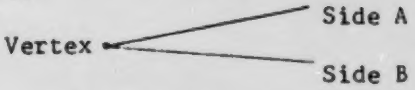
4. Telephone poles are examples of vertical lines.

5. Lines that cross each other are called intersecting lines.

Section B
BASIC TYPES OF ANGLES

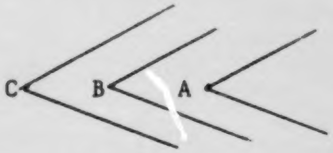
An angle is formed when two straight lines meet at a point. The lines are called the sides of the angle. The point at which the sides meet is called the vertex of the angle.

Example:



The size of an angle depends upon the amount one side has turned away from the other. The length of the sides of an angle does not determine the size.

Example:

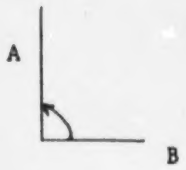


Angles C, B, and A are equal

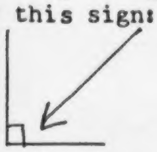
The unit of measure used in measuring an angle is the degree.° The symbol used the represent angle is $\angle$.

1. Right Angle If one side of an angle turns a quarter of a complete circle away from the other side. The angle that is formed is a right angle.

Example:

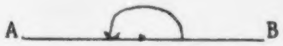


The right angle contains 90° and i represented by this sign:



We also refer to lines that meet to form right angles as being perpendicular lines.

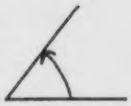
2. Straight Angle If one side of an angle turns half of a complete circle away from the other side, the angle that is formed is a straight angle. The sides of a straight angle lie in the same straight line.
Example:



Notice that a straight angle is twice the size of the right angle since in a straight angle the side has made half a complete turn or two quarter turns. The number of degrees in a straight angle is 180°.

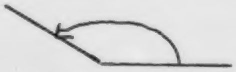
more than
less than

3. ACute Angle If one side of an angle turns less than a quarter of a circle away from the other side, the angle formed is an acute angle. An acute angle, therefore, is smaller than a right angle, or less than 90°. Example:



$<90^\circ$

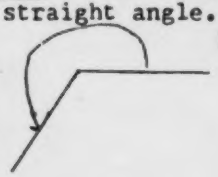
4. Obtuse Angle If one side of an angle turns more than a quarter circle but less than a half circle, the angle formed is an obtuse angle. Therefore, an obtuse angle is greater than a right angle but smaller than a straight angle. It contains more than 90° but less than 180°. Example:



>90

<180

5. Reflex Angle If one side of an angle turns more than half a circle (180°) but less than a complete circle (360°) away from the other side, the angle formed is a reflex angle. Therefore, a reflex angle is greater than a straight angle.
Example:



>180
<360

SECTION B
EXERCISES

Complete the following exercises. If you miss any questions, discuss those questions and any other questions you feel unsure about with your instructor. If you do not miss any questions, and if you feel comfortable with the material you have covered, go on to Section C.

1. How many degrees are there in a right angle? 90°
2. Write the symbol that represents a right angle. ∟

3. A straight angle contains 180°.

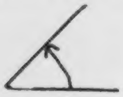
4. An acute angle contains less than 90°.

5. An obtuse angle contains more than 90° but less than 180°.

6. An angle that contains more than 180° but less than 360° is called a/an reflex angle.

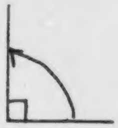
7. In each of these figures in this question, tell what type of an angle it is by writing its name under it.

(A)



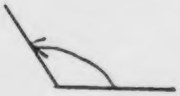
Acute

(B)



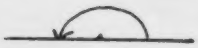
Right

(C)



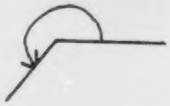
Obtuse

(D)



Straight

(E)



Reflex

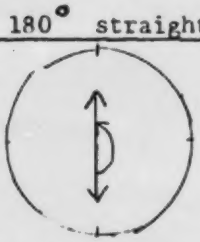
8. What type of angle is in each corner of a square? right or 90°

9. What kind of angle is each corner of an octagonal stop sign? Obtuse

10. Lines that meet to form right angles are called perpendicular lines.

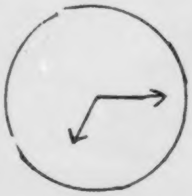
11. Name the type of angle that is formed by the hands of a clock at each of the following hours and write the number of degrees it represents - such as - less than 90°, 180°, or greater than 90°, 180°.

A. 6:00 180° straight

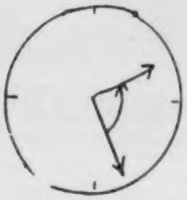


>90°

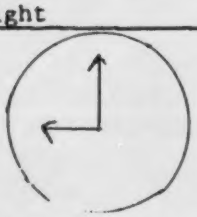
B. 7:15 ∠ 180° obtuse



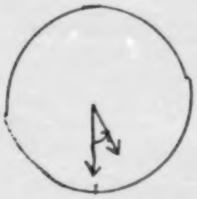
C. 2:25 90° acute



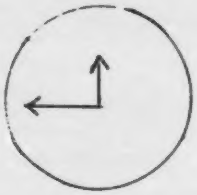
D. 9:00 90° right



E. 4:30 90° acute



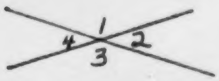
F. 12:45 90° right



Section C
PAIRS OF ANGLES

1. Vertical angles or opposite angles. When two lines cross each other four angles are formed.

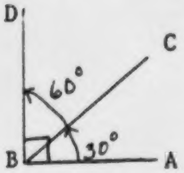
Example:



Since opposite angles have been formed by the same amount of turning, they are equal. Therefore, $\angle 1 = \angle 3$ and $\angle 2 = \angle 4$.

2. Complementary angles. When two angles put together form a right angle, and thus their sum of 90° , the angles are complementary.

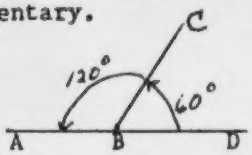
Example:



Angle ABC is the complement of angle CBD since their sum ($60^\circ + 30^\circ$) equals 90° .

3. Supplementary Angles. When the sum of two angles is 180° , the angles are said to be supplementary.

Example:



Angle ABC is the supplement of angle CBD since their sum ($120^\circ + 60^\circ$) is 180° .

It was already mentioned that the basic unit of measure for angles is a degree. However, each degree may be subdivided into 60 minutes and each minute may be further subdivided into 60 seconds. The symbol for minutes is a single quotation mark (') and for seconds is a double quotation mark ("). Therefore, when we think of degrees, we can associate it with time.

Now, lets try breaking down an angle of 90° (complementary) into minutes and seconds.

1st. $90^\circ = 89^\circ\ 60'$

Note: In this process a degree was taken and changed into minutes.

2nd. $90^\circ = 89^\circ\ 59'\ 60''$

Therefore, we see that $89^\circ\ 59'\ 60'' = 90$.

3rd. The same process is used when breaking down an angle of 180° (supplementary) into minutes and seconds.

$180^\circ = 179^\circ\ 60'$

$180^\circ = 179^\circ\ 59'\ 60''$

This is necessary to know when you are to find a complement or supplement of a given angle.

Let's try a couple for practice.

We are asked to find the complement of 28° 14' 58". We have already learned that two complementary angles equal 90° . Therefore we have to subtract 28° 14' 58" from 90°.

Yes, before we can do this we have to break down the 90° into minutes and seconds so we can subtract. Let's do that now.

$90^\circ = 89^\circ\ 59'\ 60''$

Now we can find the complement by subtracting 28° 14' 58" from 89° 59' 60". Let's do that now.

$$\begin{array}{rrrr} & 89^\circ & 59' & 60'' \\ - & \underline{28^\circ} & \underline{14'} & \underline{58''} \\ & 61^\circ & 45'' & 2'' \end{array} \qquad \begin{array}{lll} 89^\circ & 59' & \\ \cancel{90}^\circ & \cancel{60} & 60'' \end{array}$$

Now we have found the complement of 28° 14' 58" to be 61° 45' 2". Remember if we add them together we will get 90° because two complementary angles equal 90°.

$$\text{See: } \begin{array}{rrr} +28^\circ & 14' & 58'' \\ 61^\circ & 45' & 2'' \\ \hline 89^\circ & 59' & 60'' \end{array} = 90^\circ$$

We use the same process for finding the supplement of an angle. Let's try one:

We are asked to find the supplement of 28° 14' 58". Remember, supplementary angles equal 180°.

1st. We have to change 180° into minutes and seconds.

$$180^\circ = 179^\circ\ 59'\ 60''$$

Now we can find the supplement of 28° 14' 58" by subtracting it from 179° 59' 60". Let's do that:

$$\begin{array}{rrrr} & 179^\circ & 59' & 60'' \\ - & 28^\circ & 14' & 58'' \\ \hline & 151^\circ & 45' & 2'' \end{array} \qquad \begin{array}{ccc} 179^\circ & 59' & \\ \cancel{180} & \cancel{60} & 60'' \end{array}$$

Now we have found the supplement of 28° 14' 58" to be 151° 45' 2". Remember, if we add them together we should get 180°, because two supplementary angles equal 180°.

$$\text{See: } \begin{array}{rrr} 28^\circ & 14' & 58'' \\ +151^\circ & 45' & 2'' \\ \hline 179^\circ & 59' & 60'' \end{array} = 180^\circ$$

Complete the following exercises. If you miss any questions, discuss those questions and any other questions you feel unsure about with your instructor. If you do not miss any questions and if you feel comfortable with the material you have covered go on to Section D.

1. When two lines cross each other, four angles are formed and the opposite angles are equal.

2. When two complementary angles are put together, they form a right angle which contains 90°.

3. When two supplementary angles are put together, they form a straight angle which contains 180°.

4. Find the complement of 43° 18' 25".

$$\begin{array}{r} 89^\circ\ 59'\ 60'' \\ \underline{43^\circ\ 18'\ 25''} \\ 46^\circ\ 41'\ 35'' \end{array}$$

Ans. 40° 41' 35"

5. Find the supplement of 124° 42' 10"

$$\begin{array}{r} 179^\circ\ 59'\ 60'' \\ \underline{124^\circ\ 42'\ 10''} \\ 55^\circ\ 17'\ 50'' \end{array}$$

Ans. 55° 17' 50"

6. Find the complement of 62° 25' 12"

$$\begin{array}{r} 89^\circ\ 59'\ 60'' \\ -\ \underline{62^\circ\ 25'\ 12''} \\ 27^\circ\ 34'\ 48'' \end{array}$$

Ans. 27° 34' 48"

7. Find the supplement of 62° 25' 12"

$$\begin{array}{r} 179^\circ\ 59'\ 60'' \\ -\ \underline{62^\circ\ 25'\ 12''} \\ 117^\circ\ 34'\ 48'' \end{array}$$

Ans. 117° 34' 48"

8. (') is a symbol that represents minutes.

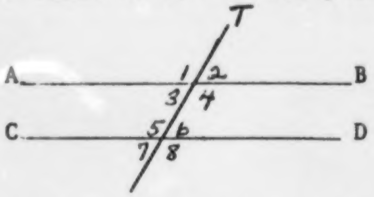
9. (") is a symbol that represents seconds.

10. True or False? Supplementary angles are twice as large as the complementary angles? true

PARALLEL LINES AND THEIR ANGLES

As was stated earlier, two or more lines are parallel if they never meet, no matter how far they are extended, and the symbol $\parallel$ means "is parallel to".

A line drawn cutting across parallel lines is called a transversal. In the diagram below lines AB and CD are parallel, and are cut by the transversal,



When parallel lines are cut by a transversal:

1. The sum of the angles across from each other equals 180°.

   $\angle 1 + \angle 2 = 180^\circ$
   $\angle 3 + \angle 4 = 180^\circ$
   $\angle 5 + \angle 6 = 180^\circ$
   $\angle 7 + \angle 8 = 180^\circ$

2. The corresponding (matching angles) are equal.

   $\angle 2 = \angle 6$
   $\angle 4 = \angle 8$
   $\angle 1 = \angle 5$
   $\angle 3 = \angle 7$

3. The alternate interior angles are equal.

   $\angle 3 = \angle 6$
   $\angle 4 = \angle 5$

4. The alternate exterior angles are equal.

   $\angle 1 = \angle 8$
   $\angle 2 = \angle 7$

Remember: It is the position of the angles that counts.
The numbers are used for labeling.

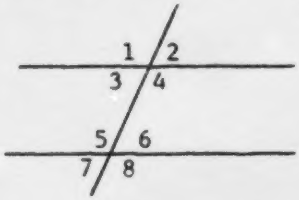
5. The consecutive interior angles are supplementary.

$$\angle 4 + \angle 6 = 180$$
$$\angle 3 + \angle 5 = 180$$

From this information given, if we know the degrees of one angle we can find the degrees of all the angles of parallel lines cut by a transversal.

Let's do one for fun.

In the figure below it was given to us that $\angle 3 = 60$. Let's see what the other angles equal.



1st. We know $\angle 3 + \angle 4 = 180$; therefore, we can subtract 60° (which was given to us) from 180° and get $\angle 4$.

$$\begin{array}{r} 180^\circ \\ -\ 60^\circ \\ \hline 120^\circ \end{array}$$

We see $\angle 4 = 120^\circ$

2nd. Since we know that opposite angles are equal we can figure $\angle 1$ and $\angle 2$. $\angle 1$ is opposite $\angle 4$ so it equals 120° . $\angle 2$ is opposite $\angle 3$ so it equals 60° .

3rd. We also know that corresponding (matching) angles are equal; therefore, we know

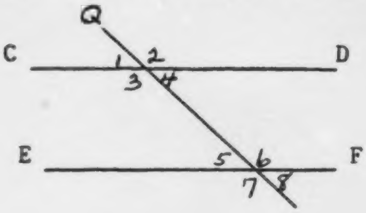
$\angle 2 = \angle 6$, $\angle 6 = 60^\circ$
$\angle 4 = \angle 8$, $\angle 8 = 120^\circ$
$\angle 1 = \angle 5$, $\angle 5 = 120^\circ$
$\angle 3 = \angle 7$, $\angle 7 = 60^\circ$

See: by knowing one angle you can find all the angles of parallel lines cut by a transversal.

EXERCISES

Complete the following exercises. If you miss any questions, discuss those questions and any other questions you feel unsure about with your instructor. If you don't miss any questions and if you feel comfortable with the material you have covered then you may go on to the next section.

In the diagram below, lines CD and EF are parallel and are cut by transversal Q.



Given: ∠ 5 = ∠ 45 Find the degrees for the rest of the angles.

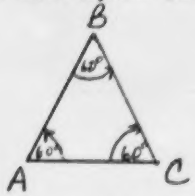
1. ∠ 1 = 45°
2. ∠ 2 = 135°
3. ∠ 3 = 135°
4. ∠4 = 45°
5. ∠ 6 = 135°
6. ∠ 7 = 135°
7. ∠ 8 = 45°

TYPES OF TRIANGLES

A triangle is a plane figure bounded by three straight lines and containing three angles. The sum of the angles of every triangle is 180°.

1. An equalateral triangle is a triangle having three equal sides. All three of its angles are equal each containing 60 .

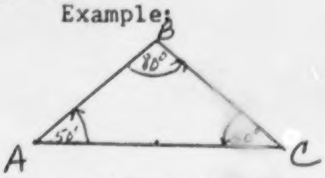
Example:



2. An Isosceles triangle has two equal sides. The equal sides are called the legs (AB & BC). The third side (AC) is called the base. The angles opposite the equal sides called base angles, are also equal.

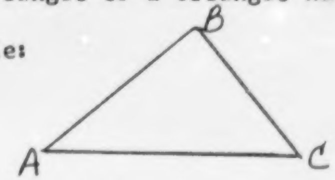
Example:

AB = BC
∠BCA = ∠ BAC



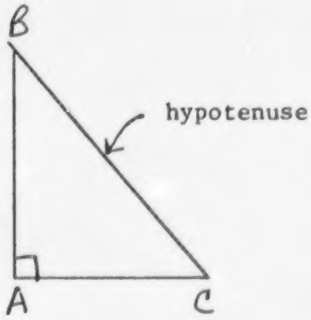
3. A scalene triangle is a triangle having no equal sides.

Example:



4. A right triangle is a triangle which contains one right angle (90). No more than one angle of a triangle can be a right angle. In a right triangle, the side opposite the right angle is called the hypotenuse.

Example:



Complete the following exercises. If you miss any questions discuss those questions and any other questions you feel unsure about with your instructor. If you don't miss any questions and if you feel comfortable with the material you have covered, then you may go on to the next section.

Match Column II with Column I. There is one definition for each term.

I	II
(3) A. Scalene triangle	1. All three sides and angles are equal.
(5) B. Right triangle	2. The sum of its angles is 180.
(4) C. Isosceles triangle	3. No two sides are equal.
(1) D. Equalateral triangle	4. Two sides only are equal and two angles are equal.
(2) E. Triangle	5. It contains an angle of 90 and has a side called the hypotenuse.

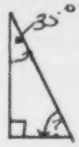
FINDING THE DEGREES OF AN ANGLE IN A TRIANGLE

1st. In order to find the degrees of one angle in a triangle, you need to know the degrees contained in the other two angles. This rule holds true for all triangles except the right triangle and the isosceles triangle.

In a right triangle you already know that one angle contains 90 ; therefore, you just need to know the degrees contained in one other angle in order to find the third angle.

In the isosceles △ , you only need to know the degrees of one base angle in order to find the third angle. Remember, in an isosceles △ , base angles are equal.

Now let's try finding the degrees contained in angle C of the below.



Given: △ABC is a right △ ∠B = 35

1st. It was given to us that △ ABC is a right △ . Therefore, we know ∠ A contains 90°. It was also given to us that ∠ B contains 35° . Now, since we know two angles, we can find the degrees contained in the third angle. Do you know how?

2nd. Since we know a △ contains 180° , we can add the two known angles together and subtract that from 180° which will give us the degrees contained in the unknown angle. (∠C)
Let's look at the process:

$$90^\circ + 35^\circ = 125^\circ$$
$$180^\circ - 125^\circ = 55^\circ$$

3rd. We have found that ∠C contain 55° . Remember all three angles have to equal 180° ; therefore, we can check our answer by adding all three angles to see if our total is 180° .
Let's check our answer.

$$90^\circ + 35^\circ + 55^\circ = 180^\circ$$

See how easy it is to check your answer.

Remember you always need to know two angles of a △ in order to find the third ∠ . Work your practice exercises to make sure you understand the process just presented to you.

EXERCISES

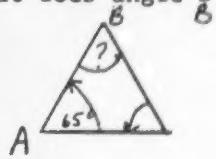
Complete the following exercises. If you miss any questions, discuss those questions and any other questions you feel unsure about with your instructor. If you don't miss any questions, and if you feel comfortable with the material you have covered, then you may go on to the next section.

1. Find the third angle of a triangle if the first two angles contain 14° and 127°.

   39°

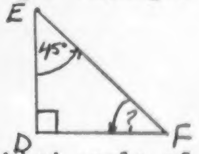
2. △ ABC is an isosceles △ . ∠A which is a base angle and equals 65°. What does angle B equal?

   50°



3. DEF is a right ~~triangle~~. Angle E equals 45°. Find angle F

   45°



4. Find the third angle of a △ if the first two angles contain 42° and 78°.

   60°

5. Find the third angle of a △ if the first two angles contain 55° and 40°.

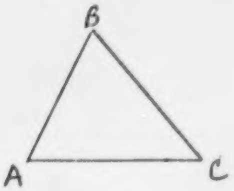
   85°

SIMILAR TRIANGLES AND CONGRUENT TRIANGLES

A. Similar Triangles

Two triangles are similar if the three angles of one triangle are equal to the three corresponding angles of another triangle. Similar triangles have identical shapes, but different sizes. Look at the examples below:

1.



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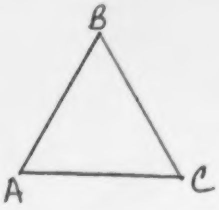

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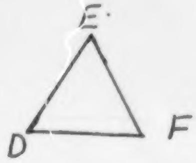
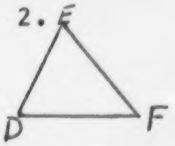


2.



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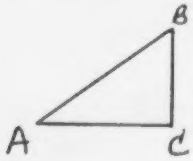

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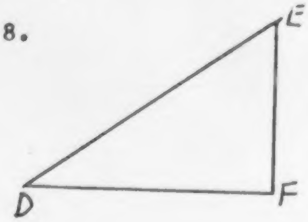

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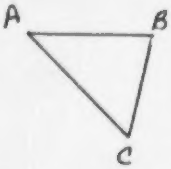

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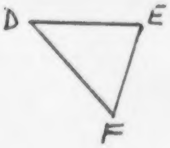
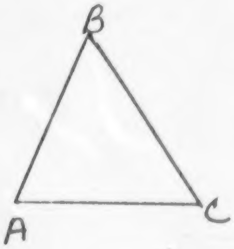
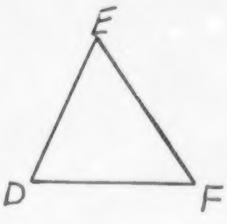


Can you tell that Δ's 1 and 2 are similar; 3 and 4 are similar; 5 and 6 are similar; and 7 and 8 are similar?

B. Congruent Triangles

Δ's are congruent when each side and each angle of one triangle is equal to the corresponding side and the corresponding angle of the other triangle. If a Δ is placed on top of another and they fit exactly in every respect, they are congruent. The sign used to show that two angles are congruent is ≅ . Look at the example below:



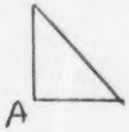


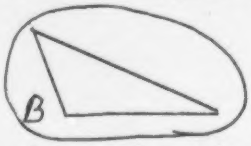
Congruent Δ 's differ from similar Δ 's in the respect that their sides are equal as well as their angles.

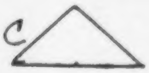
EXERCISES

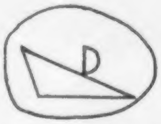
Complete the following exercises. If you miss any questions, discuss those questions and any other questions you feel unsure about with your instructor. If you don't miss any questions and if you feel comfortable with the material you have covered, then you may go on to the next section.

1. Circle the two Δ 's below that are similar.









2. What is the sign used to show that Δ 's are congruent? $\cong$

3. Two Δ 's are similar (equal) if their corresponding angles are equal.

4. How do congruent Δ 's differ from similar Δ 's?
All sides and all angles are equal.

Geometry II

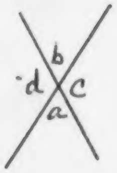
This hand-out is meant to provide additional practice with geometry and algebra. You will be combining skills of both.

Some things you must know:

1. vertical angles
2. supplementary angles
3. complementary angles
4. parallel lines
    - corresponding angles
    - alternate interior and exterior angles

1. and 2.

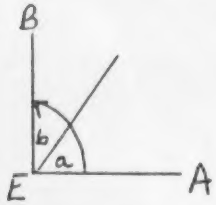
1)



2) $\angle a + \angle c = 180$
$\angle c + \angle b = 180$
$\angle b + \angle d = 180$
$\angle d + \angle a = 180$

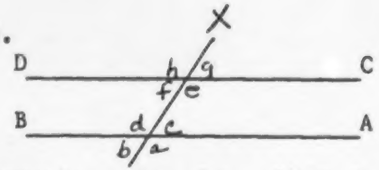
$\angle a = \angle b$
$\angle c = \angle d$

3.



$\angle AEB$ is right angle = 90°
$\angle a + \angle b = 90^\circ$

4.

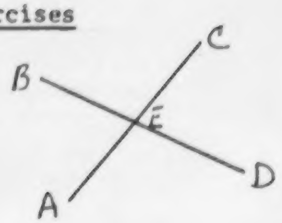


alternate $\angle d = \angle e$ $\angle b = \angle g$ alternate
interior $\angle c = \angle f$ $\angle a = \angle h$ exterior

corresponding $\angle a = \angle e$ $\angle d = \angle h$
$\angle b = \angle f$ $\angle c = \angle g$

AB is parallel to CD
X is the transversal

Exercises

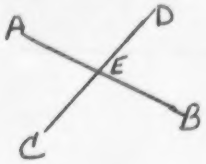


1. Line AC and BD intersect at E
If $\angle AED = 7x$ and $\angle BEC = 9x - 8^\circ$, find the number of degrees in $\angle BEC$. 28°

2. If $\angle AED = 55^\circ\ 17'\ 5''$, find $\angle DEC$. $124^\circ\ 42'\ 55''$

3. If $\angle AEB = 116^\circ\ 34'\ 29''$, find $\angle BEC$. $63^\circ\ 25'\ 31''$

For problems 4-6, Lines AB and CD intersect at E.



4. $\angle AED = 42^\circ$, find the number of degrees in $\angle CEB$. 42°

5. $\angle DEB = (4y + 7)^\circ$ $\angle CEA = (7y - 2)^\circ$, find the degrees in $\angle DEB$. 19°

6. $\angle CEA = 123^\circ 30'\ 29''$, find the degrees in $\angle BEC$. $56^\circ\ 29'\ 31''$

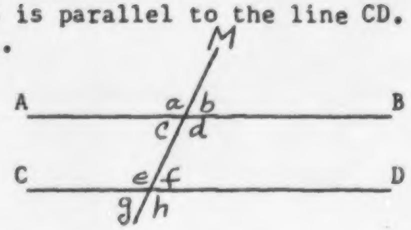
7. Find the complement of $(90-t)^\circ$ t°

8. Find the complement of $82^\circ\ 14'\ 59''$ $7^\circ\ 45'\ 1''$

9. Find the supplement of $(a + 170)^\circ$. $10-a$

10. Find the supplement of $108^\circ\ 14'\ 51''$. $71^\circ\ 45'\ 9''$

For problems 1-7, Line AB is parallel to the line CD.
Line M is the transversal.



1. Indicate all pairs of angles which are equal.

a. $\angle a = \angle d$
b. $\angle b = \angle c$
c. $\angle e = \angle h$
d. $\angle f = \angle g$

e. $\angle b = \angle f$
f. $\angle d = \angle h$
g. $\angle a = \angle e$
h. $\angle c = \angle g$

i. $\angle d = \angle e$
j. $\angle c = \angle f$
k. $\angle b = \angle g$
l. $\angle a = \angle h$

2. If $\angle f = 47^\circ$, find $\angle d$ 133°

3. If $\angle f = 47^\circ$, find $\angle c$ 47°

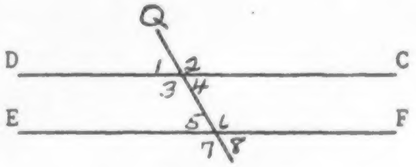
4. $\angle f + \angle h =$ 180°

5. $\angle a + \angle b =$ 180°

6. $\angle c + \angle d =$ 180°

7. $\angle e + \angle g =$ 180°

Problems 1-5



1. If $\angle 2 = X + 30^\circ$, and $\angle 8 = X^\circ$, find the degrees in both angles.
$2 = 105^\circ$ $8 = 75^\circ$

2. If $\angle 8 = 1/3 \angle 2$, find the degrees in both angles. $2 = 135^\circ$ $8 = 45^\circ$

3. If $\angle 1 = 63^\circ$, how many degrees are in $\angle 8$? $8 = 63^\circ$

4. If $\angle 2 = 3X + 10$, and $\angle 1 = 6X - 10$ find both angles. $2 = 70^\circ$ $1 = 110^\circ$

5. If $\angle 6$ is 12° more than two times $\angle 5$, find the degrees in both angles.
$5 = 56^\circ$ $6 = 124^\circ$

6. One angle is five times larger than another. If the angles are complementary, find the degrees in each angle. First angle = 15°
Second angle = 75°

7. The complement of an angle is 6° less than the angle. Find the number of degrees in the angles. 1st = 48° 2nd = 42°

8. The supplement of an angle is 68° more than the angle. Find the number of degrees in the angles. 1st = 56° 2nd = 124°

Triangles

Equilateral triangle	3 equal sides 3 equal angles = 180° Each angle = 60°	
Isosceles	2 equal sides angles opposite the equal sides are equal	
Scalene	no equal sides no equal angles	
Right triangle	has 1 right angle = 90°	

* The sum of the angles of any triangle is 180°

Example I

Find the third angle of a triangle if the first 2 angles contain 42° and 78°.

$42^\circ + 78^\circ = 120^\circ$

$180^\circ - 120^\circ = 60^\circ$

in whole

Example II

One angle is 63°. The difference between the other two angles is 13°. How many degrees are there in each of the other two angles?

$\angle 1 = 63^\circ$
$\angle 2 = X = 65^\circ$
$\angle 3 = X - 13 = 52^\circ$

$\angle 1 + \angle 2 + \angle 3 = 180^\circ$
$63^\circ + X^\circ + X - 13^\circ = 180^\circ$
$2X + 50^\circ = 180^\circ$
$2X = 180^\circ - 50^\circ$
$2X = 130^\circ$
$X = 65^\circ$

PRACTICE EXERCISE 1:
Triangles and their Angles

1. Find the third angle of a triangle if the first two angles contain 14° and 127°. 39°

2. Find the number of degrees in each angle of a triangle if the angles are in the ratio 4:9:5. 40°; 90°; 50°

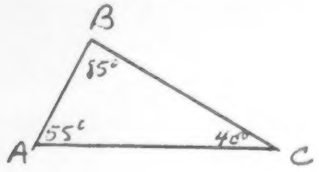
3. One angle of a triangle is 20°. The second angle is 10° larger than the third angle. Find the number of degrees in the second and third angles. 2nd 85° 3rd 25°

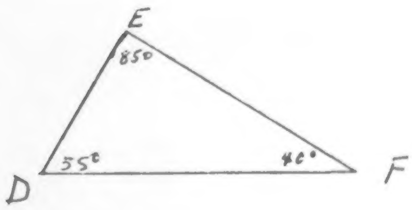
4. In triangle BCD, angle B is 5 times as large as angle D and angle C is 3 times as large as angle D. Find the number of degrees in each angle of triangle. B = 100°; C = 60°; D = 20°

5. In triangle ABC, angle A is 1/2 as large as angle C, while B is 6 times as large as angle C. Find the number of degrees in each angle of the triangle. A = 12; B = 144; C = 24°

SIMILAR TRIANGLES

Two triangles are similar if the three angles of one triangle are equal to the three corresponding angles of another triangle. Similar triangles have identical shapes, but different sizes.



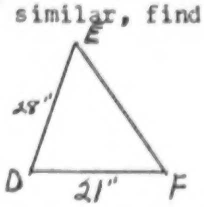


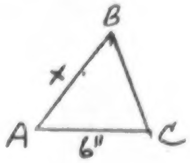
Triangles ABC and DEF are similar since all of their corresponding angles are equal.
The corresponding sides of similar triangles are those pairs of sides which are opposite equal angles. Thus, in the triangles above, AB corresponds to DE, AC corresponds to DF, and BC corresponds to EF.

Corresponding sides of similar triangles are in proportion.

EXAMPLE 1:

If the two triangles below are similar, find the length of AB.





Let AB = x. Since corresponding sides of similar triangles are in proportion:

$$\frac{x}{28} = \frac{6}{21}$$

By cross multiplication we get

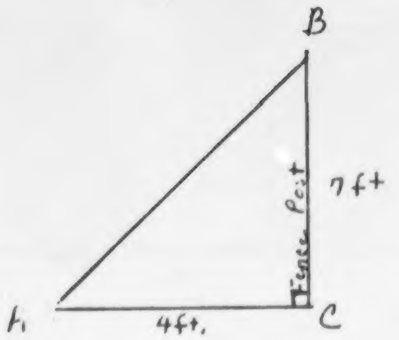
$$21x = 6\ (28)$$

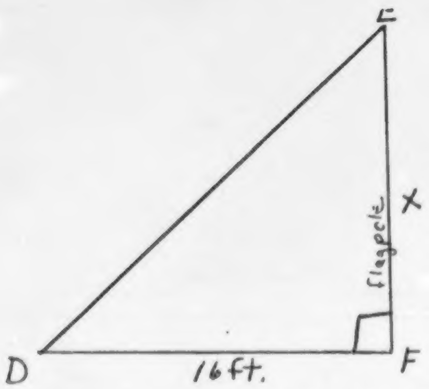
$$\frac{21x}{21} = \frac{168}{21}$$

$$x = 8$$

EXAMPLE 2:

If the sun casts a shadow 4 ft. long from a fence post 7 ft. tall and the shadow from a nearby flagpole is 16 ft. long, how tall is the flagpole? Assume that both fence post and flagpole are at right angles to the ground.





Since corresponding sides of similar triangles are in

$$\frac{7}{x} = \frac{4}{16}$$

By cross-multiplication we get

$$4x = 7\,(16)$$
$$\frac{4x}{4} = \frac{112}{4}$$
$$x = 28 \text{ ft.}$$

PRACTICE EXERCISE 2:

Similar Triangles

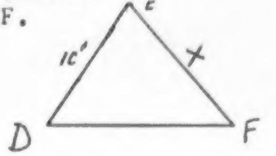
Answers to practice exercises appear at the end of the chapter

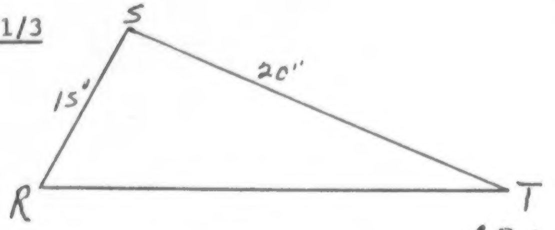
1. If the triangles below are similar, find the length of EF.

$$\frac{10}{15} = \frac{x}{20}$$

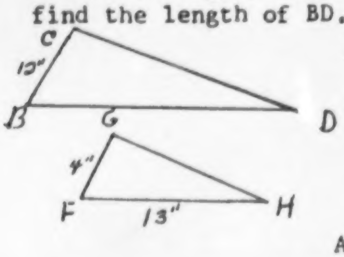
$13\ 1/3 = x$

Answer: 13 1/3





2. If the triangles below are similar, find the length of BD.



$$\frac{12}{4} = \frac{x}{13}$$

Answer: 39"

3. A 12" ruler when held vertically casts a shadow 18" long. How long will the shadow of a vertically held yardstick be at the same time of day?

$\frac{12}{18} = \frac{36}{x}$ Answer: 54"

4. The Empire State Building casts a shadow 360 ft. long at the same time that a nearby skyscraper 500 ft. tall casts a shadow 150 ft. long. How tall is the Empire State Building?

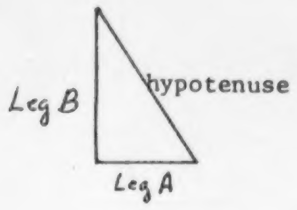
$\frac{x}{3600} = \frac{500}{150}$ Answer: 1200 ft.

5. A tree casts a shadow 54 ft. long. At the same time Steven, who is 5'8" tall, casts a shadow 9 ft. long. How tall is the tree?

$\frac{x}{54} = \frac{5\ 2/3}{9}$ Answer: 34 ft.

The Pythagorean Theorem

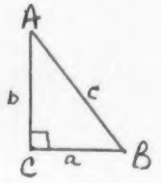
Pythagoras, a Greek mathematician, discovered a special relationship among the three sides of all right triangles.



The side opposite the right angle is called the hypotenuse. The other two sides are called legs.

The theorem states:

In any right triangle, the square of the hypotenuse (C^2) is equal to the sum of the square of the legs ($a^2 + b^2$)



For right triangle ABC,
$a^2 + b^2 = c^2$

By using your knowledge of how to solve equations and how to find square roots, it is possible to solve two basic types of problems involving the use of the Pythagorean theorem:

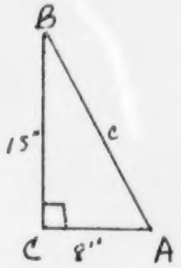
1. To find the length of the hypotenuse when the lengths of the two legs are given.

2. To find the length of one leg when the lengths of the hypotenuse and the other leg are given.

EXAMPLE 1:

Find the hypotenuse of a right triangle whose legs are 8" and 15".

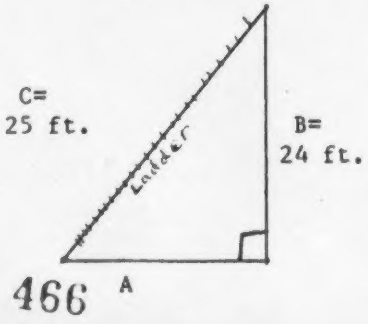
$$\begin{aligned} a^2 + b^2 &= c^2 \\ 8^2 + 15^2 &= c^2 \\ 64 + 225 &= c^2 \\ 289 &= c^2 \\ \sqrt{289} &= c \\ 17 &= c \end{aligned}$$



EXAMPLE 2:

A ladder leaned against a building at a point 24 ft. from the ground. If the ladder is 25 ft. long, how far is the foot of the ladder from the base of the building?

$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 24^2 &= 25^2 \\ a^2 + 576 &= 625 \\ -576 &= -576 \\ \hline a^2 &= 49 \\ a &= \sqrt{49} \\ a &= 7 \text{ ft.} \end{aligned}$$



Practice Exercise 3:
The Pythagorean Theorem

1. Find the hypotenuse of a right triangle whose legs are 12" and 16" long.

   20"

2. If the hypotenuse of a right triangle is 13" and one of the legs is 5", find the length of the other leg.

   5"

3. If Joy travels 18 miles north and then 24 miles west, how far will she be from her starting point?

   30 miles

4. A wire 26 feel long stretches from the top of a pole to a point on the ground 10 feet from the base of the pole. Find the height of the pole.

   24"

5. Find the length of the diagonal of a rectangle whose width is 7" and whose length is 24".

   25"

6. The diagonal of a children's pool table is 34" and its width is 16". Find its length.

   30"